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A Theoretical Explanation of the Firm's Hedging Policy

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Comments Welcome

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Abstract

The paper develops a models for evaluating the optimal hedging and trading strategies under default risk mitigation policies. It deals with an exchange traded derivative instrument, i.e. future contracts. Futures can succeed in locking the asset value, but they may create two additional risks such as default and liquidity risk. Default risk can be mitigated via collateralization, i.e. the margin account. This type of collateral is highly formalized (from the clearing house) and it does not depend on counterparty credit quality. The liquidity risk may arise from the inability to fund a hedging strategy (to top up the margin account to the initial margin in case of a margin call), due to borrowing constraints. A rational investor knows that a hedging strategy comes with a borrowing strategy. We are interested in the implications of the marking to market mechanism that characterize futures in presence of endogenous borrowing constraints. Therefore we characterize the optimal hedge in a dynamic framework by providing analytical solutions. We show that the magnitude of hedging of an investor facing borrowing constraint is always lower than the one of an unconstrained investor. The model captures the empirically observed fact, reported in Brown et al. (2003), that the hedge magnitude increases when the prices that need to be hedged move in their favor and decrease when prices moves against them. Third, the model also predicts that the hedge increases as the volatility of the underlying exposure increases. Other results follows.

Keywords: Risk Managment, Law and Finance, Corporate Finance

1 Introduction

Future contracts provide a hedge against a risk exposure. Empirical findings and common observations in practice, conclude that investors (especially firms) use little hedge through derivatives. We argue that the default risk mitigation policy and the presence of borrowing constraint do reduce the magnitude of hedging. The aim of this paper is to give, under the assumption of a default risk minimizing policy and liquidity concerns, a characterization for the problem of investor's optimal hedging policy.

Futures contract are exchange traded instruments where the parties exposed to the future risk, agrees to buy or to sell the asset at a certain time at a future at a certain price. While these contracts can succeed in locking in the asset price, they may create two additional risks such as default and liquidity risk. Default risk can be mitigated to a certain extent via the margin account. The way it works is as follows. In order to prevent that one of the parties fails to honor the agreement, the clearinghouse of the exchange asks for a margin account. It means that the parties are asked to deposit a certain amount of money (cash or T Bills). Given that the futures prices are subject to fluctuations of the asset underlying the contract, the margin account is adjusted such as to reflect the gains and the losses at the end of each trading day. This procedure is known as the marking to market the account. Furthermore there is a maintenance margin set at a lower level than the margin account. If the future's price drop such as the margin account becomes lower than the maintenance margin, the clearinghouse requires the respective party (the one that owns the margin account) to top up the margin account to the initial margin within a short period of time. To put it differently, the party receives a margin call. The liquidity risk may arise from the borrowing constraint. In other words the investor may not be able to raise enough funds to meet his margin requirements. Failure to do so may lead to financial distress situation. The later one may be characterized by a loss of the future contracts or in the worst case by a loss of the futures plus the exposure to the underlying asset. Consequently, the optimal hedge strategy of the investor who hedges the asset's risk via futures, should take into account the fact that he is subject to borrowing constraints. Hence he should avoid that hedging policy in per se is leading toward a distress situation. Note that credit risk is not part of the modelling in the sense there is no initial debt and a distress situation implies that he loses his portfolio of futures and the asset to be hedged.

There is an extensive literature on investors hedging policy. The first stream deals with the question of why investors hedge. The following answers are provided: a) financial distress, b) costly external financing, c) managerial incentives, d) tax reason (Smith and Stulz (1985)). The second stream of these literature looks at the impact of hedging on variables such as investor value, firm value, cash flow volatility etc (see, e.g., Allayannis and Weston (2001)). The third stream addresses the question of how should a investor hedge (Brown and Toft (2002)). Lastly, the fourth stream provides empirical support to the above theories (see, e.g., Nance, Smith and Smithson (1993), Guay and Kothari (2000), etc.). Having provided an empirical support to the theory of hedging,

some empirical papers are concerned by the magnitude of hedging. More precisely, Guay and Kothari (2000) explore the magnitude of the exposure hedged via derivatives in a non-financial investors sample set. They found that "the derivative securities are unlikely to have a significant impact on entity level interest rate exposures, the exchange rate exposure, or the stock return volatility". In this line Brown et al. (2003) report a hedge ratio of a magnitude 0.215. Other empirical facts come from the agriculture sector as mentioned in Arias et al. (2000). Thus, there is need to provide theoretical models able to explain why the magnitude of investor's hedging is low relative to their risk exposure. Alternatively these models include basis risk, yield risk or production risk as plausible explanation of the low hedging ratio (Lapan and Moschini (1995)), as well as tax liabilities, bankruptcy cost and liquidity cost as in Arias et al. (2000). Deep (2002) focuses on the impact of borrowing constraints on future contracts under a perfect correlation case. Hanka (2002) looks at a similar problem, in a simplified static and dynamic framework. Firm hedges as to protect liquidity reserves against shocks. Thus, the firm must adopt a dynamic futures hedging strategy such as to avoid external financing required to meet the unexpected margin calls. Mello and Parson (2000) study the alternative hedging strategies for financially constrained firms (the firm's ability to honor the funding requirement is limited by its liquidation value). Their main conclusion is that the optimal hedging strategy is the one with a borrowing strategy that imposes the lowest financing cost on the firm¹.

The paper makes the following contribution to the existing literature. First, it says that one of the reasons for which companies hedge a little, i.e. low hedge ratio, is that they do not want to incur a financial distress situation. This result goes in the same line with the empirical evidence (Mian (1996)) that big firms having easier access to external financing are less affected from a plausible financial distress situation than smaller firms which are more credit rationed. Second, the model captures the empirically observed fact stated in Brown et al. (2003) that the hedge magnitude increases when the prices that need to be hedged move in their favor and decreases when prices move against them. In this context the model predicts a positive relation between the state of the economy and the hedging magnitude. Third, the model also predicts that the hedge increases as the volatility of the underlying exposure increases.

This paper is in the spirit of Mello and Parsons (2000), in the sense that it endogenizes for the borrowing constraint. However it is based on the model proposed by Deep (2002). The difference between the two is that while the former assumes a constant borrowing constraint we propose an endogenous borrowing constraint. Furthermore this paper's theoretical explanation of low hedging magnitude, is represented by analytical expression (thanks to the convex duality approach), which is not the case in Deep (2002), Mello and Parson (2001). Lastly our paper carries a sensitivity analysis of the determinants of hedging and trading strategies. While analytical solution helps in better understanding the various links between the parameters defining the futures price and the un-

¹Mello and Parson (2000), pg 129.

derlying asset with the hedging and trading strategies, the framework is flexible enough to include a number of extensions such as a probabilistic constraints (see Basak and Shapiro (2001)), another more realistic definition of the borrowing constraints as well as credit risk.

The paper is organized as follows. Section two discusses the issue of hedging via future contracts under the perfect correlation case and in absence of borrowing constraints. Section three releases this assumption and its allows for borrowing constraints. Section fourth analyses the empirical consequences of the model. Last section concludes. Technical details provided in appendix.

