Working Paper
Series

National Centre of Competence in Research
Financial Valuation and Risk Management

Working Paper No. 536

Arbitrage Free Price Bounds for Property Derivatives

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First version: June 2009
Current version: June 2009

This research has been carried out within the NCCR FINRISK project on
"Credit Risk and Non-Standard Sources of Risk in Finance"

Arbitrage Free Price Bounds for Property Derivatives

June 9, 2009

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Abstract

Market frictions inhibit perfect replication of property derivatives and define the property spread as a price measure in the incomplete real estate market. We identify transaction costs, transaction time and short sale constraints as the main frictions in this market. Based on these frictions, we set up a framework of arbitrage free price bounds for property derivatives. In turn, we use observed derivative prices to determine the implied cost of the frictions. Finally, we verify these values using other research and so confirm the accuracy of our framework.

JEL Classification: G10, G12

Keywords: Property Derivatives; Property Spread; Arbitrage Free Price Bounds; Market Frictions; Halifax House Price Index

[1]Acknowledgement: This research has been carried out within (the project on "Credit Risk and Non-standard sources of risk in finance") the National Centre of Competence in Research "Financial Valuation and Risk Management" (NCCR FINRISK). The NCCR FINRISK is a research instrument of the Swiss National Science Foundation."

Introduction

In a swap that pays the total return of a property index, the rate that balances the swap can deviate significantly from LIBOR. We call this difference between the property swap rate and LIBOR *the property spread*, quoted on an annual basis. The swap payer pays property performance and in return gets LIBOR plus the property spread. If the spread is negative and its absolute value exceeds LIBOR, the swap payer pays on the interest leg of the swap but expects to receive the negative performance of the property leg.

In the UK, property derivatives were traded at a substantially positive spread until the end of 2006. However, the spread fell in 2007 and quickly turned negative. Quotes obtained from market participants who trade swaps on property indexes differ considerably from prices computed using models based on arbitrage arguments. Buttimer, Kau, and Slawson (1997) develop a two-state model for pricing a total return swap on a property index. Bjoerk and Clapham (2002) present an arbitrage free model that is more general than the Buttimer, Kau, and Slawson model. Patel and Pereira (2008) extend the Bjoerk and Clapham model by including counterparty default risk. However, none of these models explains the spreads observed in the market.

In contrast to property returns, equity returns are swapped against LIBOR without a spread. The reason is that a no-arbitrage argument is sufficient to price equity derivatives. A trader can sell short equities at virtually no transaction cost and invest the proceeds in an instrument returning LIBOR. It would thus be a "free lunch" to receive a rate higher than LIBOR. This standard no-arbitrage argument used in modern finance assumes that the market is virtually frictionless, that the underlying asset can be instantaneously bought or sold at no cost.

However, a no-arbitrage argument alone is not sufficient to price property derivatives because the underlying market exhibits frictions. The index and its components cannot be traded continuously and instantly at the prevailing spot price without transaction costs. This leads to a property spread.

Observed property spreads vary with the maturity of the swap. Fig. 1 shows the observed spreads against maturities, implied by Halifax House Price Index (HPI) derivative contracts in February 2007 and one year later.[2] Fig. 2 shows the development of the property spread for selected maturities.

[Insert Fig. 1 about here]

[2] We obtain prices for Contracts-for-Difference (CFDs) on the Halifax HPI. To convert them into property spread levels on a total return basis, we use LIBOR and swap rates as well as a fixed rental rate of 5%. This level corresponds to the average of UK house rental returns from 2004 to 2007, according to *The ARLA History of Buy to Let Investment*. The quarterly rental returns are very stable in percentage terms (range: 4.9% to 5.1%) and it is reasonable to assume a fixed percentage rate.

The cause of the shape of the term structures of property spreads is not obvious. As liquid and cost efficient instruments, property derivatives are beneficial to both investors and hedgers. Given the significant transaction cost advantages, it is clear that market participants looking for a short-to-medium term property exposure or hedge can benefit from the use of property derivatives. For long-term investment horizons, the impact of one-off transaction costs is less significant, making a physical purchase or sale a viable alternative to a property swap. Thus the short end of the term structure of property spreads is expected to be more volatile than the long end.

[Insert Fig. 2 about here]

A common explanation for the shape of the property spread term structure follows a classical cash and carry arbitrage argument. Cash and carry arbitrage is a strategy whereby an investor buys the underlying assets, sells the derivative and holds both positions until maturity. According to this argument, a property derivative should be priced in such a way that there exists no arbitrage opportunity when the derivative is replicated by buying actual property. In an efficient market, when investors are seeking to buy property, the price of the derivative should reflect the costs that would arise from a physical purchase. These transaction costs of say 7% should be amortized over the investment horizon. This cash and carry approach implies an inverse spread curve against maturity. The cash and carry argument is an intuitive starting point in explaining the property spread and could partly be reflected by the inverse spread curve observed in the rather bullish market in February 2007. However, the cash and carry argument alone clearly does not explain the curve prevailing in February 2008.

