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A Note on Skewness Seeking: An Experimental Analysis

Tobias Brünner* René Levínský‡† Jianying Qiu‡

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Abstract

In this paper we experimentally test skewness seeking at the individual level. Several prospects that can be ordered with respect to the third-degree stochastic dominance (3SD) criterion are ranked by the participants of the experiment. We find that the skewness of a distribution has a significant impact on the decisions. Yet, while skewness has an impact, its direction differs substantially across subjects: 39% of our subjects act in accordance with skewness seeking and 10% seem to avoid skewness. On the level of individual decisions we find that the variance of the prospects and subjects' experience increase the probability of their choosing the lottery with greater skewness.

JEL classification: D81, C91, G11

Keywords: Skewness, Stochastic dominance, Decision making under uncertainty

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1 Introduction

Choice under uncertainty, especially in the portfolio choice literature, is still dominated by the mean-variance framework. This is surprising given that already Borch (1969) and Feldstein (1969) point to the inability of the mean-variance approach to consistently order risky assets with non-normal returns, and it is well known that asset return distributions cannot be fully characterized by the mean and variance; empirical studies typically find that many stock returns exhibit positive skewness and excess kurtosis [Chunhachinda, Dandapani, Hamid, and Prakash (1997), Kahneman and Tversky (1979), and Post and van Vliet (2006)]. When commenting on the potential role of higher order moments on individual decision making, Tsiang (1972) notes that

> . . . skewness preference must be a fairly prevalent pattern of investor's behavior, for modern financial institutions provide a number of devices for investors to increase the positive skewness of the returns of their investments: for example, the organization of limited liability joint stock companies, prearranged stop-loss sales on the stock and commodity markets, puts and calls in stocks, etc., which otherwise would perhaps not have been developed.

Consequently, there is a considerable interest in exploring the effects of the third order central moment on investors' decision making. Regressing the mean rate of return on investments with the sample estimates of moments of higher order, Arditti (1967) finds the coefficient for the second moment to be positive and the impact of the third moment to be negative, whereas coefficients for higher orders are insignificant. Kraus and Litzenberger (1976) and Harvey and Siddique (2000) extend capital asset pricing models by (conditional) skewness and show that this significantly increases explanatory power. Due to several reasons outlined in Brockett and Garven

(1998), however, these results should be treated with caution.

The aim of our paper is to shed light on this issue by experimentally testing skewness seeking at the individual level. In order to test our hypothesis of skewness seeking properly, and to distill it from other phenomena, we employ the third-degree stochastic dominance criterion. To avoid the certainty effect and to minimize subjective probability distortion, we choose prospects with probabilities in the range $[0.1, 0.9]$; there is no certain investment possibility in our framework.

The rest of the paper is organized as follows: Section 2 discusses decision making under uncertainty and develops a theoretical approach capable of identifying skewness seeking in the observed choice set. Section 3 presents the experimental protocol, section 4 analyzes the data and disentangles the principal determinants of the individual decisions. Finally, section 5 concludes by summarizing the main findings.

2 Methodology

Since it is impossible to obtain the utility function directly, one typically needs to rely on the preference revealed through choices, and this will be the main approach of this paper as well. But what kind of choice pattern can be unambiguously related to skewness preference? One possible way is to investigate the choice pattern on risky alternatives with the same mean and variance, but with different levels of skewness. This is based on the following observation.

Expanding the utility function, $U(\cdot)$, in a Taylor series around the mean μ and taking expectation, one gets

$$E[U(X)] = U(\mu) + \frac{\sigma^2}{2!}U''(\mu) + \frac{m_3}{3!}U'''(\mu) + R_4\,, \tag{1}$$

where X is a random variable representing the investor's future wealth, $\mu = E(X)$, $\sigma^2 = Var(X)$, $R_4 = \sum_4^{\infty}(m_i/i!)U^{(i)}(\mu)$, $m_i = E[(X-\mu)^i]$ is the i^{th} central moment, and $U^{(i)}$ is the i^{th} derivative of the utility function. Common assumptions on investors' preferences are that they prefer more to less ($U'(x) > 0$) and are risk averse ($U'' \leq 0$). Moreover, Tsiang (1972) shows that nonincreasing (absolute or relative) risk aversion implies $U'''(x) \geq 0$. Thus, assuming convergence of the series and truncating the Taylor series after the third moment, one can order two distributions based on the first three moments (where $U'''(x) > 0$ implies skewness preference). Tsiang (1972) provides a condition that justifies the truncation of the Taylor series. He shows that the truncation will be a good approximation if the risk, measured by the standard error assumed by the investor, remains a small fraction of her total wealth.[1] This will be true if the investor, when considering a risky asset, always integrates it into her total wealth portfolio.

