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# Agency Conflicts and Risk Management\*

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## Abstract

This paper analyzes the relation between agency conflicts and risk management in a contingent claims model of the firm. In contrast to previous contributions, our analysis incorporates not only stockholder-debtholder conflicts but also manager-stockholder conflicts. In particular we consider a setting in which the manager derives perquisites from investment and overinvests when the firm has free cash flow. The paper shows that when investment policy is endogenous and depends on the selected financing and hedging policies, shareholders generally have no incentives to engage in asset substitution once debt has been issued. The paper also examines managers' risk-shifting incentives and provides an analysis of the benefits associated with risk management in different economic environments.

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## I. Introduction

Since the seminal paper by Jensen and Meckling (1976) agency conflicts have received considerable attention in financial economics. One of the most cited result in this literature is that the option-like nature of equity may induce shareholders in a levered firm to increase risk at the expense of debtholders. This particular conflict of interests between shareholders and debtholders is typically referred to as *the asset substitution problem* and has been examined in a number of papers on asset pricing or security design.

Despite the compelling theoretical arguments underlying its foundations, asset substitution lacks empirical support. Indeed, while some studies document the use bond covenants that reduce the asset substitution problem [see e.g. Chava, Kumar and Warga (2005)], a variety of others find relatively little support of asset substitution [see e.g. Titman and Wessels (1988)]. Moreover, a recent survey by Graham and Harvey (2001) finds little evidence that corporate executives are concerned with this potential conflict of interest when making financing decisions.

More generally, the empirical literature seems to find little support for most theories on risk management. One potential explanation for this lack of empirical support is that the models developed so far are very stylized and do not incorporate a number of determinants of hedging policies. In particular, while this literature has modelled shareholder-debtholder conflicts with care, it has not examined the impact manager-stockholder conflicts on risk management. This is relatively surprising since the recent “law and finance” literature following Shleifer and Vishny (1997) argues that the expropriation of shareholders by managers is at the core of agency conflicts in most countries. In this paper, we build a contingent claims model in which the manager behaves opportunistically and examine the impact of agency conflicts on financing and hedging policies. Our analysis demonstrates that the relation between risk management and debt financing is more complex than prior discussions have recognized. It also suggests reinterpretation of some of the empirical evidence on corporate risk management.

Specifically, we consider a setting in which the manager derives perquisites from investment and overinvests when the firm has free cash flow. In this environment, we examine the impact of the opportunistic behavior of the manager on financing and hedging decisions. In the paper, we presume that investment policy is not contractible for two reasons. First, it is difficult to specify the array of projects that will constitute the firm’s investment opportunity set in the future. Second, a commitment to take positive NPV projects and reject negative NPV projects would depend on managers’ proprietary information and, for that reason, would not be verifiable. However, financing and hedging are observable and

contractible; the board has to approve financial decisions. As a result, while managers typically have discretion over investment policy, shareholders generally have discretion over financing and hedging policies. Based on this allocation of control rights, the paper derives valuation formulas for corporate debt and equity. These expressions are then used to analyze the interaction between investment, financing, and hedging policies.

One of our main contributions in this paper is to show that financing and hedging policies affect not only shareholder-debtholder conflicts but also manager-shareholder conflicts. Following Jensen (1986), several authors have recognized the role of debt in limiting the manager's ability to invest in negative NPV projects. However, the role of other financial policies in controlling the free cash flow problem generally has not been explored. We argue that hedging not only can control shareholders' underinvestment incentives but managers' ability to overinvest as well. While there exists a number of papers that address issues relating to investment choices and risk management, these papers have more narrowly focused on the impact of risk management on underinvestment costs [see Mayers and Smith (1982, 1987), Bessembinder (1991), and Froot, Scharfstein, and Stein (1993)]. Within our model, optimal leverage reflects a trade-off between under- and overinvestment costs. Risk management improves firm value and affects optimal leverage by controlling both of these costs.

In section II, we present our basic model. Overinvestment results from owner-manager conflicts; underinvestment results from bondholder-stockholder conflicts. We examine the impact of financial policies on investment policy and show that risk management controls both overinvestment and underinvestment problems. In section III, we examine the ability to commit to a specific hedging policy using an intertemporal version of the basic model in the spirit of Leland (1994, 1998). We show that cash flow volatility is costly for shareholders since it induces distortions in investment policy. Thus, when shareholders have discretion over financial policies, they generally have incentives to maintain the firm's hedging policy once debt has been issued. When managers have discretion over financial policies, managers' risk-shifting incentives balance two opposite effects of hedging on the firm's investment policy. First, hedging allows for a better control of the free cash flow problem and thus reduces the level of investment in the short run. Second, hedging reduces the firm's default probability and thus increases the level of investment in the long run. We show that even though hedging constrains investment policy in the short run, managers' commitment to hedge cash flows, and thus to credit enhancement, is credible. In section V, we examine the benefits associated with risk management in different economic environments. We offer our conclusions in section VI.

## II. The basic model

This section presents a model that captures the conflicts of interest between management and shareholders within a particularly simple setting. Throughout the section, we assume that agents are risk neutral and that equilibrium interest rates are zero. We construct a two-period model and, hence, focus on three dates. To finance an initial project at date  $t = 0$ , a firm acquires external funding either from shareholders or from bondholders. This initial project yields a cash flow at  $t = 1$  which is positively related to the price of a commodity, that we denote by  $x$ . Notably, the firm cash flow is given by

$$f(x) = x - c,$$

where  $x = \mu + \phi$ ,  $\phi$  is uniformly distributed on  $[-e, e]$ , and  $c$  is a constant cost. The cash flow from this project may be reinvested at  $t = 1$ . Following Morellec (2004) and Barclay et al. (2005), we presume that the marginal product of investment is decreasing and given by a step function. Specifically, the payoff at  $t = 2$  from investment is given by  $H$ ,  $H > 1$ , per unit for the first  $I^*$  units and  $L$  per unit in excess of  $I^*$ ,  $L < 1$ . Thus, investment in the first type of project has a positive NPV whereas investment in the second type has a negative NPV.

Although manager-stockholder conflicts may take different forms, we introduce such conflicts by presuming that the manager receives private benefits from investment. We consider an extreme setting in which it always is optimal for the manager to invest. We also presume that perquisites increase with the projects' NPV. Thus, the manager exhausts available positive NPV projects before investing in negative NPV projects. This specification allows us to capture Jensen's idea that the overinvestment problem is more severe for firms with larger cash flows [see Harford (1999) for empirical evidence].

Agency costs of managerial discretion typically depend on the allocation of control rights within the firm. Throughout the paper, shareholders are represented by a board of directors that acts in their best interests. The board cannot dictate investment policy because investment decisions, although observable, are not verifiable and, thus, are not contractible.<sup>1</sup> Therefore, the board has to delegate decision-making authority with respect to investment. However, financing policy is observable and contractible; the board has to approve financing decisions. Thus, the board can use its control of financing policy to constrain investment policy.<sup>2</sup>

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<sup>1</sup>To capture this aspect of the contracting process in a more realistic way, we consider in the Appendix a setting in which the number of positive NPV projects is unobservable to investors.

<sup>2</sup>See Zwiebel (1996) and Morellec (2004) for an analysis of the role of the external market for corporate control in a set up similar to ours.

## A. Firm valuation and optimal leverage

We start by analyzing financing policies. The current value of the firm is the sum of the values of its assets in place and its investment opportunities. Since investment decisions are not contractible, the value of the firm's investment opportunities depends on the investment policy selected by the manager and, thus, on the level of the resources available for investment. As Jensen (1986) argues, debt can control the free cash flow problem by limiting these resources.

We assume that debt issued at date  $t = 0$  matures at date  $t = 1$ . Proceeds from the debt issue may be paid as a dividend at date  $t = 0$  or used to finance assets in place. Within our model, debt financing affects firm value in two ways. First, by reducing the resources available for investment at  $t = 1$ , debt constrains the manager from investing in negative NPV projects, thereby controlling the free cash flow problem. Second, by changing the default policy of the firm, debt reduces the number of states of nature where the firm invests in positive NPV projects, thereby inducing underinvestment.