The main reason why standard arbitrage free pricing models, including the classical cash and carry approach, are not sufficient to price property derivatives is that they assume the possibility of perfect replication. In the property market, we observe severe frictions that inhibit perfect replication.

As a consequence, the pricing of property derivatives must be based on arbitrage free price bounds rather than on a single arbitrage free price. Any price, i.e. any property spread, within these bounds satisfies the no-arbitrage condition.

The rest of the paper is organized as follows. First, we identify and describe the frictions in the property market that inhibit perfect replication of derivatives. Next, we define a framework for arbitrage free price bounds for the property spread. Based on this framework, we determine the implied cost of the frictions using observed property derivatives prices. Verification with other research and observations indicates that the values we find are reasonable and confirms the accuracy of our framework. Finally, we draw some overall conclusions.

Property market frictions

The property spread exists because property derivatives cannot be perfectly replicated by trading actual property. The frictions that inhibit perfect replication bear investigation. These frictions define arbitrage free price bounds for the property spread. Consider the following three basic market frictions:

- Transaction costs
- Transaction time
- Short sale constraint

We look at property exclusively from an investment perspective without a consumption component. To examine the effect of each of these frictions on the willingness of market participants to pay, we introduce two counterparties: an investor and a hedger. The investor buys property and sells at a given investment horizon. The hedger, on the other hand, owns property but is worried about a market downturn. Hence he sells his property and buys them back at the end of the hedge period.

If these market participants engage in property derivatives rather than physical assets, the price considerations are as follows: the investor, buying a derivative contract, is concerned about paying too high a property spread. The hedger, selling the derivative contract, is worried about too low a property spread. Keeping this in mind, we consider each friction and its implication on the investor and the hedger.

Transaction costs

For both the investor and the hedger, it is costly to trade physical property.[3] Transaction costs typically include agent's fees, taxes, legal fees and registration fees. For the US, aggregate agent's fees for housing transactions range from 3% to 6% as in DiPasquale and Wheaton (1996). Furthermore, stamp duty is levied in many US states and in the UK, and similar ad valorem taxes are levied in most other jurisdictions. Moreover, in both the US and the UK, lawyers perform conveyancing, and substantial legal fees can be incurred. Finally, registration fees are levied by local governments. In OECD countries, roundtrip transaction costs are generally estimated to range from 6% to 12% as in Quigley (2002), Cunningham and Hendershott (1984) and Malatesta and Hess (1986). However, technologies such as online marketplaces have already begun to reduce some of these costs for homeowners. In 2007, practitioners

[3] Management and maintenance costs of physical property are generally reflected in the income component of property indexes and are not considered here.

estimate the total transaction cost for property in the UK to be about 3% to 9% including property taxes. Transaction costs for derivatives on the other hand are negligible.

Transaction time

It is not only costly but also time consuming to trade physical property. Due diligence processes, price negotiations and the closing of a contract are time consuming. Furthermore, there are extensive search efforts to find the counterparties with the best offer. Also, the heterogeneity of properties and their unique spatial component suggest it is time consuming to identify suitable objects. It is a fact that the property market is not quite transparent. In many regions and for some sub-markets, very few comparable transactions can be observed to indicate a price level. Consequently, uncertainty about demand and price for an individual object is high. The result is often a large difference between the prices of bidders and sellers since both sides want to be compensated for the price uncertainty. In other words, transaction time reflects market illiquidity. In the UK, it takes on average three months to find a counterparty and another three months to finalize a transaction.[4] Derivatives in contrast can be traded almost instantly.

Short sale constraint

Shorting an asset is not simply the mirror image of buying an asset for various legal and institutional reasons. To be able to sell an asset short one must borrow it. The borrower pays a lending fee to the lender.

To illustrate the effect of a short sale constraint on the no-arbitrage condition, consider the cash and carry relationship. For a stock, cash and carry arbitrage works as follows: the fair price of a forward contract must equal the price of the underlying stock plus financing costs minus forgone dividends. If this relation does not hold, then cash and carry arbitrage can be achieved. For a forward price lower than the fair value, the arbitrageur would enter a long position in the forward contract, sell the stock short and lend the proceeds to earn interest. This second case involves the need to sell the underlying asset short. Short selling is generally not possible for the properties that make up an index, since one cannot lend and sell short buildings. Because of the short sale constraint, arbitrageurs can only refrain from buying overpriced assets but cannot exploit mispricings. Miller (1977) describes how short sale constraints can cause prices to reflect only the views of optimistic investors.