Unfortunately, as suggested by numerous studies [e.g., Kahneman and Tversky (1979), Thaler and Johnson (1990), Kahneman and Tversky (1992), and Thaler (1999)], investors often evaluate a risky asset independently. This implies that the truncation of the Taylor series may lead to incorrect results since the neglected part might be larger than the part kept, as shown in several examples in Brockett and Kahane (1992).

Scott and Horvath (1980) and Ingersoll (1987) show that for a strictly risk averse investor ($U' > 0$ and $U'' < 0$) with a strictly consistent direction of the third derivative ($U'''(x)$ is positive, negative, or zero for all x) $U''' > 0$ must hold. However, the common interpretation of $U''' > 0$ as a preference for skewness is wrong. To see this, note that $U'''(\mu) > 0$ in (1) does not imply $\partial E[U(X)]/\partial m_3 > 0$. The partial derivative is not applicable here since, as Brockett and Garven (1998) show, a change

[1]See also Kraus and Litzenberger (1976), Scott and Horvath (1980), and Conine and Tamarkin (1981) for more detailed discussions about the reasons for ignoring higher moments.

in the third moment leads to changes in other moments, too.

In this paper we experimentally test whether individuals' utility functions possess the three regularity properties ($U'(x) > 0$, $U''(x) \leq 0$, and $U'''(x) \geq 0$) suggested by economic theory. To avoid potential problems as mentioned above, we use the criterion of third-degree stochastic dominance proposed by Whitmore (1970). This criterion (incompletely) orders the distributions, and it implies the individual's preference relation between them, *whatever* the explicit form of the utility function; the fact that the individual's utility function fulfills the three above mentioned, general properties is sufficient. Proposing to subjects a sequence of several risky alternatives, which can be ordered with respect to the third-degree stochastic dominance criterion, we can link preferences, which are revealed by choice, directly to the shape of the utility function.

2.1 Third-degree stochastic dominance

The third-degree stochastic dominance is proposed by Whitmore (1970). Let $F(X)$ and $G(X)$ be two less-than cumulative probability distributions, where X is a continuous or discrete random variable bounded in the range $X \in [a, b]$ and representing the outcome of a prospect. Prospect $F(X)$ is said to third-degree stochastically dominate (henceforth $\succ_{3SD}$) prospect $G(X)$ if and only if

$$\int_a^x \int_a^y [G(z) - F(z)]dzdy \geq 0 \quad \forall\, x \in [a, b] \tag{2}$$

$$\int_a^b [G(x) - F(x)]dx \geq 0 \tag{3}$$

and inequality (2) holds strictly for at least one $x \in [a, b]$. Let $\mathbb{U}_3$ denote the set of utility functions satisfying $U'(x) > 0$, $U''(x) \leq 0$, and $U'''(x) \geq 0$. Whitmore (1970) shows that if $F(X) \succ_{3SD} G(X)$, the prospect $F(X)$ yields higher expected utility than $G(X)$ for *all* utility functions in $\mathbb{U}_3$.