2 Case I: Perfect Correlation

The information structure is defined as follows: (Ω, G, P) in a time span $[0, T]$ where; a positive constant T called as terminal time, P is the physical probability space, an element of Ω defined as ω is a state of nature and $\{G(t)\}_{0 < t < T}$ is the augmentation of the filtration $\{G^w(t)\}_{0 < t < T}$ generated by $w(\cdot)$, where $\{w(t), G(t); 0 \leq t \leq T\}$ is an one dimensional Brownian Motion. A progressively measurable; risk free rate satisfying $\int_0^T r(t) dt < \infty$ almost surely and constant $r(t) = r$. The risk free $B(t)$ as well as the risky $S(t)$ follow the diffusions, i.e. satisfy:

$$dB_t = rB_t dt \quad (1)$$

$$dS_t = S_t (u dt + \sigma dw_S(t)) \quad (2)$$

We set $B_0 = 1$. The parameters are the constant drift coefficients u and volatility σ . A positive and constant initial asset price $S_0 > 0$. We assume that the future contracts are written on another asset P_t , which price follows the dynamics

$$\frac{dP_t}{P_t} = m dt + b dw_{pt} \quad (3)$$

$E[dw_{pt} dw_{st}] = \rho dt$ and have constant drift and volatility. In order to provide analytical solutions, we need to have complet market. For this reason we set the absolute value of ρ to be equal to one. By the non-arbitrage argument $F_t = P_t \exp^{r(T-t)}$ and the use of Ito's Lemma yield the following futures price:

$$dF_t = F_t (m - r) dt + F_t b dw_{pt} \quad (4)$$

Moreover $\frac{m-r}{b} = \frac{u-r}{\sigma}$ and $w_{St} = w_{pt}$. Note that from now afterwards we will denote $w_{St} = w_{pt} = w_t$. The process θ_t specifies the hedge on the futures position. We further assume that the future contracts are of instantaneous maturity, as it is often done in the hedging literature. The margin account is denoted by X_t and is equal to

$$X_t = \int_0^t \exp(r(t-s)) \theta_s dF_s \quad (5)$$

The diffusion process for the margin account follows²

$$dX_t = (\theta_t F_t (m - r) + rX) dt + \theta_t F_t b dw_t \quad (6)$$

The following term $\eta(t)$ is the Radon-Nikodym derivative (satisfying the necessary conditions), under which P is equivalent to the martingale probability measure Q (note two probability measure are equivalent if they have the same null sets). The Radon-Nykodin derivative $\eta_t = E \left[\frac{dQ}{dP} \mid G_t \right]$ is unique.

$$\eta_t = \eta_0 \exp \left[- \int_0^t \frac{1}{2} k^2 ds - \int_0^t k dw_s \right] \quad (7)$$

for $d\eta_t = -\eta_t k dw_{st}$ where $k = \frac{m-r}{b} = \frac{u-r}{\sigma}$. By Girsanov's theorem $w_t^* = w_t + kt$, defines a standard Brownian Motion under Q for a constant coefficient case. Given the market completeness, the state price density is also unique:

$$\xi_t = \frac{1}{B_t} \eta_t \quad (8)$$

The state price density can be rewritten as

$$\xi_t = \xi_0 \exp \left[- \int_0^t \left(r + \frac{1}{2} k^2 \right) ds - \int_0^t k dw_s \right] \quad (9)$$

We define a new variable such as

$$z_t = \frac{1}{\xi_t} \quad (10)$$

for $z_0 = 1$. The diffusion process is a straightforward application of the Ito's Lemma on z_t .

$$dz_t = z_t \left((r + k^2) dt + k dw_t \right) \quad (11)$$

The investor's wealth W_t is given by the sum of αS_t , where α is the quantity (number of shares), plus the margin account X_t plus units Q_t invested in riskless asset $Q_t \exp(rt)$. The wealth process is defined as

$$dW_t = \alpha dS_t + dX_t + Q_t r \exp^{rt} dt \quad (12)$$

Note that the initial wealth $W_0 = X_0 + \alpha S_0 + Q_0$ is divided between the asset exposure αS_0 and the initial margin account $X_0 = h\alpha S_0$ for h being the fraction on the exposure, assuming that the investor is not borrowing to fulfill his margin requirement, hence $Q_0 = 0$. Whereas the wealth process is as follows

$$dW_t = (\alpha S_t u + \theta_t F_t (m - r) + r (X_t + Q_t \exp^{rt})) dt + (\alpha S_t \sigma + \theta_t F_t b) dw_t \quad (13)$$

² $dX_t = \exp(r(t-t)) \theta_t dF_t * 1 - \exp(r(t-0)) \theta_t dF_t * 0 + \int_0^t r \exp(r(t-s)) \theta_s dF_s dt$

2.1 The derivation of the optimal wealth and trading strategy

We introduce the case of an investor facing no borrowing constraints. S/he maximizes the portfolio value at some finite horizon T . The preference structure is given from a power utility function with relative risk aversion $RRA = -\frac{xu''(x)}{u'(x)}$ constant across different wealth levels and $ARA = -\frac{u''(x)}{u'(x)}$. The optimization of the static problem under a positive relative risk aversion $y > 0$ is as follows:

$$P_1 = \begin{cases} \max Eu(\alpha S_T + X_T) \\ \text{subject to} \\ E\left[\frac{W_T}{z_T}\right] \leq \frac{W_0}{z_0} \end{cases} \quad (14)$$

First, the investor solves for the optimal terminal wealth. Second, he derives the optimal trading and hedging³ strategy optimizing for his terminal wealth. Under the perfect correlation case the underlying asset to be hedged and the underlying of the futures matches (S_t).

Proposition 1 *The optimal terminal wealth of an unconstrained investor, whose preference structure follows a power utility function, is given as*

$$W_T = \left(\frac{\lambda}{z_T}\right)^{-\frac{1}{y}} \quad (15)$$

Whereas, the time t wealth is

$$W_t = \left(\frac{\lambda}{z_t}\right)^{-\frac{1}{y}} Y_t \quad (16)$$

for

$$Y_t = \exp\left(\left(\frac{1-y}{y}\left(r + \frac{k^2}{2}\right) + \left(\frac{1-y}{y}\right)^2 \frac{k^2}{2}\right)(T-t)\right) \quad (17)$$

The langrangean multiplier λ is defined as

$$\lambda = \left(W_0 \exp\left(-\left(\frac{1-y}{y}\left(r + \frac{k^2}{2}\right) + \left(\frac{1-y}{y}\right)^2 \frac{k^2}{2}\right)T\right)\right)^{-y} \quad (18)$$

The optimal trading strategy, in terms of the number of future contracts is.

$$\theta_t = \exp(-r(T-t)) \left(\frac{1}{y}\left(\frac{\lambda}{z_t}\right)^{-\frac{1}{y}} \frac{Y_t}{P_t} \frac{u-r}{\sigma b} - \alpha \frac{\sigma}{b} \frac{S_t}{P_t}\right) \quad (19)$$

³Brown et al (2003) state that companies actively adjust derivative positions on derivative positions based on their market views.

where α is the number of shares in the asset exposed to future risk. The hedge portfolio is

$$\theta_{hp1,t} = -\exp(-r(T-t))\alpha\frac{S_t}{P_t} \quad (20)$$

Proof. See Appendix B.1 ■

The optimal trading strategy on future consists on the tangency and the hedge portfolio. The tangency portfolio is due to stochastic fluctuations of the asset underlying the future contract (P_t). The hedge portfolio provides protection against fluctuations in the asset to be hedged (S_t). The hedge component provides perfect insurance and it is represented by $-\alpha\frac{S_t}{P_t}\exp^{-r(T-t)}$. The hedge ratio is

$$hr_1 = \frac{-\alpha}{\alpha} = -1 \quad (21)$$

which means short futures contract with the same number of units of asset to be hedged if the $S_t = P_t$ (if the drift and the volatilities are the same).