[4] According to experts of the investment bank Calyon and of the brokerage firm TFS, 2nd Pan-European Property Derivatives Conference, May 8/9, 2008, Barcelona, Spain.

An instrument which mimics property performance and which can be sold short is clearly of value to a hedger as it replaces an actual sale. Also, as it is impossible to perform arbitrage from overvalued properties or to bet on a market downturn without property derivatives, speculators and arbitrageurs clearly value the possibility to do so using these new instruments.

In summary, the investor is willing to pay a premium to avoid transaction costs and transaction time. The investor does not engage in a hedge and hence does not need short sales. From the hedger's perspective, avoiding transaction costs and transaction time is also valuable. In addition, the possibility of a short sale is necessary for the establishment of a hedge, hence the hedger is willing to pay a premium for this. The arbitrage free price bounds are defined by all three basic market frictions: transaction costs, transaction time and the short sale constraint. Due to the existence of frictions, the model provides only arbitrage free price bounds instead of a single arbitrage free price. To value these frictions, they are embedded in a no-arbitrage pricing model.

Arbitrage free price bounds

A property spread can be attributed to the frictions inherent in the market. Given the frictions, bounds of arbitrage free prices follow rather than one single arbitrage free price for property derivatives. The arbitrage free price bounds are a function of the price of the underlying instrument and of market frictions. Only if prices are outside these bounds can arbitrage be achieved using actual property.

We set the framework to derive analytical arbitrage free price bounds as follows. For any given property spread p, there is an upper arbitrage free price bound $\overline{p}$ and a lower arbitrage free price bound $\underline{p}$. The upper bound is the maximum spread an investor is willing to pay for a derivative instead of buying actual property and is only affected by buyer and seller transaction costs k_{1b} and k_{1s} and by transaction time k_2. If the property spread lies above the upper arbitrage free price bound, it is more attractive to buy actual property than to buy derivatives. Unlike the upper bound, the lower bound also reflects the value of the short sale constraint k_3.

For a given investment horizon $T - t = \tau$, the actual property investor (API) initially endowed with wealth W_t has at time T a wealth of

$$W_{\mathrm{API},T} = (W_t - (1 + k_{1b} + k_2)S_t)\mathrm{e}^{r\tau} + (1 - k_{1s} - k_2)S_T, \tag{1}$$

where S_t is the value of a property portfolio or property index on a total return basis in t. Furthermore, r represents the risk free interest rate. Transaction costs k_{1b} or k_{1s} as well as the cost of transaction time k_2 incur for both a purchase and a sale and are defined for a one way transaction in percentage terms. Furthermore, the forward price for a property derivative is

$$F_{t,T} = S_t \mathrm{e}^{(r+p_\tau)\tau}, \tag{2}$$

where p_τ is the property spread for a contract with a lifespan τ. If the investor buys a property derivative investment (PDI) instead of actual property, he gets

$$\begin{aligned} W_{\mathrm{PDI},T} &= W_t \mathrm{e}^{r\tau} + (S_T - F_{t,T}) \\ &= W_t \mathrm{e}^{r\tau} + S_T - S_t \mathrm{e}^{(r+p_\tau)\tau}. \end{aligned} \tag{3}$$

As a forward contract is unfunded, the initial wealth earns interest, captured by the first term in Eq. (3). For the no-arbitrage condition to hold, the expected wealth using a derivative investment must be equal or greater than the expected wealth using a actual property investment:

$$\mathbb{E}_t^{\mathbb{Q}}[W_{\mathrm{API},T}] \leq \mathbb{E}_t^{\mathbb{Q}}[W_{\mathrm{PDI},T}], \tag{4}$$

or

$$(W_t - (1 + k_{1b} + k_2)S_t)\mathrm{e}^{r\tau} + (1 - k_{1s} - k_2)\mathbb{E}_t^{\mathbb{Q}}[S_T] \leq W_t \mathrm{e}^{r\tau} + \mathbb{E}_t^{\mathbb{Q}}[S_T] - S_t \mathrm{e}^{(r+p)\tau}, \tag{5}$$

where $\mathbb{E}^{\mathbb{Q}}[\cdot]$ is the expected value under a martingale measure $\mathbb{Q}$. Following Bjoerk (1998) let the market choose this measure. We assume that S_t follows a geometric Brownian motion with drift μ, variance σ^2 and a Wiener process Z_t. As we will see in the next section, property indexes often exhibit significant autocorrelation in the short term but the impact of autocorrelation on the expected value decreases exponentially and affects the expected values that are relevant in the context of arbitrage free price bounds only marginally. We thus consider a geometric Brownian motion as reasonable approximation that is analytically manageable. Let