We denote the values of corporate debt and equity when the firm has issued debt with face value  $D$  by  $d(D)$  and  $e(D)$ . Shareholders have limited-liability and thus the option to default on their obligations. The optimal policy for shareholders is to default when the expected (present) value of the cash flows to be received is less than the firm's outstanding liabilities. If the firm can raise outside equity to finance positive NPV investment decisions, the present value of the cash flows from assets in place and investment projects is  $x - c + I^*(H - 1)$  for  $x - c \leq I^*$ . As a result, it is optimal for shareholders to default whenever  $\phi < c + D - \mu - I^*(H - 1)$ . Thus, if absolute priority is enforced upon default, the values of equity and debt respectively satisfy

$$e(D) = \int_{c+D-I^*(H-1)-\mu}^e \frac{a + I^*(H-1)}{2e} d\phi + (L-1) \int_{D+I^*-\mu}^e \frac{a - I^*}{2e} d\phi,$$

and

$$d(D) = \int_{c+D-I^*(H-1)-\mu}^e \frac{D}{2e} d\phi + \int_{-e}^{c+D-I^*(H-1)-\mu} \frac{\mu + \phi}{2e} d\phi,$$

for  $a = \mu + \phi - c - D$  and where  $c + D - I^*(H - 1) > \mu - e$ . In this specification, the value of equity equals the cash flow from assets in place in the non-default states plus the NPV of the investment opportunities. This NPV depends on the firm's investment policy. Three cases are possible: (i) underinvestment for  $x \in [\mu - e, c + D - I^*(H - 1))$ , (ii) optimal investment for  $x \in [c + D - I^*(H - 1), c + D + I^*]$ , and (iii) overinvestment for  $\phi \in (c + D + I^*, \mu + e]$ .

Optimal capital structure is defined by  $D \in \arg \max_{[0,b]} v(D)$  where firm value satisfies  $v(D) = e(D) + d(D)$ . We then have the following result.

**Proposition 1** *The debt level that maximizes firm value satisfies*

$$D^* = \max \left\{ \mu + e - c - I^* \frac{H - L}{1 - L}, 0 \right\}. \quad (1)$$

*The higher the overinvestment cost (the lower  $L$ ), the higher optimal leverage. The higher the underinvestment cost (the higher  $H$ ), the lower optimal leverage.*

As argued by Jensen (1986), debt policy can control the free cash flow problem. However, standard debt contracts promise the same payoff in all the states of nature. As a result, although debt financing reduces overinvestment when cash flows are high, it also induces underinvestment when the firm's cash flow are low. While several authors have recognized the role of debt in controlling the free cash flow problem, the role of other financial policies generally has not been explored. We now turn to analyzing the impact of hedging policy on investment policy and firm value.

## B. Risk management and investment policy

The Modigliani-Miller theorem implies that hedging policy increases firm value if it lowers taxes, reduces contracting costs, or improves investment policy. Mayers and Smith (1987) and Bessembinder (1991) show that hedging can control the underinvestment problem identified by Myers (1977); Froot, Scharfstein and Stein (1993) demonstrate that hedging can reduce the underinvestment cost associated with financing constraints. In this literature, the agency cost of free cash flow has been largely ignored.

To motivate the following analysis, we start by examining the impact of cash flow volatility on investment policy. For any given debt policy, firm value is given by

$$v(D) = \mu - c + I^*(H - 1) \frac{\mu + e + I^*(H - 1) - c - D}{2e} + (L - 1) \frac{(\mu + e - I^* - c - D)^2}{4e},$$

and is maximized for  $D = D^*$ . Using equations (1) and (2), firm value at optimal leverage is

$$v(D^*) = \mu - c + \frac{(H - 1)(I^*)^2}{2e} \left[ H + \frac{1}{2} \frac{H - 1}{1 - L} \right].$$

The second term on the right hand side of this equation decreases with the variance of cash flows from assets in place, as reflected by  $e$ . In other words, higher volatility in operating cash flows implies larger costs of both overinvestment and underinvestment. Thus, firm value increases when cash flow volatility is reduced

To illustrate the impact of hedging policy on investment decisions, we consider that the firm has access to both debt financing and forward contracts in the form of bundled hedging. In particular, we presume that the firm can float hybrid debt contracts with promised payment  $\gamma + \theta x$ , with  $\gamma \geq 0$  and  $\theta \in [0, 1]$ . The case  $\theta = 0$  represents fixed-rate debt studied above. The case  $\theta > 0$  represents commodity-linked debt: Promised payment to debtholders depends on the current commodity price  $x$ .<sup>3</sup> We also assume that the firm simultaneously selects its hedging and financing policies as well as the dividend payments to shareholders. These are determined jointly before selection of investment  $I$ . These assumptions imply that it is not possible to precommit to an investment policy one period from now at date 1, but that it is possible to commit to hedging and financing policies now at date 0. (We examine the commitment issue in section III.)

As discussed earlier, standard debt contracts, while potentially reducing the cost of overinvestment, increase the probability of default and, thus, underinvestment costs. While the promised payments on fixed-rate bonds are state independent, the promised payment on the commodity-linked bond rises and falls with the price of the commodity and, therefore, with the firm's ability to pay. Thus, commodity-linked debt might allow the firm to better control the costs of overinvestment and underinvestment in comparison with traditional debt contracts. Assume that the firm issues debt with promised payment  $\gamma + \theta x$  to be repaid at  $t = 1$ . Firm value then is given by

$$v(\gamma, \theta) = \mu - c + I^*(H - 1) \frac{(\mu + e)(1 - \theta) + I^*(H - 1) - c - \gamma}{2e(1 - \theta)} - (1 - L) \frac{[(\mu + e)(1 - \theta) - (c + \gamma + I^*)]^2}{4e(1 - \theta)}.$$

Using simple algebra, it is possible to show that for any  $\theta \in \left[0, 1 - \frac{I}{2e}\left(H + \frac{H-1}{2(1-L)}\right)\right)$ , the optimal fixed-rate payment on the hybrid debt contract satisfies

$$\gamma^* = \max \left\{ (\mu + e)(1 - \theta) - c - I^* \frac{H - L}{1 - L}, 0 \right\}.$$

When the manager implements this policy, firm value satisfies

$$v(\gamma^*, \theta) = \mu - c + \frac{(H - 1)(I^*)^2}{2e(1 - \theta)} \left[ H + \frac{1}{2} \frac{H - 1}{1 - L} \right] > v(D^*).$$

From this expression, one can see that the value of the firm with hedging,  $v(\gamma^*, \theta)$ , is larger than the value of the firm without hedging,  $v(D^*)$ . Therefore, by engaging in hedging activities, the manager increases firm value.

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<sup>3</sup>The analysis generally applies to any type of security with cash flows indexed to an exogenous source of risk, such as floating rate debt or foreign currency denominated debt.

The above arguments show that debt financing using hybrid securities reduces the manager's ability to overinvest as well as shareholders' incentives to underinvest. By combining standard debt contracts with forward contracts, hybrid debt financing increases firm value in comparison with standard debt financing. One question that arises naturally when dealing with agency conflicts is the ability to precommit to particular policy choices. Although standard debt covenants allow the manager to precommit to hedging through the use of insurance contracts, little progress has been made in the development of covenants that allow the firm to precommit to manage interest rate, foreign currency or commodity risk using forward, futures, options or swaps over the life of the debt issue. This suggests that one reason managers employ hybrid debt is that such instruments bundle the hedging instrument with the debt, thus precommitting the firm to maintain the position over the life of the issue. The next section examines the ability to precommit to a specific hedging policy within an intertemporal version of the above model.

### III. The intertemporal model

Throughout this section, we presume that the default-free term structure is flat with an instantaneous risk-free rate  $r$ , at which investors may lend and borrow freely. Time is continuous and defined over  $[0, \infty)$ . The per period operating income of the firm is given by

$$f(x_t) = x_t - c, \tag{2}$$

where changes in the commodity price are governed under the risk-neutral probability measure  $Q$  by the process

$$dx_t = (r - \delta) x_t dt + \sigma x_t dZ_t, \quad x_0 > 0. \tag{3}$$

In this equation,  $(Z_t)_{t \geq 0}$  is a standard Brownian motion,  $\delta > 0$  is the convenience yield from owning an additional unit of the commodity, and  $\sigma$  is a constant volatility parameter.

At each time  $t$ , cash flows from assets in place may be reinvested in new projects. As in section II, we denote the number of positive NPV projects available to the firm by  $I^*$  and presume that the marginal product of investment is  $H > 1$  per unit for the first  $I^*$  units and  $L < 1$  per unit for investment levels beyond  $I^*$ . Although the firm may operate its assets forever, it can also choose to abandon operations. By abandoning operations, the firm loses its investment opportunities. Moreover, we consider that the abandonment decision is irreversible. As a result, this decision may be viewed as the decision to liquidate the firm. We denote the abandonment value of the firm by  $A(x_t)$  and the proportional default costs by  $\alpha$ .