3 Case II: Hedging in presence of borrowing constraint

So far the optimal hedge and the optimal trading strategy is derived under the assumption that the investor is always able to raise additional amount of money to meet any margin call within a short period of time. Nevertheless in reality the investor faces borrowing constraints (hereinafter the constrained investor). Remember that we assume that the investor does not want to run into a liquidity problem, which can trigger a financial distress situation. Hence her/his optimal trading as well as hedging strategy in futures should avoid that the a fall in margin account triggers a financial distress situation. Alternatively one may think of a situation where the futures contract are seized against a large drop in the margin account, where large is defined in terms of a lower threshold. The lower bound is defined as $\beta\alpha S_T$ for $\beta \in [0, 1]$, which says that it is proportional to the exposure. Furthermore the lower bound is at least as large as the maintenance margin. In practice the maintenance margin is around 75% of the initial margin level. The initial margin level in itself is between $h \in (5\%, 20\%)$ of the contract value. Hence it closely follows the asset's exposure to risk. If we assume that the bound defining the borrowing constraint and the maintenance level matches, then the investor's hedging strategy will avoid any possible margin call. If we assume that it is lower, is as if we allow only for margin calls of low magnitudes that can be easily fulfilled via external financing (riskless debt Mello and Parsons (2000)) with no impact on her/his hedging and trading strategy.

The way it is modeled delivers the following relation. A higher exposure, in terms of number of shares, as well a higher asset price S_t leads to a higher threshold level. β represents the degree of the borrowing constraints, that is a

low β implies a low bound and the contrary. We express the bound level as dependent on the asset whose exposure we are trying to hedge. Consequently, the bound per se becomes stochastic due to the dependence on S_t . The intuition for this type of modelling is as follows. In this framework, financial distress is triggered if the margin account falls below the lower bound, in the sense that he is not able to raise external funds.

$$X_t \leq \beta \alpha S_t \quad (22)$$

Similarly this condition is equivalent to say that

$$W_t \leq (1 + \beta) \alpha S_t \quad (23)$$

his wealth should be larger than a multiple $(1 + \beta)$ of the exposure on the asset S_t , otherwise his financial situation makes impossible any additional external financing. Hence contrary to Deep (2002) that assume a constant bound K , we assume a more flexible one, proportional to the investor's total wealth. Thus in some way we are indirectly building a relation between the investor's financial position and the market credit rationing.

The margin account is subject to the following modification, due to the fact that margin wealth earns risk free interest rate only for the balances (cash) exceeding $\beta \alpha S_t$

$$dX_t = (\theta_t F (m - r) + r (X_t - \beta \alpha S_t)) dt + \theta_t F_t b dw_t \quad (24)$$

The wealth process reflecting the borrowing constraint is

$$dW_t = (\alpha S_t u + \theta_t F_t (m - r) + r (X_t - \beta \alpha S_t + Q_t \exp^{rt})) dt + (\alpha S_t \sigma + \theta_t F_t b) dw_t \quad (25)$$

3.1 The optimal wealth and the optimal trading strategy

The optimization problem includes the additional constraint, hence it is modified as follows:

$$P_1 = \left\{ \begin{array}{l} \max Eu(\alpha S_T + X_T) \\ \text{subject to} \\ E \left[\frac{\dot{W}_T}{z_T} \right] \leq \frac{\dot{W}_0}{z_0} \\ \text{and} \\ X_T \geq \beta \alpha S_T \end{array} \right. \quad (26)$$

for $\beta \in [0, 1]$. Where $X_T \geq \beta \alpha S_T$ represents the borrowing constraints⁴. This type of constraint says that the margin account would not fall below the threshold for any t with a probability equal to one. \hat{W}_t stands for the wealth of the investor who is facing borrowing constraints (constrained investor), whereas W_t for the unconstrained investor's wealth. The relation between the two is as follows

$$\begin{aligned}\hat{W}_T &= \max(W_T, (1 + \beta) \alpha S_T) \\ &= W_T + ((1 + \beta) \alpha S_T - W_T) \mathbf{1}_{W_T \leq (1 + \beta) \alpha S_T}\end{aligned}\quad (27)$$

The second term is a put option with a stochastic strike price $(1 + \beta) \alpha S_T$ driven by S_T . The expression can be rewritten as

$$\hat{W}_T = W_T \mathbf{1}_{W_T > (1 + \beta) \alpha S_T} + (1 + \beta) \alpha S_T \mathbf{1}_{W_T \leq (1 + \beta) \alpha S_T} \quad (28)$$

Proposition 2 *The time t optimal wealth of an investor facing borrowing constraints, whose preference structure follows utility power law, is*

$$\hat{W}_t = \left(\frac{\lambda}{z_t}\right)^{-\frac{1}{y}} Y_t N(\bar{d}) + (1 + \beta) \alpha S_t N(-\tilde{d}) \quad (29)$$

which is equivalent to

$$\hat{W}_t = W_t N(\bar{d}) + (1 + \beta) \alpha S_t N(-\tilde{d}) \quad (30)$$

for \hat{W}_t and W_t corresponding to the optimal wealth in the constrained and the unconstrained case. The optimal trading strategy, in terms of units of futures contracts, is as follows

$$\begin{aligned}\theta_t &= \exp(-r(T-t)) \left[\frac{u-r}{\sigma^2} \frac{1}{y} \left(\frac{\lambda}{z_t}\right)^{-\frac{1}{y}} Y_t J_3 - (1 + \beta) \alpha \frac{S_t}{P_t} J_2 \right] \\ &\quad + \frac{\sigma}{b} \left[-\frac{Y_t}{P_t} \left(\frac{\lambda}{z_t}\right)^{-\frac{1}{y}} J_1 + (1 + \beta) \alpha \frac{S_t}{P_t} J_4 \right] - \alpha \frac{\sigma}{b} \frac{S_t}{P_t}\end{aligned}\quad (31)$$

for

$$\bar{Y}_t = \exp\left(\left(\frac{1-y}{y} \left(r + \frac{k^2}{2}\right) + \left(\frac{1-y}{y}\right)^2 \frac{k^2}{2}\right)(T-t)\right) \quad (32)$$

and the λ for the upper region, is given as in the eq.(18). Furthermore

$$J_1 = \frac{n(\bar{d})}{\left(\frac{1-y}{y} k - \sigma\right) \sqrt{T-t}} \quad (33)$$

⁴Interpret this constraint as a commitment solution in the sense that the investor would not revert to the unconstrained trading strategy after the initial date, given that the optimal trading strategy of a constrained investor is decided at the initial date.

$$J_2 = \frac{n(\tilde{d})}{\left(\frac{1}{y}k - \sigma\right)\sqrt{T-t}} \quad (34)$$

$$J_3 = N(\bar{d}) + J_1 \quad (35)$$

$$J_4 = N(-\tilde{d}) + J_2 \quad (36)$$

For

$$d = \frac{\ln \frac{\left(\frac{\lambda}{z_t}\right)^{-\frac{1}{y}}}{(1+\beta)\alpha S_t} + \left(\frac{1-2y}{y}r + \frac{\sigma^2}{2} + \frac{1-y}{y}\frac{k^2}{2}\right)(T-t)}{\left(\frac{1}{y}k - \sigma\right)\sqrt{T-t}} \quad (37)$$

$N(\cdot)$ represents the risk adjusted probabilities of not incurring financial distress where

$$\bar{d} = d + \frac{1-y}{y}k\sqrt{T-t} \quad (38)$$

$$\tilde{d} = d + (\sigma - k)\sqrt{T-t} \quad (39)$$

The hedge portfolio in terms of the number of futures to be bought is

$$\theta_{hp3,t} = \exp(-r(T-t)) \frac{\sigma}{b} \left(-\frac{Y_t}{P_t} \left(\frac{\lambda}{z_t}\right)^{-\frac{1}{y}} J_1 + (1+\beta)\alpha \frac{S_t}{P_t} J_4 - \alpha \frac{S_t}{P_t} \right) \quad (40)$$

Proof. see appendix B. 3 ■

This framework delivers a complex result. The optimal wealth is a function of the optimal wealth realized in the unconstrained case weighted by the probability of a non-distress situation, plus an additional term multiplying the probability of distress. The latter one is the insurance component of the optimal constrained strategy [see, e.g., Cox and Huang (1989), Basak (1995) etc].