$$X_t = S_t \mathrm{e}^{-r\tau} \tag{6}$$

be the discounted index value. Then consider

$$\mathbb{E}_t^{\mathbb{Q}}[X_T], \tag{7}$$

with a martingale measure $\mathbb{Q}$ that is not unique because of the market frictions. Every other martingale measure $\widetilde{\mathbb{Q}}$ is related to $\mathbb{Q}$ by the density formula

$$\frac{\mathrm{d}\mathbb{Q}_t}{\mathrm{d}\widetilde{\mathbb{Q}}_t} = \mathrm{e}^{X_t - \frac{1}{2}[X]_t} = Z_t, \tag{8}$$

where $[X]_t$ is the quadratic variation process. For geometric Brownian motion,

$$X_t = \int_0^t \frac{r-\alpha}{\sigma} \mathrm{d}Z = \frac{r-\alpha}{\sigma} Z_t, \tag{9}$$

where the parameter α can be positive or negative. In the incomplete market, the parameter α is not specified in an arbitrage free model. Hence, let the market choose the parameter α in the following way. Define the theoretical price

$$\Pi_{t,T}(\alpha) = \mathbb{E}_t^{\mathbb{Q}}[S_T] = S_t \mathrm{e}^{(r+\alpha)\tau}. \tag{10}$$

In other words, for any martingale measure $\widetilde{\mathbb{Q}}$ there exists a parameter $\tilde{\alpha}$ where $\mathbb{E}_t^{\widetilde{\mathbb{Q}}}[S_T] = S_t \mathrm{e}^{(r+\tilde{\alpha})\tau}$. We follow Bjoerk (1998) to determine α by solving the least squares minimization problem

$$\alpha^* = \mathrm{argmin}_\alpha \left[\sum_{i=1}^{n} (\Pi_i(\alpha) - \Pi_i^*)^2 \right], \tag{11}$$

where Π_i^* are n historically observed prices. The minimization problem ensures that the theoretical and historic prices are as close as possible.

The parameter α is assumed to be constant. Allowing for a time dependent α would generalize the approach.

The upper bound of the property spread $\overline{p}$ then follows

$$\overline{p}_\tau = \frac{\ln[(1+k_{1b}+k_2)\mathrm{e}^{r\tau} + (k_{1s}+k_2)\mathrm{e}^{(r+\alpha^*)\tau}]}{\tau} - r. \tag{12}$$

Next we consider the lower bound of the property spread, $\underline{p}$. The actual property hedger (APH), the seller, has an initial wealth of W_t tied up in the properties S_t that he sells at t. After expiry of the hedge horizon T, he buys the properties back and gets

$$\begin{aligned} W_{\mathrm{APH},T} &= (1-k_{1s}-k_2-k_3)S_t\mathrm{e}^{r\tau} - (1+k_{1b}+k_2)S_T + S_T \\ &= (1-k_{1s}-k_2-k_3)S_t\mathrm{e}^{r\tau} - (k_{1b}+k_2)S_T, \end{aligned} \tag{13}$$

where k_3 is the cost of the short sale constraint and $\mathrm{e}^{r\tau}$ reflects the interest earned on the cash proceeds from the sale over the hedging time τ.

If, for a given hedge horizon, the hedger holds on to the portfolio S_t and sells a property derivative (PDH) instead of the physical properties, he gets

$$\begin{aligned} W_{\mathrm{PDH},T} &= S_T + (F_{t,T} - S_T) \\ &= S_T + S_t\mathrm{e}^{(r+p_\tau)\tau} - S_T \\ &= S_t\mathrm{e}^{(r+p_\tau)\tau} \end{aligned} \tag{14}$$

For the no-arbitrage condition, the expected wealth using the derivative hedge must be equal to or greater than the expected wealth after a actual sale:

$$\mathbb{E}^{\mathbb{Q}}[W_{\mathrm{APH},T}] \leq \mathbb{E}^{\mathbb{Q}}[W_{\mathrm{PDH},T}], \tag{15}$$

or

$$(1 - k_{1s} - k_2 - k_3)S_t\mathrm{e}^{r\tau} - (k_{1b} + k_2)\mathbb{E}^{\mathbb{Q}}[S_T] \leq S_t\mathrm{e}^{(r+p_\tau)\tau} \tag{16}$$

It follows that the lower bound of the property spread $\underline{p}$ is

$$\underline{p}_\tau = \frac{\ln[(1 - k_{1s} - k_2 - k_3)\mathrm{e}^{r\tau} - (k_{1b} + k_2)\mathrm{e}^{(r+\alpha^*)\tau}]}{\tau} - r. \tag{17}$$

Empirical results

Empirically observed derivative prices allow us to quantify the cost of the frictions in our arbitrage free price bound framework.