In our study we focus on the role of the third moment in individual decision making, i.e., we would like to establish that

$$E_F[U(X)] > E_G[U(X)] \Rightarrow U'''(x) > 0\,. \tag{4}$$

Therefore, the influence of the third moment on decision making cannot be over-weighted by the first two moments. To ensure this, let $\Delta_n(x) = \int_a^x \Delta_{n-1}(y)dy$ for $n > 1$, where $\Delta_1(x) = F(x) - G(x)$. Noticing that $\Delta_1(b) = 0$ and $\Delta_n(a) = 0$, we get

$$\begin{aligned} E_F[U(X)] - E_G[U(X)] &= \int_a^b U(x)d\Delta_1(x) \\ &= -U'(b)\Delta_2(b) + U''(b)\Delta_3(b) - \int_a^b U'''(x)\Delta_3 dx\,. \end{aligned}$$

According to the definition of third-degree stochastic dominance, $F(X) \succ_{3SD} G(X)$ implies that $\Delta_2(b) \leq 0$ and $\Delta_3(x) \leq 0\ \forall x \in [a,b]$. We can easily see that if $U'(x) > 0$ and $U''(x) \leq 0$, $U'''(x) < 0$ is only a sufficient condition for $E_F[U(X)] - E_G[U(X)] > 0$, but not a necessary condition. Since, if $\Delta_2(b) < 0$, $\Delta_3(b) < 0$, and $|U'(x)|$ or $|U''(x)|$ is sufficiently larger than $|U'''(x)|$, we might still have $E_F[U(X)] - E_G[U(X)] > 0$ even when $U'''(x) < 0$. To establish the explicit relation between the choice and shape of the utility, we impose the following conditions on all pairs of risky alternatives ranked during the experiment:

$$\Delta_2(b) = 0 \quad \text{and} \quad \Delta_3(b) = 0\,.$$

By Stone (1973)'s finding that

$$\int_a^b (b-x)^k dF(X) = k!F_k(b)\,,$$

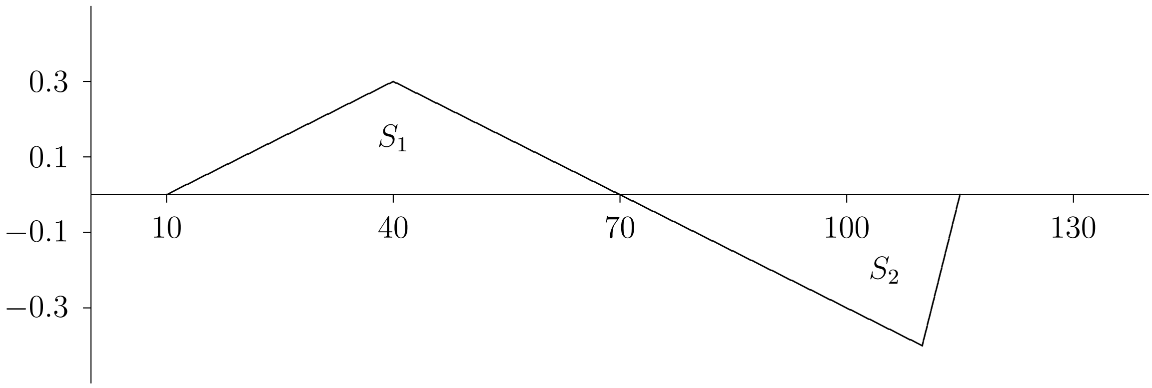
it can easily be shown that

$$\Delta_2(b) = 0 \Leftrightarrow \mu_F = \mu_G \quad \text{and} \quad \Delta_3(b) = 0 \Leftrightarrow \sigma_F^2 = \sigma_G^2\,. \tag{5}$$

In other words, the conditions (5) guarantee the desired implication

$$F(X) \succ_{3SD} G(X) \quad \text{and} \quad E_F[U(X)] > E_G[U(X)] \Rightarrow U'''(x) > 0\,. \tag{6}$$