Concerning the optimal trading strategy both, the tangency and the hedge portfolio have gone through substantial changes, incorporating parameters such as the fraction β defining the level of the borrowing constraint as well as the probability of a financial distress situation. The hedge portfolio is composed of three terms. The first term $e^{-r(T-t)} \frac{\sigma}{b} \left(-\frac{Y_t}{P_t} \left(\frac{\lambda}{z_t}\right)^{-\frac{1}{y}} J_1 \right)$ is of negative sign and its magnitude is positively related to the state of the economy. High states of the economy induce a high threshold (the strike price of our put insurance). Consequently, this term aims at taking profit by shorting even more the position in futures. The middle term $e^{-r(T-t)} \frac{\sigma}{b} (1+\beta)\alpha \frac{S_t}{P_t} J_4$ is due to the fact that the investor does not want to go beyond the lower threshold, which may trigger financial distress. Hence, when the probability of hitting the threshold is high ($N(-\tilde{d})$ incorporated on J_4), the investor increases the risk position, hence the volatility of his portfolio as to avoid the financial distress situation. This will mitigate the overall position in futures as it carries a positive sign. The last term is the one we have seen in the unconstrained portfolio and it equals

$-e^{-r(T-t)} \frac{\sigma}{b} \frac{S_t}{P_t} \alpha$. It says that the futures contract do offer perfect hedge against S_t price movements even in a constrained investor problem framework.

The hedge ratio is more complex than the one seen in the previous cases and defined as

$$hr_3 = \frac{\frac{\sigma}{b} \left(-\frac{Y_t}{P_t} \left(\frac{\lambda}{z_t} \right)^{-\frac{1}{y}} J_1 + (1 + \beta) \alpha \frac{S_t}{P_t} J_4 - \alpha \frac{S_t}{P_t} \right)}{\alpha} \quad (41)$$

Unlike the hedge ratios observed in cases on no borrowing constraint, the one defined in Eq. (41) depends on the various parameters as well as probability density functions.

4 Empirical consequences

This section provides graphical illustration of the results obtained so far. We set the base case parameters such the futures price F_t volatility $b = 0.3$ while the drift m is endogenously determined given the absence of arbitrage. The initial value is $P_0 = 1$. The underlying asset's S_t parameters are $u = 0.1$ for the drift and $\sigma = 0.3$ for the volatility. The correlation coefficient between the underlying asset S_t and the underlying of the futures P_t (hence the futures price F_t itself) is $\rho = 1$, while assuming a risk aversion of $y = 2$. Note that for hedging purposes, a negative correlation requires a long position in futures, whereas a positive correlation a short position. The fraction h determining the level of initial margin account, is taken exogenously $h = 0.1$ given that this type of collateral is highly formalized and does not depend on the counterparty (here investor) credit quality. We assume that the initial exposure is $\alpha S_0 = 1$ given $S_0 = 1$ and $\alpha = 1$. Then for $X_0 = h\alpha S_0$ we have the initial margin account $X_0 = 0.1$ and initial wealth $W_0 = 1.1$, while assuming that s/he distributes all her/his money between the αS_0 and X_0 . Consequently, there are no units invested at risk free asset $Q_0 = 0$ at time zero. We introduce an additional parameter $g \left(= \frac{\beta}{h} \right)$ representing the ratio of the fraction β defining the degree of the borrowing constraint with the fraction required for the initial margin account h . For graphical purposes we name g as the "% of the barrier" in the sense that it gives the fraction applied to h . Given that $\beta < h$ the ratio $g < 1$. Lastly, the risk free rate is taken constant at $r = 0.05$, the maturity $T = 1yr$.

Figure 1 displays the interaction of the terms composing the hedge as well as the trading portfolio of the constrained investor. More specifically the magnitude of the hedging portfolio for the investor facing borrowing constraints is always lower than the one corresponding to the unconstrained investor across all states of the economy. This result constitute one of the major findings of this paper. A second major finding of this paper is concerned with the positive relation between the state of the economy and the hedging magnitude. Given that in this framework, low state (recession) it implies also lower asset prices, the model predicts that the investors will hedge less when the price that they need to hedge

is also getting lower. In other word a firm trying to lock S_T selling price by shorting the exposure via P_T as hedges (correlation $\rho = 1$) will actually reduce the hedge portfolio as the S_t and P_t is going down. Empirically this phenomena is reported in Brown et al. (2003). Graphically, we see the discrepancy between the two is larger not only when the economy is experiencing a downturn. This is due to the following reason. First, while the futures contracts still offer a hedge to the S_t exposure (the last term of the hedging portfolio), the probability of incurring a financial distress in economic downturns increases. This is due to the fact that the restriction becomes more important in downturns as the total wealth of the investor decreases. The leads to an increase of the overall risk position of the portfolio and mainly the asset underlying the futures contract P_t as to avoid financial distress situation. Increasing the overall risk position means increasing the volatility of the investors portfolio. This effect is induced from the mid term of the hedging portfolio leads to even positive overall hedge portfolio in very low state of the economy. The same conclusion is not true for the unconstrained investor where the hedge portfolio dominates the tangency portfolio.

Figure 2 displays the hedge $\theta_{hp3,t}$ portfolio (in terms of number of units hold in P_t) of the constrained investor (the meshed surface) versus the ones of an unconstrained investor (the plain surface) in a y and g (% barrier) space. The top panel of this figure reveals the following facts. First, the magnitude of the hedge portfolio of the investor facing borrowing constraints is lower than the one corresponding to the unrestricted investor. Second, the magnitude of hedging is positively related with the relative risk aversion and with the degree of borrowing constraints. A higher borrowing constraint (higher g) is equivalent to lower available external financing and may still induce higher hedging. However this impact is quite negligible and it is less and less important for higher maturities. The hedge portfolio of an unconstrained investor is as expected insensitive to these two parameters.

Figure 3 shows that the trading (tangency + hedge) portfolios for the constrained and unconstrained investor decrease for higher risk aversion. This is due to the fact that high risk averse investors hold more in futures for hedging purposes. On the other hand an increase in the g reduces (from -0.1 to -0.05) the magnitude of trading portfolio only for the constrained investor. The effect is mainly due to the hedge portfolio. More importantly we see that the trading portfolio of a constrained investor tends to be lower (in magnitude) than the one of an unconstrained investor.

Figure 4 and Figure 5 are in the same line with the above result the hedging and the trading strategy of an investor facing borrowing constraints is lower than the respective ones for the unconstrained investor once plotted in a volatilities space. More specifically less hedge is needed if the underlying of the future contracts P_t is highly volatile. The contrary is true for the magnitude of the hedging position versus the volatility of the asset to be hedged S_t . The same results prevail for the trading strategy.