In July 2008, three year notes on the Halifax HPI were traded at a discount such that the breakeven on the notes corresponds to a 45% decline in UK house prices. In contrast, the biggest peak to trough move in the history of the Halifax HPI was -14.7%, from July 1989 to February 1993.

It is obvious that market participants had a negative view on property prices and were ready to pay a significant premium to sell a property index derivative short. Hedge funds were strong sellers of property derivatives as they bought distressed subprime credit portfolios and hedged their collateral risk by selling property derivatives short. Property derivatives still offer the only way to hedge this property risk.

For empirical analysis, we consider quotes for derivatives on the Halifax HPI, a transaction-based hedonic index reflecting monthly quality adjusted house price development in the UK. In contrast to appraisal based indexes such as the often referenced IPD indexes, a transaction based index does not suffer smoothing effects that distort derivative prices. Smoothing effects of appraisal-based property indexes are described in Geltner, MacGregor, and Schwann (2003).

The Halifax HPI derivatives market provides daily liquidity. The nonseasonally adjusted all buyers monthly index is the reference instrument of the Halifax HPI family. Daily bid and ask quotes are from the brokerage firm Tradition Financial Services (TFS) for Halifax HPI forward contracts with maturities from one to thirty years, from February 2007 to August 2008. Even though the market for property derivatives is very young, their prices experienced strong fluctuations. Fig. 3 displays all observed forward prices against their maturities. The highest levels were reached in May 2007, the lowest in July 2008.

[Insert Fig. 3 about here]

Only contracts with maturities up to ten years are traded regularly and are quoted at reasonably narrow bid-ask spreads. We concentrate on the data of these contracts to avoid data not updated for several days or weeks.

The data set allows assessing the market implied cost of the frictions that impact derivative prices. First, we calculate the property spreads from the forward prices according to Eq. (2). We again use LIBOR and swap rates as well as a 5% rental rate as described above. The resulting property spreads are shown in Fig. 4.

[Insert Fig. 4 about here]

Furthermore, we estimate the parameter α using historical index values. To get a reasonable result, we include one full market cycle. As of July 2008, the market is 11 months after the most recent peak of house prices. Going back a full market cycle, we find June 1990 as the corresponding point in time 11 months after the market peaked last time in July 1989. For this period, monthly index values for the Halifax HPI as well as 1-month LIBOR rates are obtained. Using this monthly data, the parameter α is 3.98% p.a. according to the criterion in Eq. (11).

Next, consider the values of the market frictions. The transaction costs k_{1b} and k_{1s} can be observed exogenously as they reflect cash out costs. There is survey evidence on the magnitude of transaction costs for UK property. Table 1 shows the ranges for the transaction cost components according to *The Global Property Guide, 2007.* Looking for boundary values, we take the maximum of the ranges. The maximum value of the buyer's transaction cost range k_{1b} is 5.15% while the maximum value of the seller's range k_{1s} is 4.11%.

[Insert Table 1 about here]

To assign a value to the frictions k_2 and k_3 we fit both the upper and lower bound to observed prices. In particular, we solve for the value of the frictions such that the bounds are as narrow as possible but all spread observations p^* just remain within the arbitrage free price bounds, i.e.

$$\min_{k_2,k_3}[\overline{p}(k_2) - \underline{p}(k_2, k_3)] \tag{18}$$

subject to

$$\underline{p}(k_2, k_3) \leq p^* \leq \overline{p}(k_2). \tag{19}$$

This process leads to a market implied cost of 4.46% for transaction time k_2 and one of 11.83% for the short sale constraint k_3. Fig. 5 plots the historical trajectory of the Halifax HPI and the arbitrage free price bounds for its forward prices, using the obtained values for the market frictions.

[Insert Fig. 5 about here]

Verification of friction costs

The next step is to verify the obtained values for the implied friction costs k_2 and k_3.

The cost of transaction time k_2 is a measure of marketability for illiquid assets. Longstaff (1995) derives an analytical upper bound on the value of marketability using option pricing theory. Dyl and Jiang (2008) use the Longstaff model to value illiquid common stock. The model requires only two inputs: the volatility of the considered asset and the length of time the asset is illiquid or the time it takes to sell the illiquid asset. The model's fundamental insight is that marketability is properly construed as the option to sell an asset at the time of one's own choosing. In particular, Longstaff uses a lookback option that pays the difference between the maximum asset value during the marketability period and the asset value at the end of the period. As transacting at the maximum value assumes perfect market timing, the option price is an upper bound for the value of marketability. This upper bound is an increasing function of the length of marketability restriction as well as asset volatility. This property is intuitive since the longer it takes to sell an asset and the more volatile its price, the higher is the opportunity cost of not being able to trade. Longstaff mentions that this upper bound can also be viewed as the maximum amount that any investor would be willing to pay to obtain immediacy in liquidating an asset position. He finally assesses whether the bound is consistent with the empirical studies of Pratt (1989) and Silber (1991) who estimate the value of the lack of marketability of restricted stock and private equity. He finds that the model could actually provide a tight bound, representing a useful approximation of the value of marketability. Its closed form solution is