## A. Financial policies and firm value

As in section II, we consider hybrid debt contracts that are characterized by (i) a perpetual flow of coupon payments  $\gamma + \theta x_t$  and (ii) a commitment that if the firm defaults on its obligations, it is immediately liquidated.<sup>4</sup> Within the present model, debt financing has three effects on firm value. First, debt increases the default probability of the firm and the present value of bankruptcy costs. Second, debt induces underinvestment in positive NPV projects. Third, debt reduces the resources available for investment and thus improves the investment policy of the firm when the commodity price is high.

We denote the values of equity and debt by  $e(x, \gamma, \theta)$  and  $d(x, \gamma, \theta)$ . Because of the limited liability principle, shareholders have the option to default on their debt obligations. We denote the default threshold by  $x_d$ , the first passage time of  $(x_t)_{t \geq 0}$  at  $y$  by  $T_y$ , and the expectation operator associated with the probability measure  $Q$  conditional on the commodity price starting at the level  $x_0$  by  $E_Q^{x_0}$ . Assuming that absolute priority is enforced upon default, we can write the value of equity as:

$$e(x, \gamma, \theta) = E_Q^{x_0} \left\{ \int_0^{T_{x_d}} e^{-rt} [(1 - \theta)x_t - c - \gamma + (H - 1)I^*] dt \right\} + E_Q^{x_0} \left\{ \int_0^{T_{x_d}} e^{-rt} (L - 1) [(1 - \theta)x_t - c - \gamma - I^*] \mathbf{1}_{(1 - \theta)x_t > \gamma + c + I^*} dt \right\}, \quad (4)$$

where the second term on the right hand side accounts for the cost of overinvestment. The value of corporate debt is then equal to the the sum of the cash flows that debtholders receive outside of default and their cash flow in default:

$$d(x, \gamma, \theta) = E_Q^{x_0} \left\{ \int_0^{T_{x_d}} e^{-rt} (\gamma + \theta x_t) dt \right\} + (1 - \alpha) A(x_d) E_Q^{x_0} [e^{-rT_{x_d}}], \quad (5)$$

where  $\mathbf{1}_\omega$  is the indicator function of the event  $\omega$  and  $\alpha$  accounts for bankruptcy costs.

Solving these equations yields

$$e(x, \gamma, \theta) = \frac{(1 - \theta)x_0}{\delta} - \frac{(1 - \theta)x_d}{\delta} \left( \frac{x}{x_d} \right)^\vartheta + \frac{(H - 1)I^* - \gamma - c}{r} \left[ 1 - \left( \frac{x_0}{x_d} \right)^\vartheta \right] + \frac{(L - 1)\sigma((1 - \theta)x_0)^\xi (c + \gamma + I^*)^{1 - \xi}}{\lambda(\lambda - \sigma - b)(\lambda - b)} \left[ 1 - \left( \frac{x_0}{x_d} \right)^{\vartheta - \xi} \right], \quad (6)$$

and

$$d(x, \gamma, \theta) = \frac{\theta x_0}{\delta} + \frac{\gamma}{r} - \left( \frac{\theta x_d}{\delta} + \frac{\gamma}{r} \right) \left( \frac{x_0}{x_d} \right)^\vartheta + (1 - \alpha) A(x_d) \left( \frac{x_0}{x_d} \right)^\vartheta, \quad (7)$$

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<sup>4</sup>Fan and Sundaresan (2000) analyze the effects of debt renegotiation on the valuation of corporate securities and leverage choices.

where  $b = (r - \delta - \sigma^2/2) / \sigma$ ,  $\lambda = \sqrt{2r + b^2}$ ,  $\vartheta = -(b + \lambda) / \sigma$ , and  $\xi = -(b - \lambda) / \sigma$ . The first three terms on the right hand side of equation (6) represent the expected present value of the cash flows from assets in place and positive NPV projects. The fourth term is the cost of overinvestment. The first two terms of the right hand side of equation (7) represent the expected present value of the coupon payments until the default time. The third term accounts for the residual value of the firm upon default.<sup>5</sup>

We consider a stock-based definition of default whereby it is optimal for shareholders to default on their debt obligations when equity is worthless. This stock-based definition implies that the firm is insolvent on a cash flow basis at the default date. Therefore, shareholders have control rights over the decision to default. The first-best default policy for shareholders consists in selecting the default threshold that maximizes equity value. Thus, the default threshold satisfies the smooth-pasting condition [see Dumas (1991)]:

$$\left. \frac{\partial e(x, \gamma, \theta)}{\partial x} \right|_{x=x_d} = 0. \quad (8)$$

As before, the debt contract selected by shareholders maximizes the sum of the proceeds from the debt issue and the value of equity after debt has been issued. This debt contract solves

$$\max_{\{\gamma, \theta\}} v(x, \gamma, \theta) \quad (9)$$

subject to  $\{\gamma, \theta\} \in \mathbb{R}_+^2$  and  $v(x, \gamma, \theta) = e(x, \gamma, \theta) + d(x, \gamma, \theta)$ . This specification shows that while the default threshold is selected ex post to maximize equity value, the initial financing decision is determined ex ante to maximize firm value.

## B. Analysis

This section examines the predictions of the model for financial policies, investment policy and firm value. Thereafter, input parameter values are set as follows:<sup>6</sup> the risk-free interest rate  $r = 5.375\%$ , the convenience yield on the commodity price  $\delta = 2.5\%$ , the volatility of

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<sup>5</sup>These equations incorporate the benefits (reduced default risk and improvement in investment policy) associated with the hedge implicit in the commodity-linked bond over the life of the debt issue. Thus, this formulation presumes that there are incentives within the firm to maintain the firm's hedging policy after debt has been issued. Section III.C. provides an analysis of these incentives.

<sup>6</sup>These parameters roughly reflect a typical Standard and Poor's 500 firm. The risk-free rate is equal to the 30-year rate on treasury bonds quoted on June 18, 2005. The value of the volatility parameter has been chosen to match the volatility of equity returns in the US economy. Using Itô's lemma, this volatility can be computed as:  $vol(de/e) = (\partial e / \partial x)(x/e)\sigma$ . The convenience yield on the commodity price is consistent with a lease rate on metals such as gold. Bankruptcy costs are at the upper bound of recent estimates of Andrade and Kaplan (1998). Mello and Parsons (1992) assume that after default the new owners of the

returns on the commodity price  $\sigma = 25\%$ , the initial commodity price  $x_0 = 5$ , bankruptcy costs  $\alpha = 25\%$ , and the abandonment value of the firm  $A(x) = A = 10$ . The number of growth options available to the firm is  $I^* = 4$ . The return on investment is given by  $H = 1.1$  per unit for the first  $I^*$  units, and  $L = 0.8$  for investment levels beyond  $I^*$  as in Morellec (2004). The effects of changing these parameters are also examined.

## 1. Debt financing, hedging and firm value

We first start by analyzing the impact of the three financing alternatives (equity, straight debt, hybrid debt) on firm value. Figure 1 plots firm value for the three financing alternatives as a function of the number of growth options available to the firm  $I^*$  when the value-maximizing amount of each debt instrument is floated. The solid line represents a firm issuing equity. The short-dashed line represents a firm issuing fixed-rate debt. The long-dashed line represents a firm issuing commodity-linked debt.

[Insert Figure 1 Here]

Figure 1 shows that the impact of financial policies on firm value depends on the number of growth options available to firm. When the firm has few growth options, the cost of underinvestment is low while the potential cost of overinvestment is high. Issuing debt increases firm value by allowing the firm to reach a better trade-off between these costs. With additional growth options, the likelihood of overinvestment decreases, and thus the free cash flow benefits of debt fall. At the same time, the cost of underinvestment is larger. Thus, the improvement in firm value associated with debt financing decreases as the number of growth option rises.

Figure 1 also reveals that hedging creates value above and beyond debt financing. In particular, firm value is higher with commodity-linked debt than with fixed-rate debt. In fact, given any fixed-rate bond and any commodity price, it is always possible to identify a commodity-linked bond that Pareto dominates (for financial stakeholders) the fixed-rate bond, as we now illustrate [see also Mello and Parsons (1989)].