Figure 1: Difference of cumulative distributions F_1 and F_2, integrated



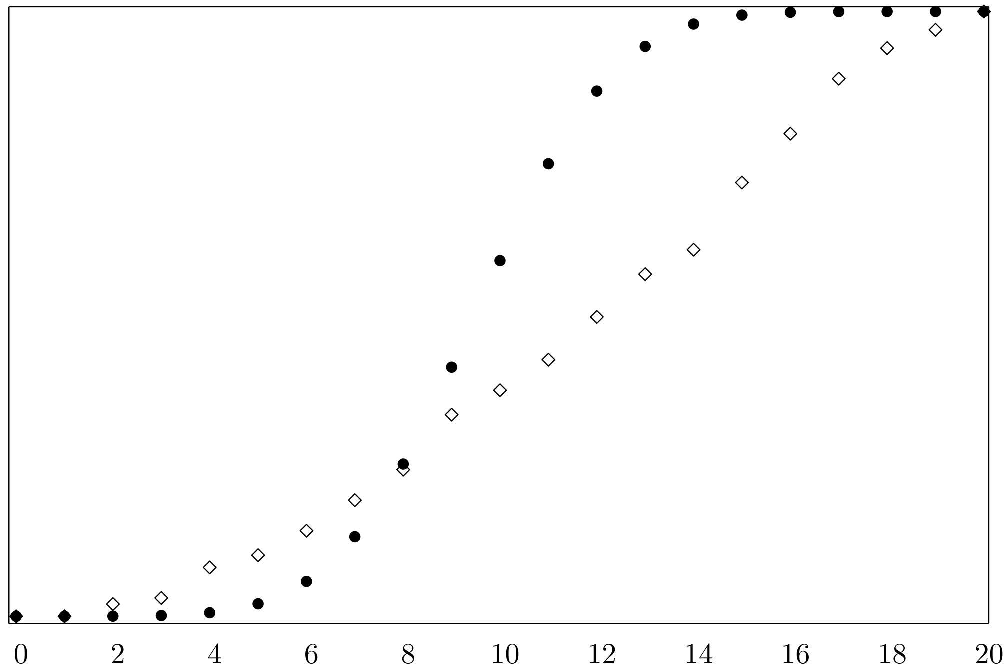
3 The experimental design

In order to test for skewness seeking, 20 pairs of prospects (see Appendix) are generated, as, e.g.,

prospect	gain	probability	mean	variance	skewness
1	10	0.1			
	110	0.9	100	900	-8/3
2	40	0.2			
	115	0.8	100	900	-3/2

In each pair, one prospect third-degree stochastically dominates the other. The graphical demonstration for the pair in the example above is presented in Figure 1. The line represents $\int_a^y (F_1(z) - F_2(z))dz$, the integral $\int_a^x \int_a^y (F_1(z) - F_2(z))dzdy$ is represented by the area between the line and the horizontal axis. Since the positive part of the area is $S_1 = \frac{1}{2}(70 - 10) \times 0.3 = 9$ and the negative part $S_2 = \frac{1}{2}(115 - 70) \times 0.4 = 9$, the integral $\int_a^x \int_a^y (F_1(z) - F_2(z))dzdy$ is nonnegative for all $x \in [10, 115]$.

Figure 2: Cumulative density function of the number of times subjects have chosen the prospect with greater skewness. ● *is the binomial distribution and* ◇ *is the actual distribution of the experimental subjects.*



Except for two pairs, which will be discussed below, the two lotteries forming a pair have the same mean and variance. Each participant successively had to choose for each of the 20 pairs of lotteries the prospect that she prefers. To minimize possible framing effects, the order in which the prospects were presented was determined randomly and independently for each participant[2].

In total, 99 subjects participated in the experiment. The subjects were students from the University of Jena, Germany. Forty-eight of the participants were students of business administration or economics, 49 participants were enrolled in other subjects and for two participants the field of study was not known. At the end of the experiment, one pair of prospects was randomly chosen and played, and participants were paid according to the prospect they had chosen in the course of the experiment.

[2]Random lottery pair design as in Holt and Laury (2002).

Table 1: Distribution of the participants

	$k \leq 5$	$5 < k < 15$	$k \geq 15$
Binomial prediction	0.02	0.96	0.02
All subjects	0.10	0.51	0.39
Econ./B.A. students	0.09	0.55	0.36
Other students	0.12	0.46	0.42

Each participant is characterized by the number of rounds in which she chose the more skewed prospect (k).

A session lasted about 45 minutes, and the payoffs ranged from 1 to 19 euros with an average payoff of 9.91 euros.

4 Results

Figure 2 shows the distribution of individuals characterized by the number of responses in accordance with skewness seeking. The dotted line follows a binomial distribution, i.e., it represents the decisions of individuals who are indifferent between the two alternatives in each of the 20 pairs and therefore choose randomly. The binomial distribution has most of its mass at the average of 10, with only 4% of the participants preferring the more skewed prospect in more than 14 or less than 6 of the 20 choices (see also Table 1).