$$D_{\max} = \left(2 + \frac{\sigma^2 T}{2}\right) \mathrm{N}(d) + \sqrt{\frac{\sigma^2 T}{2\pi}} \mathrm{e}^{-\frac{\sigma^2 T}{8}} - 1, \tag{20}$$

where T is the length of the marketability period, σ is the standard deviation of the asset under consideration, $\mathrm{N}(\cdot)$ is the cumulative normal distribution function and $d = \sqrt{\sigma^2 T}/2$.

(20) provides a discount while k_2 represents a surcharge. The reference value for the verification of k_2 is

$$k_2' = \frac{1}{1 - D_{\max}} - 1. \tag{21}$$

To apply the Longstaff model to verify k_2, values for the marketability period and for the volatility of property are required. According to market practice, it takes about six months to buy or sell a property as described above. Volatility of property indexes needs to be assessed with caution, as property prices usually exhibit inertia. Geltner and Miller (2001) propose to adjust standard deviation of property returns by a correction factor equal to $1/(1 - \mathrm{AR}(1))$ where AR(1) is the first order autoregressive coefficient. We measure standard deviation and autocorrelation for the Halifax HPI since its inception in 1983 as well as over the mentioned market cycle since 1991 and over the last ten years. Table 2 summarizes the results. While plain standard deviation increases and autocorrelation decreases over time, the adjusted standard deviation turns out to be very stable.

[Insert Table 2 about here]

For an illiquidity period of six months and an adjusted volatility level over the considered market cycle of 7.79% p.a., the Longstaff model gives a value of $k'_2 = 4.68\%$ compared to k_2 which we empirically obtained to be 4.46%. These values are very close indicating a reasonable level.

Finally, the cost of the short sale constraint has to be verified. The most direct measure of short sale costs is the lending or loan fee paid by the lender of a security to the borrower of that security.[5] This fee arises because to sell a security short, an investor must borrow shares from an investor who owns them and is willing to lend them. The lending fee serves to equilibrate supply and demand in the lending market.

While quantity data in the shorting market are readily available, price data are not. The lending market is not centralized and lending fees do not need to be disclosed.

Cohen, Diether, and Malloy (2007) report examples from a sample of proprietary stock lending data from September 1999 to August 2003. They report statistics on the full sample and on two sub-samples, large stocks and small stocks.[6] The mean lending fee from the full sample is 2.60% p.a. while the 0.75 percentile value is at 4.20%. For large stocks, the mean lending fee is 0.39% and the 0.75 percentile value is only at 0.16%. Small stocks on the other hand have much higher loan fees with a mean value at 3.94% and a 0.75 percentile value at 5.30%. The most extreme lending fees in this sample are 7.25% and 14.75% respectively.

D'Avolio (2002) describes the market for borrowing stock and finds that 91% of the stocks lent out in the investigated sample exhibit a lending fee below 1%. The remaining 9% of the stocks have a mean fee of 4.3%. Less than 1% of the sample, typically small stocks with little institutional ownership, exhibit very high fees ranging from 10% up to 79%.

Investigating boundary levels, extreme values that reflect the maximum willingness to pay for a short sale possibility are critical. Stocks with very high lending fees can be considered special cases. For example, Lamont and Thaler (2003) study 18 equity carve-outs from April 1996 to August 2000 in which the parent has stated its intention to spin off its remaining shares of an already listed subsidiary company. Most subsidiaries were overpriced compared to their parent companies and had a significantly larger short interest than the parent. In the case of Palm Inc, a temporarily heavily overpriced subsidiary of 3Com, short interest was as high as 147.6% at the peak. That is, more than all floating shares had been sold short. Borrowed shares can be sold short to an investor who then lends them again. Lamont and Thaler (2001) conclude that in the case of Palm, arbitrageurs could not find enough shares to satisfy

[5]A series of recent papers analyzes direct measures of shorting costs, e.g. Brent, Morse, and Stice (1990), D'Avolio (2002), Figlewski and Webb (1993), Lamont and Stein (2004), Ofek, Richardson, and Whitelaw (2004), Jones and Lamont (2002), Reed (2002) and Geczy, Musto, and Reed (2002).

[6]Large stocks exhibit a market capitalization above the NYSE median while small stocks are below the median.

the demand of irrational investors. The chief impediment of the arbitrage strategy of buying parents and shorting subsidiaries is short sale constraints.