[Insert Table 1 Here]

In Table 1 we list, for a range of current commodity prices, the commodity-linked bond that is bondholder-equivalent to the original fixed-rate bond. We also list the (ex post)

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firm continue to employ the assets in their current use. Using these parameter values and assuming that the debt has been issued at par, the debtholder bankruptcy recovery rate is 55.9%. This value is consistent with the recovery rate on corporate bonds reported by Hamilton, Gupton, and Berthault (2001) for the period 1981-2000 or by Keenan, Shotgrin, and Sobehart (1999) for the period 1920-1998.

levered equity values given the fixed-rate bond and given the commodity-linked bond. The difference between the value of the equity levered with the commodity-linked bond and levered with the fixed-rate bond is a direct measure of the relative deviations from value maximization associated with the two forms of debt financing. As illustrated by the Table, although the fixed-rate and commodity-linked instruments are bondholder-equivalent in the sense that  $d(x, \gamma, 0) = d(x, 0, \theta)$ , they are not equivalent for shareholders. Because of the conflicts of interests within the firm, the Modigliani-Miller theorem does not obtain:  $e(x, \gamma, 0) < e(x, 0, \Theta^d(\gamma, x))$  and  $v(x, \gamma, 0) < v(x, 0, \Theta^d(\gamma, x))$ .

## 2. Debt financing, hedging and distortions in investment policy

The above analysis reveals that financial policies can have a significant impact on firm value. Within the present model, financing policies affect firm value through two different channels. First, financial policies interact with investment policy. Second, financial policies determine the credit quality of the firm. This section examines the impact of financial policies on investment policy. The next section focuses on the analysis of credit quality.

Promised payments on commodity-linked bonds rise and fall with the price of the commodity and, therefore, with the firm's ability to pay. Promised payments are high when the firm's resources are high and the cost of overinvestment is severe. Promised payments are low when the firm's resources are low and the cost of underinvestment is important. By contrast, promised payments on fixed-rate debt are independent of the contemporaneous commodity price. Thus, commodity-linked debt should allow the firm to reduce agency costs in comparison with a traditional debt contracts.

Figure 2 plots deviations from value-maximization in investment policy as a function of the number of growth options in the firm's investment opportunity set. The solid line represents an unlevered firm. The short-dashed line represents a firm that issues fixed-rate debt. The long-dashed line represents a firm that issues commodity-linked debt.

[Insert Figure 2 Here]

Figure 2 shows that distortions in investment policy account for 10% to 20% of firm value depending on the number of growth options available to the firm. In addition, figure 2 reveals that both commodity-linked debt and fixed-rate debt allow for better control of agency costs within the firm than equity financing. Finally, the figure also indicates that (1) commodity-linked debt is more effective than standard debt in controlling those costs, and (2) the superiority of fixed-rate debt over equity decreases with the number of growth options. Again this result arises because more growth options implies less overinvestment and more underinvestment.

### 3. Debt financing, hedging and credit risk

Default risk is an important determinant of both firm value and the magnitude of distortions in investment policy associated with each of the financing alternatives. One way to measure the impact of financial policies on credit risk is to compute the probability of default over  $\tau$  years and the average lifetime of the firm for each of the financing alternatives. Both of these measures are functions of the selected default threshold, which itself depends on the selected financing policy.

Consider two debt contracts satisfying the initial funding condition:  $d(x, \gamma, \theta) = D$ . The first contract is a fixed rate bond for which  $\theta = 0$ . The second contract is a commodity-linked bond for which  $\gamma = 0$ . Each of these contracts is associated with a specific default threshold, respectively denoted by  $x_d(\gamma, 0)$  and  $x_d(0, \theta)$ . The credit-enhancement associated with the use of commodity-linked debt can be measured by computing the relative improvement in the average lifetime of the firm and the default probability. These measures satisfy

$$\frac{E_Q^{x_0} [T_{x_d(0,\theta)}]}{E_Q^{x_0} [T_{x_d(\gamma,0)}]} = \frac{\ln(x_0) - \ln(x_d(0, \theta))}{\ln(x_0) - \ln(x_d(\gamma, 0))}$$

and

$$\frac{\mathcal{P}^{x_0} [T_{x_d(\gamma,0)} < \tau]}{\mathcal{P}^{x_0} [T_{x_d(0,\theta)} < \tau]},$$

where

$$\begin{aligned} \mathcal{P}^{x_0} [T_y < \tau] &= \mathcal{N} \left( \frac{\ln(x_0^{-1}y) - [\mu - \delta - \sigma^2/2] T}{\sigma\sqrt{T}} \right) \\ &\quad - \left( \frac{y}{x_0} \right)^{\frac{2(\mu-\delta)-\sigma^2}{\sigma^2}} \mathcal{N} \left( \frac{\ln(x_0^{-1}y) + [\mu - \delta - \sigma^2/2] T}{\sigma\sqrt{T}} \right), \end{aligned}$$

and  $\mu > \delta + \sigma^2/2$  stands for the mean total return on the commodity price under the physical probability measure  $\mathcal{P}$ .

Table 2 shows that both these quantities are larger than 1, reflecting the improvement associated with commodity-linked debt in both of these dimensions. The credit-enhancement is achieved because of the positive correlation between the payments to commodity-linked bondholders and the firm's cash flows, which provides the firm with two sources of gain. First, this positive correlation reduces default risk for any given default threshold. Second, it reduces the default threshold selected by shareholders by improving the firm's investment policy (and hence by increasing the value of the firm as a going concern). Below, we show that these measures of credit risk also play an important

role in the commitment to maintain the hedge implicit in the commodity-linked bond once debt has been issued.

[Insert Table 2 Here]

An alternative expression for the superiority of commodity-linked instruments relates to the increase in the firm's debt capacity [see Mello and Parsons (1989)]. In this paper, we define debt capacity as the maximum amount of debt that can be sold against the firm's assets. We report in Table 2 the fixed rate debt capacity of the firm for different environments. We also report the commodity-linked debt capacity of the firm. Because default risk is lower with commodity-linked debt, the debt capacity of the firm is greater for commodity-linked bonds than for fixed rate bonds. In the base case environment for example, using commodity-linked debt increases the firm's debt capacity by 36%.

### C. Risk-shifting incentives

The above analysis highlights the benefits of commodity-linked debt over fixed-rate debt. These benefits are present only if shareholders and the manager have incentives to maintain the hedge implicit in the commodity-linked bond after debt has been issued.

**Shareholders** Jensen and Meckling (1976) argue that since equity in a levered firm is equivalent to a call option on the firm's assets, shareholders have an incentive to increase risk once debt has been issued. If the firm has issued commodity-linked debt, shareholders can increase risk by unwinding the hedge implicit in the commodity-linked bond.

The analysis in section III.B. suggests that even though shareholders have incentives to increase risk in a Modigliani-Miller world, this might not be the case if financing decisions have an impact on investment policy. In particular, volatility in cash flows is costly for shareholders since it induces suboptimal investment decisions. High cash flows increase the resources available for investment and hence the potential cost of overinvestment. Low cash flows increase the probability of default and the cost of underinvestment. When these effects are greater than the potential wealth transfers arising from risk-shifting, shareholders have incentives to maintain the hedge implicit in the commodity-linked bond after debt has been issued.

To analyze shareholders' incentives to undo the hedge ex post, consider a commodity linked debt with promised payment:  $s(x_t) = \gamma + \theta x_t$ . This contract can be decomposed into a fixed-rate bond and a swap composed of forward contracts maturing at every point in time  $t \in (0, \infty)$ . In particular, it is possible to identify a swap with  $n$  packages of forward contracts with price  $f$  and a fixed-rate debt contract with coupon payment  $\gamma_f$

such that:  $s(x_t) = \gamma_f + n(x_t - f)$ . This duplication strategy is composed of  $\theta$  packages of forward contracts and a fixed-rate debt contract with coupon payment  $\gamma_f = \gamma + \theta f$  where  $f$  solves:

$$E_Q^{x_0} \left\{ \int_0^{T_{x_d}} e^{-rt} (x_t - f) dt \right\} = 0, \quad (10)$$

the solution to which is

$$f = \frac{r}{\delta} \left[ x_0 - x_d (x_0/x_d)^\vartheta \right] \left[ 1 - (x/x_d)^\vartheta \right]^{-1}.$$

The forward price  $f$  is such that the market value of the swap is equal to zero at the time it is issued. Thus, this equation reflects the credit risk of the firm. It also incorporates the fact that the counterparty to an unwinding transaction would be junior to existing bondholders in bankruptcy [see Tucker (1991)] and that it is optimal for shareholders to default when the value of the firm's assets is less than the value of corporate debt.

Table 3 examines shareholders' incentives to undo the hedge implicit in the commodity-linked bond once debt has been issued. In this table, we report the values of the debt and equity claims if the firm maintains the hedge implicit in the commodity linked bond. We also report the values of these securities if shareholders unwind the hedge implicit in the commodity linked bond by taking long positions in forward contracts.