By contrast, the actual distribution, represented by diamonds, has considerably more mass on its tails. Only 51% of the participants choose the more skewed prospect between 6 and 14 times as compared to 96% predicted by the binomial distribution. This shows that the third moment does matter. Around 39% of the participants prefer the prospect with greater skewness in at least 15 of the 20 rounds. Although at the same time 10% of the participants choose the more skewed prospect in less than six rounds, this is clear evidence that for many participants skewness is a positive factor in their decision-making process. The last two lines of Table 1 show that the results for the two subgroups, students of business administration/economics and other students, are very similar, suggesting that basic knowledge of decision theory has no effect on the decisions. Although the thresholds 5 and 15 are chosen to represent the usual 5% significant level, they might seem arbitrary. Hence two other combinations are chosen for comparison: 4 and 16, and 6 and 14. These results are reported in Table 3. It can be seen that the results are robust. According to the binomial prediction, there should be around 15 times more people who choose the more skewed lottery between 7 and 13 times than people who choose the more skewed lottery 14 times or more. Table 3 shows that there are even slightly more subjects who choose the more skewed prospects more than 13 times than between 7 and 13 times.

Figure 3 shows the proportion of subjects that choose the prospect with greater skewness for each of the 20 rounds. In the first two rounds, around 40% of the subjects choose the more skewed prospect, from the third to the sixth rounds the proportion rises above 50%, and for most of the remaining rounds the proportion of subjects preferring the more skewed prospect stays above two thirds. Since the sequence in which the pairs are presented to the subjects is chosen randomly for each subject, which pair of prospects is presented in which round varies across participants. (Moreover, only one randomly determined prospect is eventually played and paid out, and therefore there are no hedging opportunities between rounds.)

The evolution of preferences follows the *discovered preference hypothesis* which Plott (1996) elaborated for the rationality in individual behavior. The agents' choices reflect a kind of myopia during the first phase of the experiment where the individuals are confronted with a new type of task. In the second phase, the individual awareness of the environment stabilizes the choices. For example, in our data set considering only the 56 subjects who reveal skewness preferences already in the first half of the experiment (choosing the skewed prospect at least six times out of ten, with an average of 7.48), we observe that these subjects increase their number of skewness-preferred choices in the second half of the experiment to 7.84. The intensification of skewness preferences is stronger for the 27 non-economics students in the subsample, who increase the number for choices by even 0.7 (from 7.59 to 8.29), while the choices of then 29 students of economics are more stable (7.38 and 7.41). This fits well to Plott's anticipation: the economics students are more familiar with the type of the presented task, and their choices do not reveal the dynamic adjustment we observe for the non-economists.

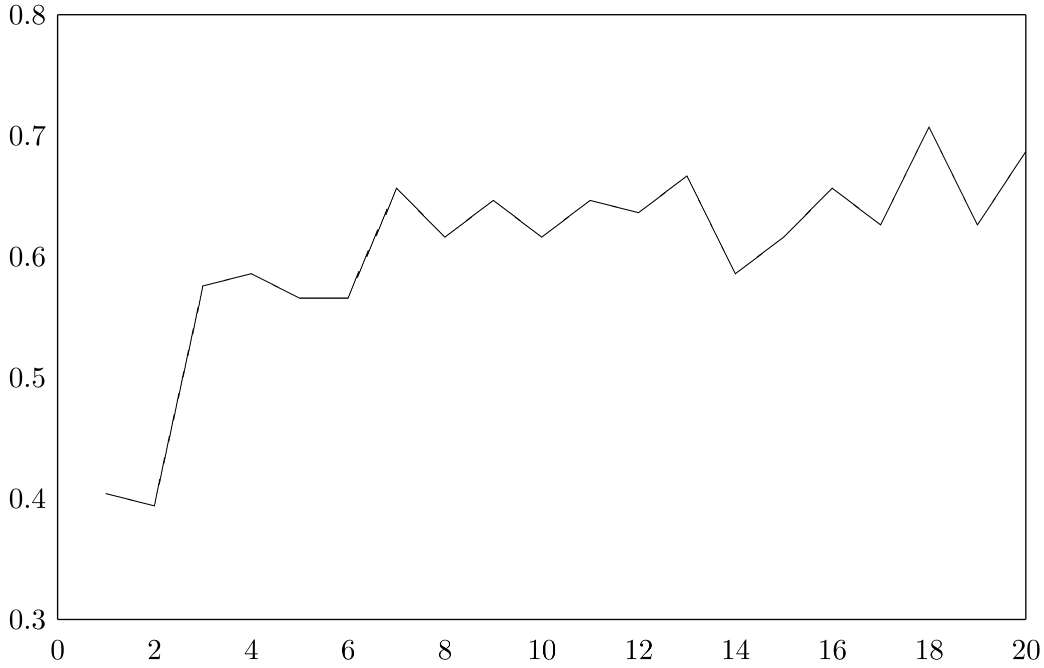
We conclude that the number of repetitions increases either the preference for skewed prospects or the ability to detect these. We observe a weak version of "learning" without feedback as, e.g., Weber (2003). However, our results will be less interesting if skewness only plays a marginal role compared to variance. To examine the relative importance of skewness and variance for subjects' choices, we include the following pair:

prospect	gain	probability	mean	variance	skewness
1	55	0.4			
	130	0.6	100	1350	-0.41
2	85	0.8			
	160	0.2	100	900	1.50

Here $2 \succ_{3SD} 1$. Notice that prospect 2 not only has larger skewness but also smaller variance.[3] If variance plays a much more important role than skewness, we would

[3]Within a pair of prospects, it is impossible that one prospect third-degree stochastically dominates another and at he same time has a larger variance.

Figure 3: Proportion of subjects choosing the prospect with greater skewness per period.



expect the more skewed prospect, prospect 2, to be chosen more frequently than the other 18 pairs with identical variance. 66% subjects choose prospect 2, which is not significantly different from the choice patterns of other pairs.

When constructing lotteries with two outcomes with the same mean and variance, the prospect with greater skewness also exhibits a higher naive expected value.[4] To investigate whether our results merely reflect the fact that subjects base their decisions on naive expectations, we include the following pair:

[4]The naive expected value is the average evaluated using equal probabilities for all outcomes; e.g., the naive means of the lotteries presented in the example at the beginning of section 3 are 60 and 77.5, respectively.

prospect	gain	probability	mean	naive mean	variance	skewness
1	40	0.5				
	160	0.5	100	100	3600	0
2	52	0.25				
	60	0.4	100	$97\ \frac{1}{3}$	3456	.62
	180	0.35				

Here prospect 2 $\succ_{3SD}$ prospect 1, but prospect 1 has a higher naive expected value. We concede that the difference of the naive means of the two prospects is not huge, unfortunately. Since the prospects have to be third-degree stochastically comparable, it is not possible to construct a pair with a more pronounced difference of naive means. And, as shown above, this small difference is unlikely to significantly affect choices. Seventy percent of the participants choose prospect 2, the prospect with greater skewness. This share does not differ from the choice patterns of other pairs and indicates that naive expectations are not the driving force behind our results.

In order to investigate the decisions taken by subjects more closely, we estimate a generalized probit model with mixed effects.[5] We tested several specifications also including interaction effects. These, however, turned out not to be robust. In the following we report the results for the specification that fitted the data best. Explanatory variables are time (t), difference of skewness between the more skewed and the less skewed one (ΔS), standard deviation of prospects (Std), and the order in which the prospects were presented on the screen ($Up = 1$: prospect with higher skewness appeared above the prospect with lower skewness and $Up = 0$: otherwise). Random effects that vary across the 99 subjects are the intercept and the coefficient of ΔS. These random effects are included to take account of the observation that our subject pool is very heterogenous. y_{it} is 1 if subject i chooses the prospect with greater skewness in period t and zero otherwise; y^*_{it} is an unobservable continuous

[5]See Pinheiro and Bates (2000) for a good reference of mixed effects models.