Figure 6 and figure 7 both focus on the time effect on the optimal hedging and trading portfolio of the two types of investors. Figure 6 basically says that the hedge portfolio of an investor facing borrowing constraint and the one of an investor facing no such borrowing constraints tend to converge as the maturity increases. This is due to the fact that the constraint is less important for large maturities than for lower ones and may be linked to the fact that the probability of incurring a financial distress is lower for larger maturity and the contrary. More specifically as the risk of incurring the financial distress⁵ is reduced, the hedge portfolio of the unconstrained investor is converging to the hedge portfolio of the unconstrained investor. However, as the maturity increases further on the two hedge portfolios exhibits the same pattern in the sense that the magnitude of hedging decreases as the discount factor increases $\exp^{r(T-t)}$ given that ($F_t = P_t \exp(r(T-t))$). Concerning the time to maturity effect it is shown that it in general⁶ the probability of financial distress is increasing as time t converges to maturity T . To conclude the hedging portfolios determines the shape and the results of the trading portfolio.

5 Conclusions

In this paper we have analyzed the optimal trading and hedging policy of an investor who is subject to default risk as well as liquidity risk. Default risk is mitigated via a required margin account. Liquidity risk may arise from the inability to raise external money to fulfill the margin call in a margin account. Given that the later one can lead to a distress situation, the optimal trading and hedging policy accounts for the liquidity risk. It is shown that the optimal trading and hedging policy of a constrained investor (the one that face borrowing constraints), is always lower than the one of an unconstrained investor. This is true across different states of the economy. The two policies collapse for large maturities, when the probability of running into a financial distress situation is quite low. This is an important result because it explains the empirical observations that firms do little hedging. Other important result is that the hedge magnitude is positively related to the price that need to be hedged. This result helps in better understanding the puzzling empirically observed fact stated in Brown et al. (2003) that "...there is a tendency from firms to decrease hedging as prices moves against them". Third, the model also predicts that the hedge and the volatility of the underlying exposure are positively related, while it is negatively related to the volatility of the asset underlying the futures contract. Fourth, the hedging magnitude decreases with the risk aversion.

The paper may be extended or improved in a number of ways. First, one can assume that the asset to be hedged is a commodity. In this case one has to assume that it follows a mean reversion diffusion process. Second, it can be extended to the case of a probabilistic maintenance level. To put it differently,

⁵Fig.8 says the distress probability is lower for longer maturities.

⁶With exception of relative risk aversion near to one.

the investor will try not to reach the maintenance level (thus avoid a margin call), with a given probability. So far we have assumed that the investor use a probability equal to one. Third, it would be quite intuitive to assume that the lower barrier would be a function of the degree of the leverage of the investor, in the sense that a more leverage leads to higher bound and vice versa.

6 Appendix B.1

6.1 Unconstraint Optimization

Proof.

$$\begin{aligned}
E_t[X_T] &= E_t \left[\left(\left(\frac{\lambda}{z_T} \right)^{-\frac{1}{y}} - \alpha S_T \right) \frac{z_t}{z_T} \right] \\
&= E_t \left[\left(\frac{\lambda}{z_t} \right)^{-\frac{1}{y}} e^{\frac{1-y}{y} \left((r + \frac{k^2}{2})(T-t) + k^2(w_T - w_t) \right)} \right] \\
&\quad - E_t \left[\alpha S_T \frac{z_t}{z_T} \right]
\end{aligned} \tag{42}$$

$$\begin{aligned}
X_t &= \left(\frac{\lambda}{z_t} \right)^{-\frac{1}{y}} e^{\left(\frac{1-y}{y} \left(r + \frac{k^2}{2} \right) + \left(\frac{1-y}{y} \right)^2 \frac{k^2}{2} \right) (T-t)} - \alpha S_t \\
Y_t &= e^{\left(\frac{1-y}{y} \left(r + \frac{k^2}{2} \right) + \left(\frac{1-y}{y} \right)^2 \frac{k^2}{2} \right) (T-t)} \\
X_t &= \left(\frac{\lambda}{z_t} \right)^{-\frac{1}{y}} Y_t - \alpha S_t
\end{aligned} \tag{43}$$

for $w_t^* = w_t + kt$. ■

6.1.1 Derivation of the optimal trading strategy

Proof. The Ito's lemma to the optimal wealth $X_t = M(S_t, z_t, t)$, leads us to the following result

$$dM = (\cdot) dt + M_s S_t \sigma dw_t + M_z z_t k dw_t \tag{44}$$

Comparing the term in front of the dw_t and the one corresponding to the wealth eq.13 the following is true

$$\begin{aligned}
\theta_t Fb &= \frac{1}{y} \left(\frac{\lambda}{z_t} \right)^{-\frac{1}{y}} Y_t k - \alpha S_t \sigma \\
\theta_t &= e^{-r(T-t)} \left(\frac{1}{y} \left(\frac{\lambda}{z_t} \right)^{-\frac{1}{y}} \frac{Y_t}{P_t} \frac{u-r}{\sigma b} - \alpha \frac{\sigma}{b} \right)
\end{aligned} \tag{45}$$

The optimal trading strategy θ_t is provided in the proposition. Note that $S_t = P_t$ given that $\frac{u-r}{\sigma} = \frac{m-r}{b}$. ■

6.2 Constraint optimization

Proof. The time t wealth is

$$\begin{aligned}
\hat{W}_t &= E \left[\left(\frac{\lambda}{z_T} \right)^{-\frac{1}{y}} \frac{z_t}{z_T} \mathbf{1}_{W_T > (1+\beta)\alpha S_T} \right] + (1+\beta) \alpha E \left[S_T \frac{z_t}{z_T} \mathbf{1}_{W_T \leq (1+\beta)\alpha S_T} \right] \\
&= \left(\frac{\lambda}{z_t} \right)^{-\frac{1}{y}} Y_t E_t \left[e^{-\left(\frac{1-y}{y}\right)^2 \frac{k^2}{2} (T-t) + \frac{1-y}{y} k (w_T - w_t)} \mathbf{1}_{W_T > (1+\beta)\alpha S_T} \right] + \\
&\quad + (1+\beta) \alpha S_t E_t \left[e^{\left(u-r-\frac{\sigma^2}{2}\right)(T-t) + \sigma \left((w_{pT}^* - w_t) - k_p (T-t) \right)} \mathbf{1}_{W_T \leq (1+\beta)\alpha S_T} \right] \\
&= \left(\frac{\lambda}{z_t} \right)^{-\frac{1}{y}} Y_t E_t^{\bar{Q}} \left[\mathbf{1}_{W_T > (1+\beta)\alpha S_T} \right] + (1+\beta) \alpha S_t E_t^{\tilde{Q}} \left[\mathbf{1}_{W_T \leq (1+\beta)\alpha S_T} \right]
\end{aligned} \tag{46}$$

Given the general relation

$$w_t^* = w_t + kt \tag{47}$$

$$\bar{w}_t = w_t - \frac{1-y}{y} kt \tag{48}$$

$$\bar{w}_t = w_t^* - \frac{k}{y} t \tag{49}$$

$$\tilde{w}_t = \bar{w}_t + \left(\frac{k_p}{y} - \sigma \right) t \tag{50}$$

The term $E_t \left[\mathbf{1}_{W_T > (1+\beta)\alpha S_T} \right]$ is equal to the probability

$$\begin{aligned}
&= P(W_T > (1+\beta)\alpha S_T) \\
&= N(\bar{d})
\end{aligned} \tag{51}$$

for

$$d = \frac{\ln \left(\frac{\lambda}{z_t} \right)^{-\frac{1}{y}} + \left(-u + \frac{r}{y} + \frac{\sigma^2}{2} + \frac{1}{y} \frac{k^2}{2} \right) (T-t)}{\left(\frac{k_p}{y} - \sigma \right) \sqrt{T-t}} \tag{52}$$