[Insert Table 3 Here]

Simulation results reported in Table 3 show that unwinding the hedge reduces the value of corporate debt and that the relative reduction in debt value increases with the firm's credit risk (as the commodity price decreases). Data in Table 3 also suggest that, even though unwinding the hedge allows for wealth transfers between bondholders and shareholders, the latter are worse off if they undo the hedge implicit in the commodity-linked debt. This result arises in the present model for two reasons. First, commodity-linked allows for a better control of deviations from value maximization in investment policy. Second, unwinding the hedge increases the commodity price at which the firm defaults, thus limiting the potential gains associated with risk-shifting. This in turn implies that shareholders' commitment to hedge cash flows, and thus to credit enhancement, is credible.

**Managers** The above analysis reveals that when shareholders have control rights over financing policies, the hedging commitment is credible since shareholders have incentives to maintain the hedge after the issuance of debt. When the manager has decision rights over the firm's financial policies, the commitment issue is related to managerial utility.

Suppose that the manager owns a fraction  $\varphi$  of the firm's equity. His expected utility is then given by

$$m(x, \gamma, \theta) \equiv \varphi e(x, \gamma, \theta) + \phi E_Q^{x_0} \left\{ \int_0^{T_{x_d}} e^{-rt} [I^* + (x_t - \theta x_t - c - \gamma - I^*)^+] dt \right\},$$

where the second term on the right hand side represent the perquisites from investment and  $x^+ = \max(0, x)$ . As before, the manager has an incentive to unwind the hedge implicit in the commodity-linked bond if  $m(x, \gamma, \theta) > m(x, \gamma_f, 0)$  where  $\gamma_f \equiv \gamma + \theta f$ .

If the manager derives utility from both a compensation package and the perquisites from investment, then undoing the hedge changes both the perquisites derived from investment and the value of the firm's equity. Simulation results reported in Table 3 show that in this case the manager never finds it optimal to undo the hedge. This comes from two separate effects. First, undoing the hedge reduces equity value, and hence the value of the manager's compensation package. Second even though commodity-linked debt better controls the level of investment when the firm's cash flows are high, it is also associated with a larger average lifetime for the firm. This larger average lifetime implies that a larger number of positive NPV projects will be undertaken by the firm, which partly offsets the constraint put on the manager by the hybrid debt instrument. Thus, even though hedging constrains investment policy in the short run, it reduces the probability of default and thus increases the investment level in the long run. This analysis reveals an important *intertemporal effect* of a firm's hedging policy. This in turn implies that managers' commitment to hedge cash flows, and thus to credit enhancement, is credible.

#### D. Supply-side analysis

The above analysis emphasizes the benefits of commodity-linked debt over fixed-rate debt, showing that hedging creates value above and beyond debt financing. In this section, we relax some of the assumptions made earlier to understand which firms are more likely to engage in hedging transactions.

**No access to external funds** There is evidence suggestive of great heterogeneity in access to external funds across firms and that this heterogeneity is important in explaining the investment behavior of firms. This heterogeneity should also be reflected in the firm's choice of a financial policy, as we now illustrate.

Suppose that the firm has no access to external funds. In that case the amount invested in new projects is capped by the cash flows from assets in place and default is triggered

by a liquidity constraint. Thus, the default threshold  $x_f$  is defined by  $(\pi - \theta)x_f \equiv c + \gamma$  and firm value satisfies (see the Appendix)

$$\begin{aligned}
v(x, \gamma, \theta) = & \left( \frac{\pi x_0}{\delta} - \frac{c}{r} \right) + \left[ (1 - \alpha) A(x_f) - \left( \frac{\pi x_f}{\delta} - \frac{c}{r} \right) \right] \left( \frac{x_0}{x_f} \right)^\vartheta \\
& + (H - 1) \left[ \frac{(\pi - \theta)x_0}{\delta} - \frac{c + \gamma}{r} - \left( \frac{(\pi - \theta)x_f}{\delta} - \frac{c + \gamma}{r} \right) \left( \frac{x_0}{x_f} \right)^\vartheta \right] \\
& + (L - H) \frac{\sigma [(\pi - \theta)x_0]^\xi (c + \gamma + I^*)^{1-\xi}}{\lambda(\lambda - \sigma - b)(\lambda - b)} \left[ 1 - \left( \frac{x_0}{x_f} \right)^{\vartheta - \xi} \right]
\end{aligned} \tag{11}$$

Equation (11) admits the following interpretation. The first term on the right hand side is the present value of a perpetual entitlement to the flow of income from the firm's assets. The second term represents the change in firm value occurring upon default weighted by the present value of \$1 contingent on default. The third term is the expected present value of the firm's growth options. This term presumes that shareholders will get a gross rate of return of  $H$  on investment. The fourth term corrects the third term to reflect the change in the marginal product of investment occurring at  $I^*$ .

Consider for example that the firm needs an initial amount of external finance of \$10 that can be raised by issuing either fixed-rate debt ( $\gamma = 0.898$ ) or commodity-linked debt ( $\theta = 0.0448$ ). Firm value with fixed-rate debt is then equal to \$109.2 whereas it is equal to \$132.5 with commodity-linked debt. Again this difference in value arises from an improvement in both credit quality and investment policy. In particular, issuing commodity-linked debt increases the expected lifetime of the firm by 70%. It also reduces deviations from value maximization in investment policy from 18.8% to 14.0% of firm value. (Note that the dollar amount of deviations is essentially the same reflecting the enhancement in the firm's average lifetime.) These results suggest that commodity-linked debt, and thus hedging, should be particularly valuable for firms with limited access to external capital markets.

**Correlated cash flows and investment opportunities** The above analysis presumes that cash flows from assets in place and growth options are not related. Yet, a number of firms have investment projects whose profitability is related to that of assets in place. For example, while the contemporaneous gold price determines the level of cash flows from assets in place for a gold mining firm, it also determines the economic rents associated with the development of new mines. In particular, a number of the proven reserves become profitable only when the gold price is high enough to offset either large entry costs or the

value of waiting to invest. This suggests that for such firms the number of positive NPV projects is positively related to the cash flows from assets in place.

Consider for example that at any date  $t$  the number of growth options in the firm's investment opportunity set is given by

$$I^* = \rho_I x_t + I, \quad (12)$$

where  $\rho_I \in (0, 1)$  and  $I$  is a positive constant. In order to analyze in a simple way the value implications of the correlation between cash flows and investment opportunities, assume that the dynamics of the commodity price are

$$dx_t = \sigma x_t dZ_t, \quad x_0 > 0. \quad (13)$$

This specification implies that the expected number of growth options is  $E(I^*) = \rho_I x_0 + I$ . Setting  $I = E(I^*) - \rho_I x_0$ , allows for an analysis of the impact of the correlation coefficient  $\rho$  on firm value for firms having the same unconditional number of growth options  $E(I^*)$  (even though  $\rho_I$  varies across firms). In this setting, the values of equity and debt satisfy equations (4) and (5) with  $I^*$  replaced everywhere by equation (12). The solution to equation (5) is unaffected. The value of equity is given in the Appendix.

Figure 3 plots firm value as a function of the correlation coefficient between cash flows from assets in place and investment opportunities. Three financing alternatives are considered: Equity, fixed-rate debt and hybrid debt. The solid line represents equity financing. The short-dashed line represents a firm issuing debt with constant contractual payments. The long-dashed line represents a firm using commodity linked debt. The figure represents firm value when the value-maximizing amount of each of these instruments is floated.

[Insert Figure 3 Here]

Figure 3 indicates that firm value increases with the correlation coefficient between cash flows and investment opportunities. This result arises because the correlation between the cash flows from assets in place and the firm's growth options acts as a natural hedge that limits deviations from value-maximization in investment policy. In particular, the firm has more growth options when cash flows from assets in place are high and the likelihood of overinvestment is greater. The firm has fewer growth options when cash flows are low and the likelihood of underinvestment is greater. Because of this natural hedge, potential improvements in firm value associated with a specific financing policy are lower. This effect is illustrated in Figure 3, which shows that the improvement in firm value due to financing and hedging policies decreases as the correlation coefficient  $\rho$  increases.

## IV. Conclusion

We present a model that highlights the relation between agency conflicts and risk management. We show that the costs of both underinvestment and overinvestment are essential in determining the firm's hedging policy. In particular, firms that derive more of their value from assets in place (lower market-to-book ratios), although having lower costs of underinvestment, generally display larger costs of overinvestment. Thus, they may be more likely to hedge to control these overinvestment incentives. Along the same line of reasoning, firms with higher leverage usually have lower agency costs of free cash flow and, hence, weaker incentives to hedge to control these costs. This suggests that the relation between the willingness of the firm to hedge or the extent of hedging and the sources of value within the firm is more complex than prior discussions have recognized.