The value of a short sale opportunity for an index that reflects a broad property portfolio is hardly directly comparable to the short sale cost of a particular stock. In sum, the level of typical lending fees is clearly below the obtained boundary value of shorting costs $k_3 = 11.86\%$ but some special cases can reach levels far outside this soft arbitrage free price bound. The relatively high value k_3 may reflect the fact that actual property can be overpriced because of some irrational investors unlike derivatives where short selling is possible.

Conclusion

Prices of property derivatives do not result from a simple no-arbitrage argument as the derivatives cannot be perfectly replicated. Transaction costs, transaction time and short sale constraints cause a property spread, a measure for prices in the incomplete real estate market. As these frictions inhibit perfect replication, they define arbitrage free price bounds for the property spread.

In the present paper we set up a framework of arbitrage free price bounds for property derivatives. Furthermore, we empirically assign values to the market frictions which affect prices of property derivatives or, equivalently, the property spread. We base our research on the UK housing market, where prices for property derivatives are readily available. In particular, we find boundary values of 5.15% for the buyer's transaction costs, 4.11% for the seller's transaction costs, 4.46% for transaction time and 11.83% for the short-sale constraint.

These market implied friction costs turn out to be consistent with other research and market observations, and confirm the accuracy of our framework. The price process within the price bounds is left for future research.

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Table 1: **Transaction costs in the UK.** The Table presents ranges for real estate transaction cost components in percentage of transaction price, according to *The Global Property Guide, 2007*.

Cost Component	Range	Who Pays?
Stamp Duty	0 - 4%	Buyer
Legal Fees	0.5% - 1%	Buyer
Land Registry Fees	0.04% - 0.15%	Buyer
Agent's Fees	2% - 3.5% (+ 17.5% VAT)	Seller
Costs Paid by Buyer	0.54% - 5.15%	
Costs Paid by Seller	2.35% - 4.11%	
Roundtrip Transaction Costs	2.89% - 9.26%	

Table 2: **Plain and adjusted standard deviations of the Halifax HPI.** The first column shows plain standard deviations of the Halifax HPI. The second column shows the first order autoregressive coefficients of the index; these are used to compute the adjusted standard deviations presented in the third column. Both plain and adjusted standard deviations are quoted on an annual basis.

Sample Period	Plain Standard Deviation	AR(1) Coefficient	Adjusted Standard Deviation
1983-2008	4.55%	0.45	8.28%
1991-2008	4.75%	0.39	7.79%
1998-2008	5.33%	0.36	8.27%

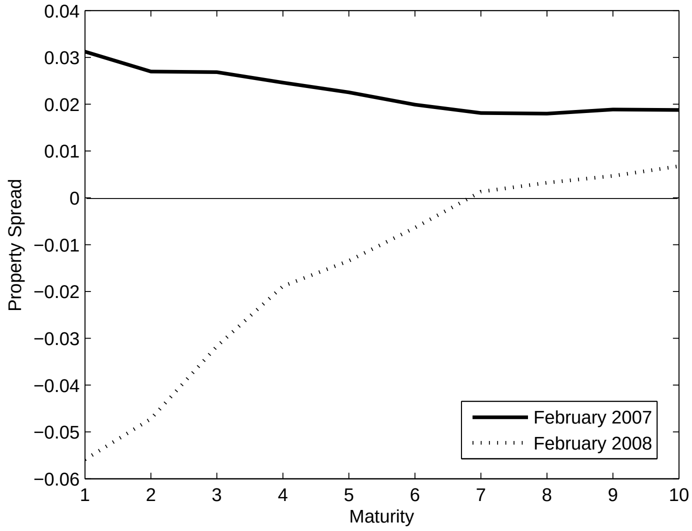

Figure 1: **The term structure of property spreads.** The lines show the property spreads of Halifax HPI contracts against maturity, based on ask prices, in February 2007 and one year later.

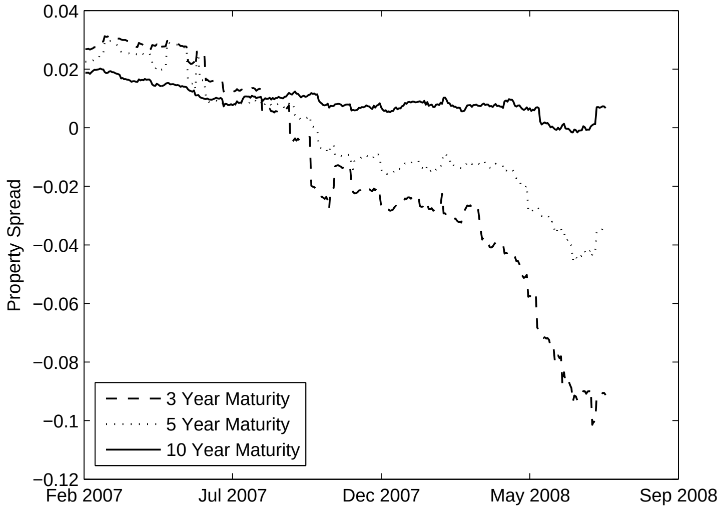

Figure 2: **Property spreads over time.** The figure plots the evolution of the property spreads of Halifax HPI 3-, 5- and 10-year contracts from February 2007 to July 2008.