The time t wealth is then defined as

$$\hat{W}_t = \left(\frac{\lambda}{z_t} \right)^{-\frac{1}{y}} Y_t * N(\bar{d}) + (1+\beta) \alpha S_t N(-\tilde{d}) \tag{53}$$

■

6.2.1 Derivation of the optimal trading strategy

Proof. The optimal trading strategy is derived from the following procedure. First we apply the Ito's Lemma to the optimal wealth level $W_t = M(S_t, z_t, t)$.

$$dM = (\cdot) dt + M_s S_t \sigma dw_t + M_z z_t k_s dw_t \tag{54}$$

Taking the first derivative wrt to $M(\cdot)$ we have

$$M_s S_t = \left[- \left(\frac{\lambda}{z_t} \right)^{-\frac{1}{y}} Y_t \frac{n(\bar{d})}{\left(\frac{k_p}{y} - \sigma \right) \sqrt{T-t}} + \right. \\ \left. + (1 + \beta) \alpha S_t \left(N(-\tilde{d}) + \frac{n(\tilde{d})}{\left(\frac{k_p}{y} - \sigma \right) \sqrt{T-t}} \right) \right]$$

and as

$$M_z z_t = \frac{1}{y} \left[\left(\frac{\lambda}{z_t} \right)^{-\frac{1}{y}} Y_t \left(N(\bar{d}) + \frac{n(\bar{d})}{\left(\frac{k_p}{y} - \sigma \right) \sqrt{T-t}} \right) + \right. \\ \left. - (1 + \beta) \alpha S_t \frac{n(\tilde{d})}{\left(\frac{k_p}{y} - \sigma \right) \sqrt{T-t}} \right] \quad (55)$$

The optimal trading strategy θ_t is given from equalizing the dw_t of eq.(13) and the term in front of dw_t in the eq.(54) the following is true

$$(M_z z_t k_p + M_s S_t \sigma) \quad (56) \\ k \frac{1}{y} \left[\left(\frac{\lambda}{z_t} \right)^{-\frac{1}{y}} Y_t \left(N(\bar{d}) + \frac{n(\bar{d})}{\left(\frac{k_p}{y} - \sigma \right) \sqrt{T-t}} \right) + \right. \\ \left. - (1 + \beta) \alpha S_t \frac{n(\tilde{d})}{\left(\frac{k_p}{y} - \sigma \right) \sqrt{T-t}} \right] + \\ + \sigma \left[- \left(\frac{\lambda}{z_t} \right)^{-\frac{1}{y}} Y_t \frac{n(\bar{d})}{\left(\frac{k_p}{y} - \sigma \right) \sqrt{T-t}} + \right. \\ \left. + (1 + \beta) \alpha S_t \left(N(-\tilde{d}) + \frac{n(\tilde{d})}{\left(\frac{k_p}{y} - \sigma \right) \sqrt{T-t}} \right) \right]$$

for simplicity set

$$J_1 = \frac{n(\bar{d})}{\left(\frac{k_p}{y} - \sigma \right) \sqrt{T-t}} \quad (57)$$

$$J_2 = \frac{n(\tilde{d})}{\left(\frac{k_p}{y} - \sigma \right) \sqrt{T-t}} \quad (58)$$

$$J_3 = N(\bar{d}) + J_1 \quad (59)$$

$$J_4 = N(-\tilde{d}) + J_2 \quad (60)$$

then

$$\begin{aligned} \alpha S_t \sigma + \theta_t F_t b &= k_p \frac{1}{y} \left(\left(\frac{\lambda}{z_t} \right)^{-\frac{1}{y}} Y_t J_3 - (1 + \beta) \alpha S_t J_2 \right) + \\ &+ \sigma \left(- \left(\frac{\lambda}{z_t} \right)^{-\frac{1}{y}} Y_t J_1 + (1 + \beta) \alpha S_t J_4 \right) \end{aligned} \quad (61)$$

then

$$\begin{aligned} \theta_t F_t &= \frac{u - r}{\sigma b} \frac{1}{y} \left(\left(\frac{\lambda}{z_t} \right)^{-\frac{1}{y}} Y_t J_3 - (1 + \beta) \alpha S_t J_2 \right) + \\ &+ \frac{\sigma}{b} \left(- \left(\frac{\lambda}{z_t} \right)^{-\frac{1}{y}} \bar{Y}_t J_1 + (1 + \beta) \alpha S_t J_4 - \alpha S_t \right) \end{aligned} \quad (62)$$

Finally

$$\begin{aligned} \theta_t &= e^{-r(T-t)} \left[\frac{m - r}{b^2} \frac{1}{y} \left(\frac{1}{P_t} \left(\frac{\lambda}{z_t} \right)^{-\frac{1}{y}} Y_t J_3 - (1 + \beta) \alpha J_2 \right) + \right. \\ &\left. + \frac{\sigma}{b} \left(- \frac{Y_t}{P_t} \left(\frac{\lambda}{z_t} \right)^{-\frac{1}{y}} J_1 + (1 + \beta) \alpha J_4 - \alpha \right) \right] \end{aligned} \quad (63)$$

■

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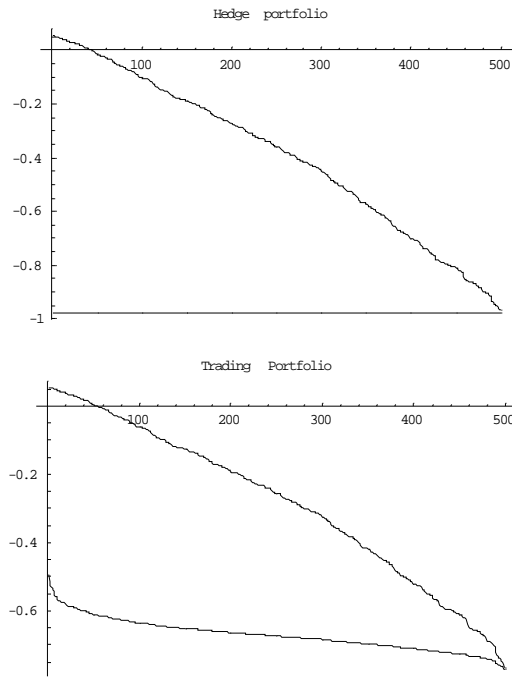


Figure 1: Top panel: The number of futures for hedging purposes of an restricted investor is always lower (the line on top) than the one of an unrestricted investor (the constant line) from the lowest to the highest state of the economy. Lower panel: The magnitude of the trading portfolio of the restricted investor is always lower (the line on top) than the one of an unrestricted investor (the lower line) from the lowest to the highest state of the economy. The base case parameters are: $u = 0.1, r = 0.05, \sigma = 0.3, b = 0.3, \rho = 1, y = 2, T = 1yr, t = 0.5, Q_0 = 0, z_0 = 1, P_0 = 1, S_0 = 1, \alpha = 1, h = 0.1, g = 0.55$. Then $m = 0.1, X_0 = 0.1, \alpha S_0 = 1, W_0 = 1.1$. The x-axis represents increasing states of the economy for 500 simulated market risk factors.