The paper also has implications for the asset substitution problem. In particular, it reveals that because greater volatility in cash flows induces greater distortions in investment policy, shareholders generally have incentives to maintain the firm's hedging policy after debt has been issued. That is, manager-stockholder conflicts help mitigate stockholder-bondholder conflicts. Also, when managers have control rights over financial policy, their incentives to maintain the firm's hedging policy increase with the number of growth options available to the firm, and thus with the firm's market-to-book ratio. Finally, the paper provides a number of comparative statics that emphasize the impact of a firm's economic environment on the benefits associated with risk management.

## Appendix

### A. Stochastic investment opportunities

Assume that the number of positive NPV projects available to the firm at  $t = 1$ , denoted by  $I$ , is uniformly distribution on  $[\underline{I}, \bar{I}]$ ,  $\underline{I} \geq 0$ ,  $\bar{I} \leq \mu + e$ . Moreover, suppose that investors know the distribution of  $I$  but cannot observe its realization. Because the number of growth options is unobservable and the manager always wants to invest, any claim by management that cash flows are too low to finance positive NPV projects is not credible. Therefore, firm value satisfies

$$v(D) = \int_{-e}^e \frac{\mu + \phi - c}{2e} d\phi + (L-1) \int_{\underline{I}}^{\bar{I}} \int_{c+D+I-\mu}^e \frac{\mu + \phi - c - D - I}{2e(\bar{I} - \underline{I})} d\phi dI \quad (\text{A.1})$$

$$+ (H-1) \int_{\underline{I}}^{\bar{I}} \frac{1}{\bar{I} - \underline{I}} \left[ \int_{c+D-\mu}^{c+D+I-\mu} \frac{\mu + \phi - c - D}{2e} d\phi + \int_{c+D+I-\mu}^e \frac{I}{2e} d\phi \right] dI$$

Using the same line of reasoning as in the main text, it is possible to show that the selected debt level is given by  $D^* = \max \left\{ \mu + e - c - E(I) \frac{H-L}{1-L}, 0 \right\}$ .

### B. Values of corporate securities

We assume throughout that the initial value of the commodity price is such that  $f(x_0) \leq I^*$ . The value of the equity is then given by

$$e(x, \gamma, \theta) = E_Q^{x_0} \left\{ \int_0^{T_{x_d}} e^{-rt} [(1-\theta)x_t - c - \gamma + (H-1)I^*] dt \right\} \quad (\text{B.1})$$

$$+ E_Q^{x_0} \left\{ \int_0^{T_{x_d}} e^{-rt} (L-1) [(1-\theta)x_t - c - \gamma - I^*] \mathbf{1}_{(1-\theta)x_t > \gamma + c + I^*} dt \right\}$$

Using standard results concerning Brownian motion, it is possible to show that the first term on the right hand side is equal to

$$(1-\theta) \left[ \frac{x_0}{\delta} - \frac{\pi x_d}{\delta} \left( \frac{x}{x_d} \right)^\vartheta \right] - \frac{c + \gamma - (H-1)I^*}{r} \left[ 1 - \left( \frac{x}{x_d} \right)^\vartheta \right], \quad (\text{B.2})$$

Consider next the second term. We have

$$E_Q^{x_0} \left\{ \int_0^{T_{x_d}} e^{-rt} [(1-\theta)x_t - c - \gamma - I^*] \mathbf{1}_{(1-\theta)x_t > \gamma + c + I^*} dt \right\} \quad (\text{B.3})$$

$$= (1-\theta) E_Q^{x_0} \left\{ \int_0^{T_{x_d}} e^{-rt} x_t \mathbf{1}_{(1-\theta)x_t > \gamma + c + I^*} dt \right\}$$

$$- (s + \gamma + I^*) E_Q^{x_0} \left\{ \int_0^{T_{x_d}} e^{-rt} \mathbf{1}_{(1-\theta)x_t > \gamma + c + I^*} dt \right\}.$$

Consider the first integral on the right hand side of (B.4). Writing  $Z_t^b = Z_t + bt$  with  $b = \frac{1}{\sigma} (r - \delta - \sigma^2/2)$  and  $a = \frac{1}{\sigma} \ln \left( \frac{c+\gamma+I^*}{(1-\theta)x_0} \right)$ , gives

$$(1 - \theta) E_Q^{x_0} \left\{ \int_0^{T_{Z_d}} e^{-rt} x_t \mathbf{1}_{f(\delta)(x_t) > I^*} dt \right\} = (1 - \theta) x_0 E_Q^{x_0} \left\{ \int_0^{T_{Z_d}} e^{-rt} e^{\sigma Z_t^b} \mathbf{1}_{Z_t^b > a} dt \right\}, \quad (\text{B.4})$$

where  $Z_d = \frac{1}{\sigma} \ln \left( \frac{x_d}{x_0} \right)$  and  $T_{Z_d} = \inf \{t \geq 0 : Z_t^b = Z_d\}$ . Applying Cameron-Martin-Girsanov theorem with  $dQ/dP|_{\mathcal{F}_t} = e^{bZ_t - \frac{b^2}{2}t}$  yields

$$(1 - \theta) x_0 E_P^{x_0} \left\{ \int_0^{+\infty} e^{(\sigma+b)Z_t^b} e^{-\frac{\lambda^2}{2}t} \mathbf{1}_{Z_t^b > a} dt \right\} - (1 - \theta) x_0 E_P^{x_0} \left\{ \int_{T_{Z_d}}^{+\infty} e^{(\sigma+b)Z_t^b} e^{-\frac{\lambda^2}{2}t} \mathbf{1}_{Z_t^b > a} dt \right\}, \quad (\text{B.5})$$

where  $\lambda = \sqrt{2r + b^2}$  and  $(Z_t^b, t \geq 0)$  is a standard Brownian motion under  $P$ . To compute these integrals, we use the following lemma [see Karatzas and Shreve (1991) pp.272].

**Lemma:** *If  $f : \mathbb{R} \rightarrow \mathbb{R}$  is a piecewise continuous function with*

$$\int_{-\infty}^{+\infty} |f(x+y)| e^{-|y|\sqrt{2\gamma}} dy < \infty; \forall x \in \mathbb{R},$$

*for some constant  $\gamma > 0$ , and  $(Z_t, t \geq 0)$  is a standard Brownian motion, the resolvent operator of Brownian motion  $K_\gamma(f)$  is defined by*

$$K_\gamma(f) \equiv E \left\{ \int_0^{+\infty} e^{-\gamma t} f(Z_t) dt \right\} = \frac{1}{\sqrt{2\gamma}} \int_{-\infty}^{+\infty} f(y) e^{-|y|\sqrt{2\gamma}} dy. \quad (\text{B.6})$$

Using the above Lemma and the strong Markov property of Brownian motion, the second expectation in expression (B.5) can be computed as follows

$$\begin{aligned} & (1 - \theta) x_0 E_P^{x_0} \left\{ \int_{T_{Z_d}}^{+\infty} e^{(\sigma+b)Z_t^b} e^{-\frac{\lambda^2}{2}t} \mathbf{1}_{Z_t^b > a} dt \right\} \quad (\text{B.7}) \\ &= (1 - \theta) x_0 E_P^{x_0} \left\{ e^{-\frac{\lambda^2}{2}T_{Z_d}} \int_0^{+\infty} e^{(\sigma+b)(Z_t^b + Z_d)} e^{-\frac{\lambda^2}{2}t} \mathbf{1}_{Z_t^b > a - Z_d} dt \right\} \\ &= (1 - \theta) x_0 e^{(\sigma+b)Z_d - |Z_d|\lambda} E_P^{x_0} \left\{ \int_0^{+\infty} e^{(\sigma+b)Z_t^b} e^{-\frac{\lambda^2}{2}t} \mathbf{1}_{Z_t^b > a - Z_d} dt \right\} \\ &= \frac{(1 - \theta) x_0}{\lambda} e^{(\sigma+b)Z_d - |Z_d|\lambda} \int_{a - Z_d}^{+\infty} e^{-|y|\lambda} e^{(\sigma+b)y} dy \\ &= \frac{c + \gamma + I^*}{\lambda(\lambda - \sigma - b)} \left( \frac{x_0}{x_d} \right)^\vartheta \left( \frac{(1 - \theta) x_d}{c + \gamma + I^*} \right)^\xi, \end{aligned}$$

where  $b = (\mu - \sigma^2/2) / \sigma$ ,  $\lambda = \sqrt{2r + b^2}$ ,  $\vartheta = -(b + \lambda) / \sigma$  and  $\xi = (\lambda - b) / \sigma$ .