Table 2: Results of probit regression

Expl. variable	coefficient	std. error	t-statistic	p-value
α	-0.5006*	0.1213	-4.1274	0.0000
Δ S	-0.0309	0.0368	-0.8409	0.4005
StD	0.0151*	0.0021	7.1101	0.0000
t	0.0334*	0.0051	6.5058	0.0000
Up	0.0571	0.0603	0.9468	0.3438
Std. dev. of the random effects	$\sigma_u = 0.4611$; $\sigma_v = 0.2663$			
Std. dev. of the error term	$\sigma_e = 0.9278$			
Number of observations	1980			

* Significant at $p = 0.01$

variable underlying the discrete decision y_{it}. Formally, the model is as follows:

$$\begin{aligned} y^*_{it} &= \alpha + u_i + (\beta_1 + v_i) \cdot \Delta S_{it} + \beta_2 \cdot StD_{it} + \beta_3 \cdot t + \beta_4 \cdot Up_{it} + \varepsilon_{it}, \qquad (7) \\ y_{it} &= 1 \quad \text{if} \quad y^*_{it} > 0 \quad \text{and 0 otherwise}, \end{aligned}$$

where $i \in \{1, \ldots, 99\}$ denotes the 99 subjects, $t \in \{1, \ldots, 20\}$ denotes the 20 rounds, $u_i \sim N(0, \sigma_u^2)$ denotes the random effects in the intercept for each participant, $v_i \sim N(0, \sigma_v^2)$ denotes the random effects in the difference of skewness for each participant, and $\varepsilon_{it} \sim N(0, \sigma_e^2)$. The results of the regression are presented in Table 2. Interestingly, the difference in skewness between two prospects does not significantly affect the probability that subjects choose the prospect with greater skewness. Instead, the variance has a significant positive impact: participants are more likely to pick the prospect with higher skewness when the prospects have high variance. This is probably because the larger the standard deviation of prospects, the larger the range of the utility function relevant for decision making. Hence the change of the shape of the utility function (i.e., the difference of the rate of change of marginal utility), which is the origin of skewness seeking, becomes easier to detect.

The results from the probit model also support our previous observation that the probability of choosing the more skewed prospect increases over time. However, after excluding the first four periods (and considering them as practice (or warm-up) rounds to acquaint subjects with the setup of the experiment), the time parameter becomes insignificant.

Finally, the order in which the two prospects of a pair are presented has no effect on the decision.

5 Concluding remarks

Our results indicate that ignoring higher moments is not justified when studying decision making under uncertainty. In line with many empirical studies in the finance literature [e.g., Arditti (1967), Kraus and Litzenberger (1976) and Harvey and Siddique (2000)], we find evidence for skewness seeking. However, our results suggest that the case for skewness seeking is not as clear-cut as these empirical studies suggest. Our pool of 99 subjects is very heterogenous with respect to the behavior toward skewness, ranging from three subjects who chose the prospect with greater skewness in all 20 periods – the probability of meeting such an individual if choices were purely random is lower than one against one million ($= 2^{-20}$) – to two subjects who avoided skewed prospects in all but two periods. Our findings are important for the construction of mean-variance-skewness efficient portfolios. So far it has been assumed that there is a positive trade-off between the expected return and skewness [see, e.g., Chunhachinda, Dandapani, Hamid, and Prakash (1997)]. Our results show that this is not true for all investors. Moreover, even for skewness seeking investors this trade-off may crucially depend on the variance of the portfolio considered.

Our last remark concerns the design of individual decision-making experiments. There are two essential research questions in individual decision making: what is the initial choice [see, e.g., Costa-Gomes, Crawford, and Broseta (2001)], and which kind of (stable) preferences evolves after a certain number of periods [Plott (1996)]?

In our experiment we employ a design similar to Costa-Gomes, Crawford, and Broseta (2001) (no feedback after any round should suppress learning as much as possible). In spite of this, we observe a clear behavioral shift over time. In the first round, only 40% of the subjects choose the more skewed prospect. Then this probability increases by about 5% per round until the fifth round, when it reaches about 65%, remaining more or less stable until the end of the experiment. Finally, we observe the convergence of preferences predicted by Plott's *discovered preference hypothesis.*

We conclude that we observe "learning" without feedback in our experiment. This questions the possibility to explore the initial individual choices in experiments where the tasks are subsequently repeated even if the subjects are not provided with information between the rounds.

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