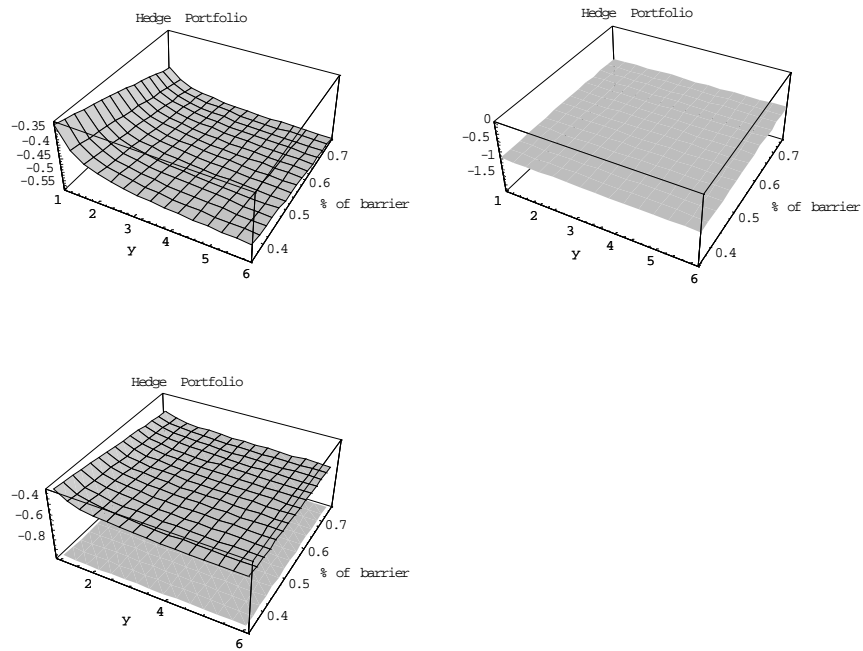


Figure 2: The number of future for hedging purposes increases with the % barrier and y . The base case parameters are: $u = 0.1, r = 0.05, \sigma = 0.3, b = 0.3, \rho = 1, y = 2, T = 1yr, t = 0.5, Q_0 = 0, z_0 = 1, P_0 = 1, S_0 = 1, \alpha = 1, h = 0.1, g = 0.55$. Then $m = 0.1, X_0 = 0.1, \alpha S_0 = 1, W_0 = 1.1$.

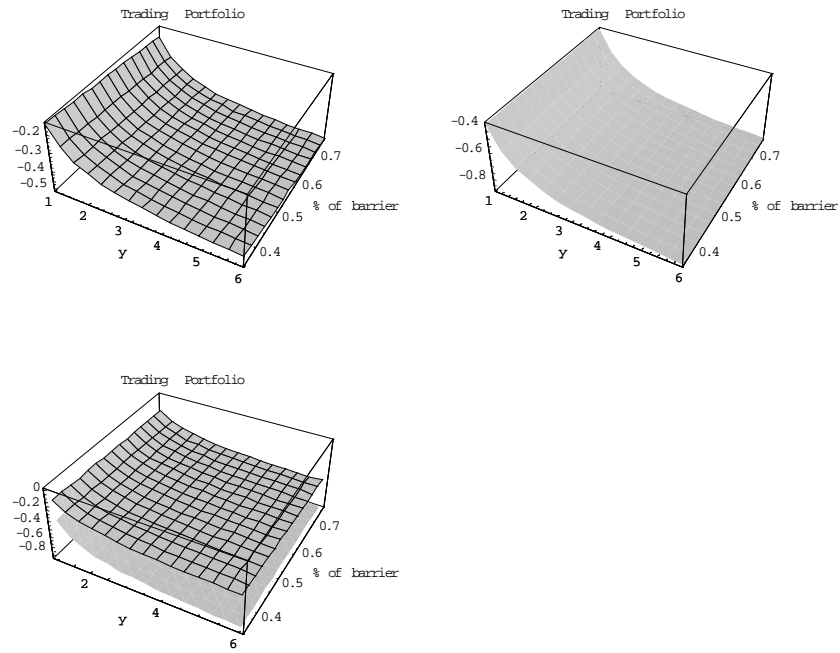


Figure 3: The number of future for trading purposes increases with the % barrier and y . The base case parameters are: $u = 0.1, r = 0.05, \sigma = 0.3, b = 0.3, \rho = 1, y = 2, T = 1yr, t = 0.5, Q_0 = 0, z_0 = 1, P_0 = 1, S_0 = 1, \alpha = 1, h = 0.1, g = 0.55$. Then $m = 0.1, X_0 = 0.1, \alpha S_0 = 1, W_0 = 1.1$.

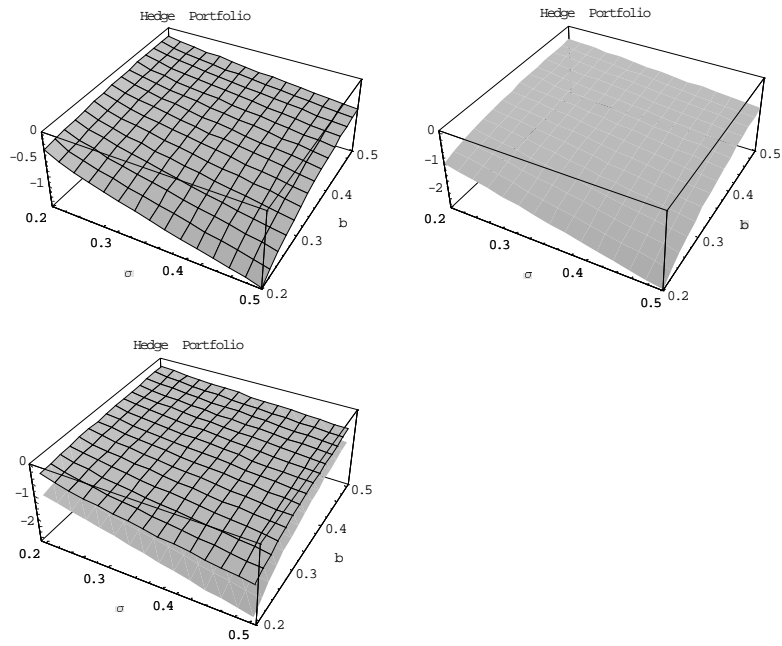


Figure 4: The number of future (in terms of units of P_t) for hedging purposes decreases with the volatility characterizing the futures, whereas it increases with respect to the asset's volatility. The base case parameters are: $u = 0.1, r = 0.05, \sigma = 0.3, b = 0.3, \rho = 1, y = 2, T = 1yr, t = 0.5, Q_0 = 0, z_0 = 1, P_0 = 1, S_0 = 1, \alpha = 1, h = 0.1, g = 0.55$. Then $m = 0.1, X_0 = 0.1, \alpha S_0 = 1, W_0 = 1.1$.