Similar calculations yield

$$(1 - \theta) x_0 E_P^{x_0} \left\{ \int_0^{+\infty} e^{(\sigma+b)Z_t^b} e^{-\frac{\lambda^2}{2}t} \mathbf{1}_{Z_t^b > a} dt \right\} = \frac{c + \gamma + I^*}{\lambda(\lambda - \sigma - b)} \left( \frac{(1 - \theta) x_d}{c + \gamma + I^*} \right)^\xi, \quad (\text{B.8})$$

Combining these two equations gives

$$(1 - \theta) E_Q^{x_0} \left\{ \int_0^{T_{x_d}} e^{-rt} x_t \mathbf{1}_{f(\delta)(x_t) > I^*} dt \right\} = \frac{c + \gamma + I^*}{\lambda(\lambda - \sigma - b)} \left( \frac{(1 - \theta) x_d}{c + \gamma + I^*} \right)^\xi \left[ 1 - \left( \frac{x_0}{x_d} \right)^{\vartheta - \xi} \right]. \quad (\text{B.9})$$

The second term in equation (B.3) is

$$E_Q^{x_0} \left\{ \int_0^{T_{x_d}} e^{-rt} \mathbf{1}_{(1-\theta)x_t > \gamma + c + I^*} dt \right\} = \frac{1}{\lambda(\lambda - b)} \left( \frac{(1 - \theta) x_d}{c + \gamma + I^*} \right)^\xi \left[ 1 - \left( \frac{x_0}{x_d} \right)^{\vartheta - \xi} \right]. \quad (\text{B.10})$$

Therefore, equation (B.3) reduces to

$$\begin{aligned} & E_Q^{x_0} \left\{ \int_0^{T_{x_d}} e^{-rt} [(1 - \theta) x_t - c - \gamma - I^*] \mathbf{1}_{(1-\theta)x_t > \gamma + c + I^*} dt \right\} \\ &= \frac{\sigma(c + \gamma + I^*)^{1-\xi} ((1 - \theta) x_0)^\xi}{\lambda(\lambda - \sigma - b)(\lambda - b)} \left[ 1 - \left( \frac{x_0}{x_d} \right)^{\vartheta - \xi} \right], \end{aligned} \quad (\text{B.13})$$

which yields the value of equity.

### C. Value of the firm without access to external resources

When the firm has no access to external resources, equity value satisfies

$$\begin{aligned} v(x, \gamma, \theta) &= E_Q^{x_0} \left\{ \int_0^{T_{x_f}} e^{-rt} [(1 - \theta) x_t - c - \gamma + (H - 1) I^* \mathbf{1}_{(1-\theta)x_t > \gamma + s + I^*}] dt \right\} \\ &+ (H - 1) E_Q^{x_0} \left\{ \int_0^{T_{x_f}} e^{-rt} [(1 - \theta) x_t - c - \gamma] \mathbf{1}_{(1-\theta)x_t \leq \gamma + s + I^*} dt \right\} \\ &+ (L - 1) E_Q^{x_0} \left\{ \int_0^{T_{x_f}} e^{-rt} [(1 - \theta) x_t - c - \gamma - I^*] \mathbf{1}_{(1-\theta)x_t > \gamma + c + I^*} dt \right\} \end{aligned} \quad (\text{C.1})$$

The first term is identical to those described above with  $x_d$  replaced by  $x_f$ . The additional terms can be computed using a similar line of reasoning.

### D. Correlated cash flows and investment opportunities

When cash flows from assets in place and investment opportunities are correlated, the value of equity can be expressed as

$$\begin{aligned} e(x, \gamma, \theta) &= E_Q^{x_0} \left\{ \int_0^{T_{x_d}} e^{-rt} [(1 - \theta) x_t - c - \gamma + (H - 1)(\rho_I x_t + I^*)] dt \right\} \\ &+ E_Q^{x_0} \left\{ \int_0^{T_{x_d}} e^{-rt} (L - 1) [(1 - \theta - \rho_I) x_t - c - \gamma - I^*] \mathbf{1}_{(1-\theta-\rho_I)x_t \geq c + \gamma + I^*} dt \right\} \end{aligned} \quad (\text{D.1})$$

the solution to which is

$$\begin{aligned}
e(x, \gamma, \theta) = & \frac{(1 - \theta + (H - 1) \rho_I) x_0}{r} - \frac{(1 - \theta) + (H - 1) \rho_I}{r} x_d \left( \frac{x}{x_d} \right)^\vartheta \\
& + \frac{(H - 1) I^* - c - \gamma}{r} \left[ 1 - \left( \frac{x_0}{x_d} \right)^\vartheta \right] \\
& + \frac{(L - 1) \sigma x_0^\xi (1 - \theta - \rho_I)^\xi (c + \gamma + I^*)^{1-\xi}}{\lambda (\lambda - \sigma - b) (\lambda - b)} \left[ 1 - \left( \frac{x_0}{x_d} \right)^{\vartheta - \xi} \right]
\end{aligned} \tag{D.2}$$

when the dynamics of the commodity price satisfy

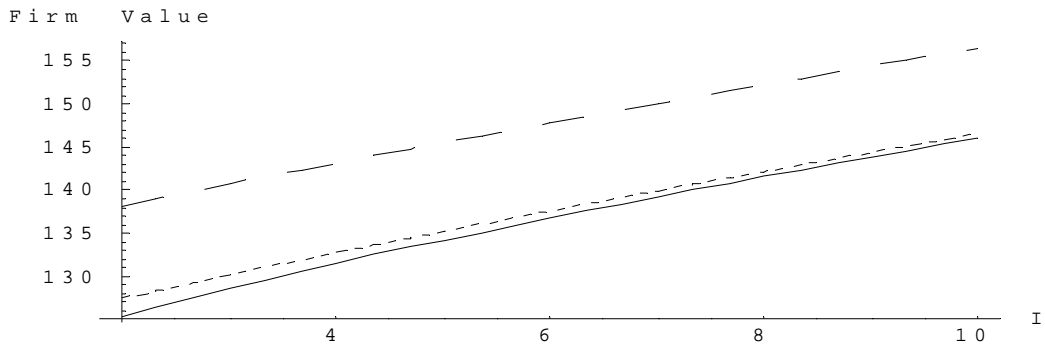
$$dx_t = \sigma x_t dZ_t, \quad x_0 > 0. \tag{D.3}$$

## References

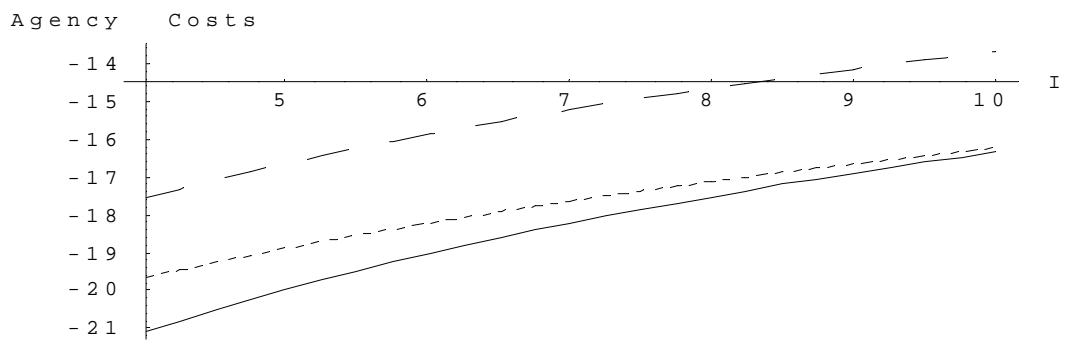
- Andrade, G., and S. Kaplan, 1998, "How Costly is Financial Distress? Evidence from Highly Leveraged Transactions that Became Distressed", *Journal of Finance* 53, 1443-1493.
- Barclay, M., E. Morellec, and C. Smith, 2005, "On the debt capacity of growth options", *Journal of Business*, Forthcoming.
- Bessembinder, H., 1991, "Forward Contracts and Firm Value: Investment Incentives and Contracting Effects", *Journal of Financial and Quantitative Analysis* 26, 519-532.
- Chava, S., P. Kumar, and A. Warga, 2005, "Agency costs and the pricing of bond covenants," working paper, University of Houston.
- Dumas, B., 1991, "Super Contact and Related Optimality Conditions", *Journal of Economic Dynamics and Control* 15, 675-685.
- Fan, H., and S. Sundaresan, 2000, "Debt Valuation, Renegotiation, and Optimal Dividend Policy", *Review of Financial Studies* 13, 1057-1099.
- Froot, K., D. Scharfstein, and J. Stein, 1993, "Risk Management: Coordinating Corporate Investment and Financing Policies", *Journal of Finance* 48, 1629-1658.
- Hamilton, D., G. Gupton, and A. Berthault, 2001, *Default and Recovery Rates of Bond Issuers: 2000*, Moody's Investor Service.
- Harford, J., 1999, "Corporate Cash Reserves and Acquisitions", *Journal of Finance* 54, 1969-1997.
- Hennessy, C, and Y. Tserlukevich, 2005, "Analyzing Callable and Convertible Bonds when the MM Assumptions Are Violated," Working Paper, University of California Berkeley.
- Jensen, M., 1986, "Agency Costs of Free Cash Flow, Corporate Finance and Takeovers", *American Economic Review* 76, 323-329.
- Jensen, M. and W. Meckling, 1976, "Theory of the Firm: Managerial Behavior, Agency Costs, and Ownership Structure," *Journal of Financial Economics* 3, 305-360.
- Keenan, S.C., I. Shotgrin, and J. Sobehart, 1999, *Historical Default Rates of Corporate Bond Issuers, 1920-1998*, Moody's Investor Service.