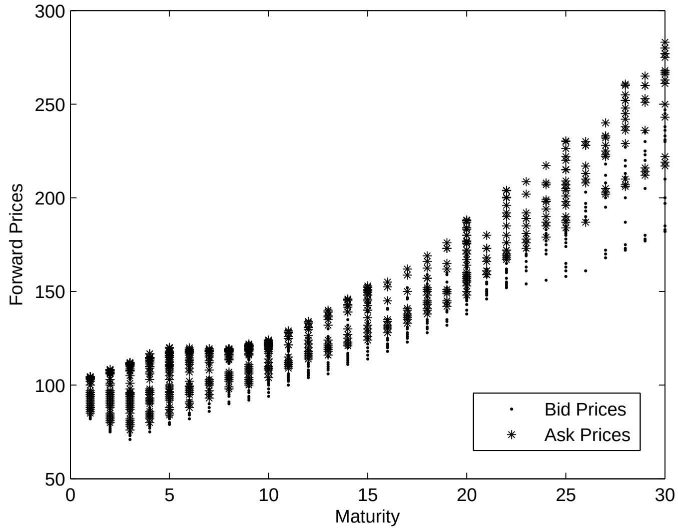

Figure 3: **Property forward prices.** The figure shows bid and ask prices for forward contracts on the Halifax HPI index for maturities ranging from one to 30 years. The spot level is always set at 100. Quotes are daily observed from February 2007 to August 2008.

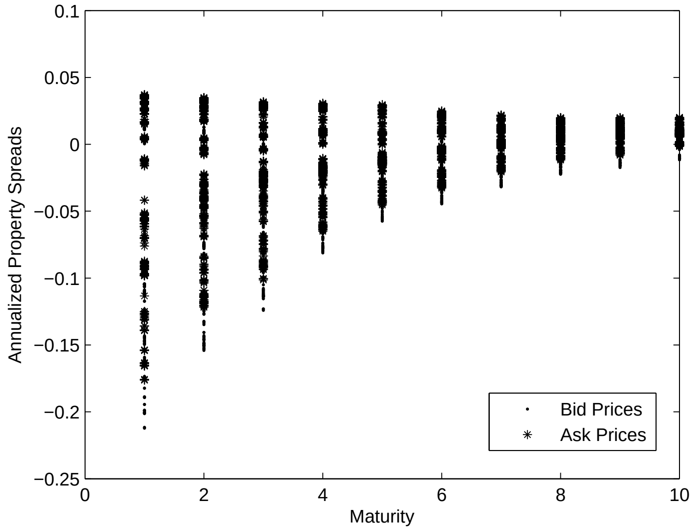

Figure 4: **Observed property spreads.** The figure shows property spreads implied by forward contracts on the Halifax HPI index against contract maturity. Quotes are daily observed from February 2007 to August 2008.

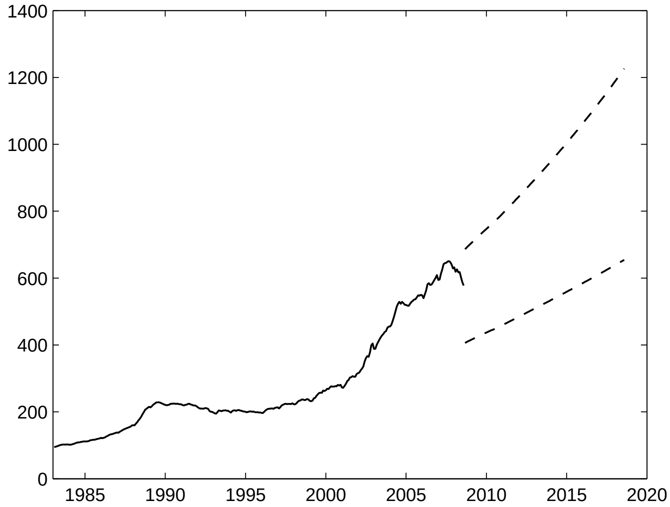

Figure 5: **Arbitrage free price bounds.** The figure plots historical levels of the Halifax HPI (1983=100) and the arbitrage free price bounds for forward prices of Halifax HPI index contracts (dashed lines).