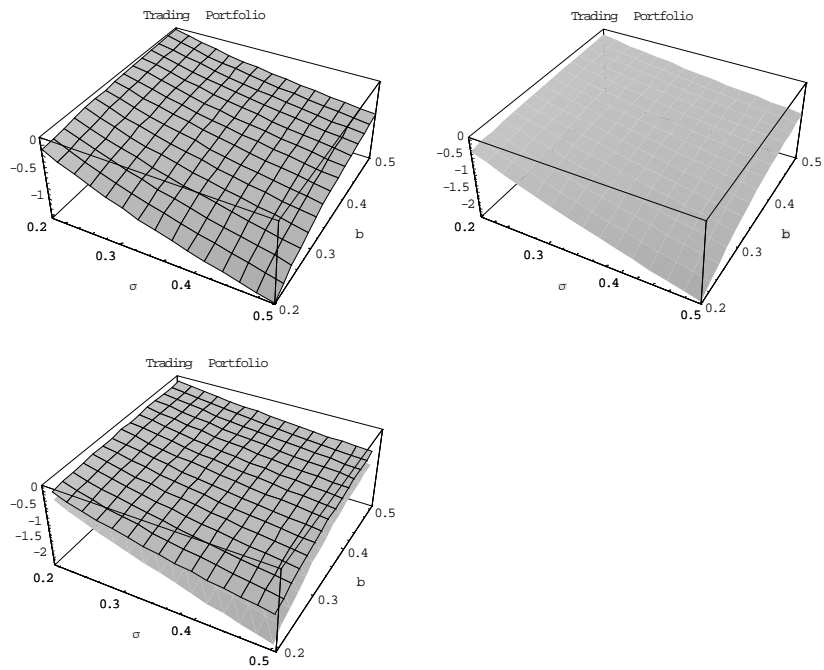


Figure 5: The trading portfolio (in terms of units of P_t) decreases with the volatility underlying the futures, whereas it increases with respect to the asset's volatility. The base case parameters are: $u = 0.1, r = 0.05, \sigma = 0.3, b = 0.3, \rho = 1, y = 2, T = 1yr, t = 0.5, Q_0 = 0, z_0 = 1, P_0 = 1, S_0 = 1, \alpha = 1, h = 0.1, g = 0.55$. Then $m = 0.1, X_0 = 0.1, \alpha S_0 = 1, W_0 = 1.1$.

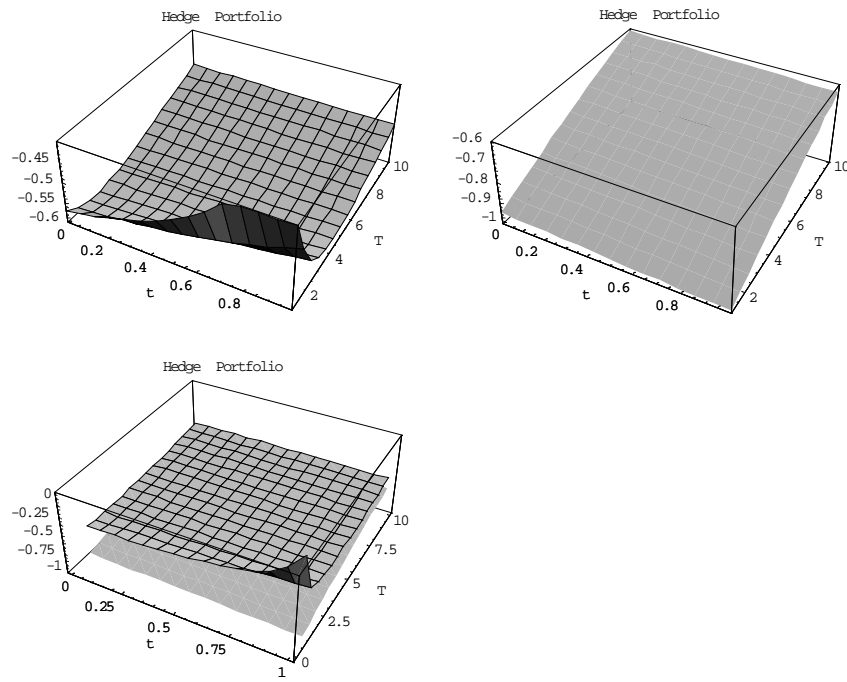


Figure 6: The term structure of the hedging portfolio of a constrained investor versus an unconstrained one. The base case parameters are: $u = 0.1, r = 0.05, \sigma = 0.3, b = 0.3, \rho = 1, y = 2, T = 1yr, t = 0.5, Q_0 = 0, z_0 = 1, P_0 = 1, S_0 = 1, \alpha = 1, h = 0.1, g = 0.55$. Then $m = 0.1, X_0 = 0.1, \alpha S_0 = 1, W_0 = 1.1$.

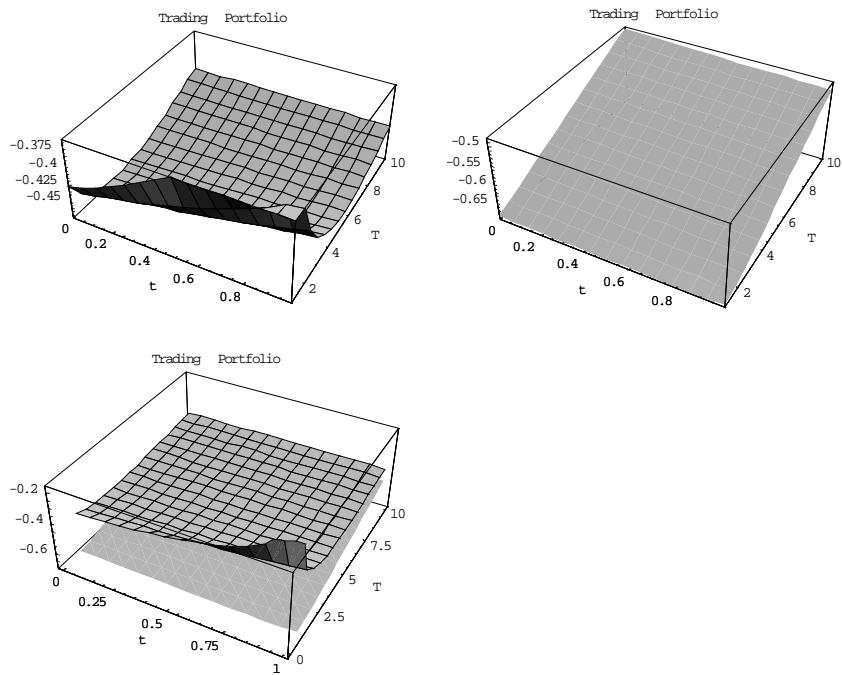


Figure 7: The term structure of the trading portfolio of a constrained investor versus an unconstrained one. The base case parameters are: $u = 0.1, r = 0.05, \sigma = 0.3, b = 0.3, \rho = 1, y = 2, T = 1yr, t = 0.5, Q_0 = 0, z_0 = 1, P_0 = 1, S_0 = 1, \alpha = 1, h = 0.1, g = 0.55$. Then $m = 0.1, X_0 = 0.1, \alpha S_0 = 1, W_0 = 1.1$.

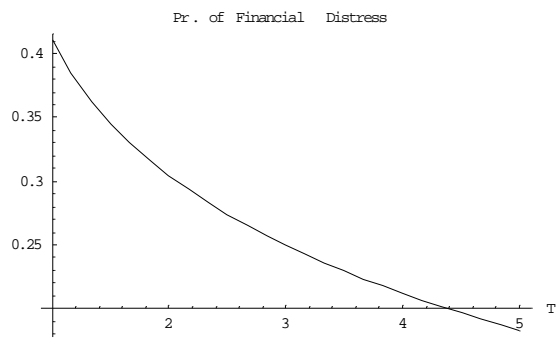


Figure 8: The term structure of the probability of financial distress. The base case parameters are: $u = 0.1, m = 0.15, r = 0.05, \sigma = 0.25, b = 0.3, \rho = 0.8, y = 2, T = 1yr, t = 0.5, Q_0 = 0, z_0 = 1, P_0 = 1, S_0 = 1, \alpha = 1, h = 0.1, g = 0.55$. Then $X_0 = 0.1, \alpha S_0 = 1, W_0 = 1.1$.