- Leland H., 1998, "Agency Costs, Risk Management, and Capital Structure," *Journal of Finance* 53, 1213-1243.
- Leland, H., 1994, "Corporate Debt Value, Bond Covenants, and Optimal Capital Structure," *Journal of Finance* 49, 1213-1252.
- Mayers, D. and C. Smith, 1987, "Corporate Insurance and the Underinvestment Problem", *Journal of Risk and Insurance* 44, 46-54.
- Mayers, D., and C. Smith, 1982, "On the Corporate Demand for Insurance," *Journal of Business* 55, 281-296.
- Mello, A., and J. Parsons, 1989, "Measuring the Agency Cost of Alternative Debt Instruments", Working Paper, MIT.
- Mello, A., and J. Parsons, 1992, "Measuring the Agency Cost of Debt", *Journal of Finance* 47, 1887-1904.
- Mello, A., and J. Parsons, 2000, "Hedging and Liquidity", *Review of Financial Studies* 13, 127-153.
- Morellec, E., 2004, "Can Managerial Discretion Explain Observed Leverage Ratios?" *Review of Financial Studies* 17, 257-294.
- Myers, S., 1977, "Determinants of Corporate Borrowing", *Journal of Financial Economics* 5, 147-175.
- Smith, C. and J. Warner, 1979, "On Financial Contracting: An Analysis of Bond Covenants", *Journal of Financial Economics* 7, 117-161.
- Titman, S., and R. Wessels, 1988, "The Determinants of Capital Structure Choice," *Journal of Finance* 43, 1-19.
- Tucker, S., 1991, "Interest Rate Swaps and the 1990 Amendments to the United States Bankruptcy Code: A Measure of Certainty Within Swap Market Contracts", *Utah Law Review* 3, 581-615.
- Zwiebel, J., 1996, "Dynamic Capital Structure under Managerial Entrenchment", *American Economic Review* 86, 1197-1215.

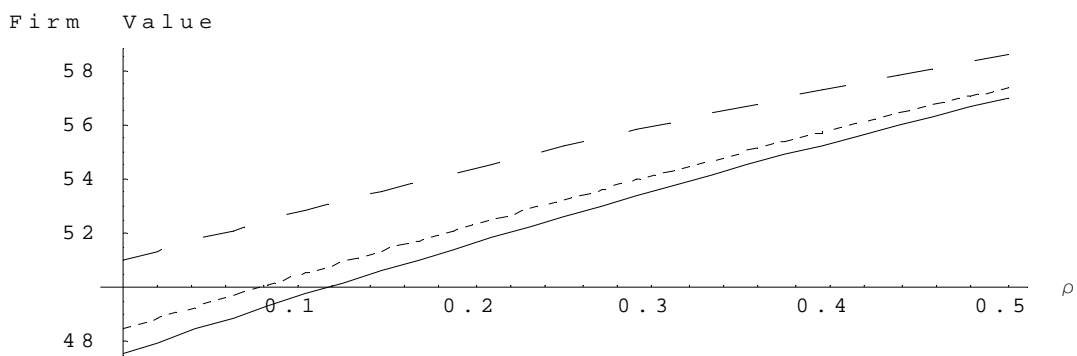
**Figure 1: Financial policies and firm value.** Figure 1 plots firm value for the three financing alternatives as a function of the number of growth options available to the firm.



**Figure 2: Deviations from value-maximization in investment policy.** Figure 2 plots deviations from value-maximization in investment policy as a function of the number of growth options available to the firm.



**Figure 3: Correlated cash flows and investment opportunities.** Figure 3 plots firm value as a function of the correlation coefficient between cash flows from assets in place and investment opportunities.



**Table 1: Pareto superior commodity-linked bond.** Table 1 lists the commodity-linked bond that is bondholder-equivalent to the original fixed-rate bond, for a range of current commodity prices. It also lists the (ex post) levered equity values. The coupon payment on fixed-rate debt is set equal to  $\gamma = 2$ .

Commodity price $x_t$	Fixed rate	Commodity	Commodity	Equity	Value
	bond	linked bond	linked bond	Fixed rate	Commodity
	value	parameter	value	debt	linked debt
	$d(x_t, \gamma, 0)$	$\Theta^d(\gamma, x_t)$	$d(x_t, 0, \Theta^d(\gamma, x))$	$e(x_t, \gamma, 0)$	$e(x, 0, \Theta^d(\gamma, x))$
2.5	25.50	0.2548	25.50	22.34	26.49
3.0	27.93	0.2302	27.93	36.73	40.82
3.5	29.58	0.2082	29.58	52.04	56.06
4.0	30.77	0.1893	30.77	67.86	71.80
4.5	31.67	0.1732	31.67	83.97	87.82
5.0	32.36	0.1594	32.36	100.25	104.01
5.5	32.92	0.1475	32.92	116.63	120.26
6.0	33.37	0.1372	33.37	133.05	136.58
6.5	33.74	0.1281	33.74	149.49	152.91

**Table 2: Credit risk and debt capacity.** Table 2 reports the main comparative statics of the model concerning the firm's default probability, the firm's average lifetime as well as its debt capacity. For each of the simulations relating to the default probability or the expected increase in lifetime, the coupon payment is adjusted so that the market value of corporate debt satisfies  $d(x, \gamma, \theta) = 25$ .

Input parameter values	Average lifetime enhancement	Default probability enhancement	Debt capacity (fixed rate)	Debt capacity (commodity)	Debt capacity enhancement
Base	22%	13%	76.2	117.2	54%
$\sigma = 0.15$	23%	18%	87.2	121.7	40%
$\sigma = 0.35$	21%	10%	72.0	116.4	62%
$r = 0.08$	29%	16%	87.1	136.5	58%
$r = 0.04$	19%	10%	69.1	104.5	51%
$I^* = 10$	23%	14%	82.6	130.6	58%
$I^* = 2$	22%	12%	74.0	112.8	52%

**Table 3: Asset substitution and equity value.** Table 3 investigates shareholders' and managers' incentives to undo the hedge in the commodity linked debt as a function of the commodity price once debt has been issued. The initial commodity price is  $x_0 = 5$  and the market value of corporate debt at the time of issuance is  $d(x, \gamma, \theta) = 30$ . As the commodity-price goes down, debt becomes more risky and thus its market value decreases. Ex-post (after debt has been issued) shareholders can either maintain the hedge implicit in the commodity-linked bond (Hedge) or unwind it by taking positions in forward contracts (Unwind). It is assumed that the manager owns 5% of the firm's equity and extracts 1.1 cent of private benefits out of every dollar of investment.

Commodity price $x_t$	Shareholders' strategy	Corporate debt value	Equity value	Firm value	Managerial Utility
5	Hedge	30.00	106.18	136.18	6.88
	Unwind	29.54	104.06	133.60	6.85
4	Hedge	24.28	77.42	101.70	5.18
	Unwind	23.75	75.80	99.55	5.14
3	Hedge	18.69	48.98	67.67	3.49
	Unwind	18.04	47.78	65.82	3.45
2	Hedge	13.47	21.61	35.08	1.81
	Unwind	12.52	20.89	34.41	1.77