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Marc ARNOLD
University of Zurich (SFI PhD Program)

Alexander F. WAGNER
University of Zurich and Swiss Finance Institute

Ramona WESTERMANN
University of Geneva (SFI PhD Program)

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Macroeconomic Conditions, Growth Options and the Cross-Section of Credit Risk

Marc Arnold, Alexander F. Wagner, Ramona Westermann *

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ABSTRACT

This paper develops a structural equilibrium model with intertemporal macroeconomic risk, incorporating the fact that firms are heterogeneous in their asset composition. Compared to firms which are mainly composed of invested assets, firms with growth options have larger costs of debt because they are more volatile and have a higher tendency to default during recession when marginal utility is high and recovery rates are low. Our model matches stylized facts regarding credit spreads, default probabilities, leverage, and investment clustering. Importantly, it also makes predictions about the cross-section of all these features.

JEL-code: G32

Keywords: Capital structure, macroeconomic risk, growth options, credit spread puzzle

*Arnold is a Ph.D. student at the Swiss Finance Institute at the University of Zurich, and Westermann a Ph.D. student at the Swiss Finance Institute at the University of Geneva. Wagner is an assistant professor of finance at the Swiss Finance Institute at the University of Zurich and a CEPR Research Affiliate. Office address: Department of Banking and Finance, Plattenstrasse 14, CH-8032 Zurich, Switzerland, Phone: +41-44-634-3963, Email: alexander.wagner@bf.uzh.ch. We thank an anonymous referee whose suggestions have greatly improved the paper. Tony Berrada, Simon Broda, Marc Chesney, Pierre Collin-Dufresne, Rajna Gibson, Michel Habib, Dirk Hackbarth, Mario Häfeli, Erwan Morellec, Gabriel Neukomm, Kjell Nyborg, Tatjana-Xenia Puhan, Alexandre Ziegler and seminar participants at the University of Zurich, the Humboldt University of Berlin, the C.R.E.D.I.T Conference in Venice, and the Research Day of the NCCR FINRISK provided helpful comments. This research was supported by the NCCR FINRISK, the Swiss Finance Institute, and the Research Priority Program "Finance and Financial Markets" of the University of Zurich.

1. Introduction

This paper examines the effect of time-varying macroeconomic conditions on credit spreads, firm value, and financial policy choices of firms with assets in place and growth options. The central thesis we develop is that expansion options react quite differently to the underlying risk in the economy than invested assets do. First, as growth options are levered claims, they are more volatile than assets in place. Second, we show that firms with lots of growth options ("growth firms") are more exposed to business cycle risk, which induces a stronger tendency to default during recession when recovery rates are low and marginal utilities high. Both effects raise the costs of debt and induce lower leverage choices of growth firms compared to those of firms with mainly invested assets.

Standard structural models of default face the challenge that they significantly underestimate credit spreads for corporate debt; this is the credit spread puzzle (see, e.g., Elton, Gruber, Agrawal, and Mann, 2001; Huang and Huang, 2003; Chen, Collin-Dufresne, and Goldstein, 2009). A related empirical regularity is that the puzzle is particularly strong for growth firms. Davydenko and Strebulaev (2007) show that, after controlling for standard credit risk factors, proxies of growth options are all positively and significantly related to credit spreads. Molina (2005) finds that firms with a higher ratio of fixed assets to total assets have lower yield spreads and higher ratings. Relatedly, firms with more growth options typically have lower leverage (Smith and Watts, 1992; Fama and French, 2002; Frank and Goyal, 2009). These cross-sectional features are not addressed by existing structural models because they consider firms with only invested assets.

Our model matches these facts. We show that allowing firms to be heterogeneous with respect to the importance of growth options in the values of their assets explains the aggregate credit spread puzzle, not only qualitatively, but also quantitatively. This is achieved while fitting historically reported asset volatilities and default rates for realistic debt maturities. Moreover, heterogeneity in the composition of assets can help explain cross-sectional variation of credit spreads and leverage. Our model is also consistent with observed default clustering, aggregate investment spikes and busts, and recovery rates. Additionally, we derive cross-sectional predictions for these features.

For our analysis, we develop a structural-equilibrium framework in the spirit of Bhamra, Kuehn, and Strebulaev (2010a). Thus, we embed a pure structural model of financial decisions into a consumption-based asset pricing model with a representative agent. Our model simultaneously incorporates both intertemporal macroeconomic risk (building on work by Hackbarth, Miao, and Morellec, 2006; Bhamra, Kuehn, and Strebulaev, 2010b; Chen, 2010), which has been shown to be important for explaining credit spreads and leverage, as well as expansion options. Macroeconomic shocks to the growth rate and volatility of earnings and to the growth rate and volatility of consumption arise due to switches between two states of the economy, boom and recession. The changes in the state of the economy are modeled via a Markov chain, a standard tool to model regime switches. The representative agent has the continuous time equivalent of Epstein-Zin-Weil preferences (Epstein and Zin, 1989; Weil, 1990). Therefore, how he prices claims depends on both his risk aversion and his elasticity of intertemporal substitution. Via the market price of consumption determined by the agent's preferences, we are able to link unobservable risk-neutral probabilities used in the structural model to historical probabilities. With this model, we can, therefore, study endogenously the effect of macroeconomic risk on credit spreads and optimal financing decisions.

We allow firms to have expansion options. These options are converted into invested assets when the underlying earnings process exceeds the investment boundary. We pinpoint the isolated effect of a firm's asset composition on credit risk and leverage by assuming, in the main analysis, that the exercise price of the growth option is financed through the sale of some assets in place, i.e., without additional funds being injected into the company. We also study equity-financing later in the paper. Default occurs when earnings are below the default threshold in a given regime. Shareholders maximize the value of equity by simultaneously choosing the optimal default and expansion option exercise policies. The capital structure is determined by trading off tax benefits of debt against default costs to maximize the ex-ante value of equity, i.e., the value of the firm.

The first result the model yields is that, like in other macroeconomic models, default boundaries are countercyclical, i.e., shareholders default earlier in recession than in boom. Thus, default is more likely in recession which, together with counter-cyclical marginal utilities and default costs, raises the costs of debt for all firms compared to a benchmark model without business cycle risk.

The central new feature of our model is that the asset composition alone matters significantly for the costs of debt. Two forces lead to the cross-sectional prediction that debt is particularly costly for firms with a high portion of expansion options in their assets' values. First, because options represent levered claims, firms with valuable growth options are more sensitive to the underlying earnings process than firms which consist of only invested assets. The volatility of the underlying earnings process would, consequently, underestimate the true default risk of growth firms. While the literature discusses this basic idea within equity-financed firms (Berk, Green, and Naik, 1999; Carlson, Fisher, and Giammarino, 2006), little is known about its impact on debt prices. Our structural model allows us to *jointly* analyze a firm's expansion policy and financial leverage. We show that the combination of these factors is critical for a full exploration of the quantitative implications of the riskiness of growth options on credit spreads.

The second driving force is that option values are more sensitive to macroeconomic regime changes than are assets in place. This higher sensitivity is, to some extent, another consequence of the idea that options represent levered claims. Importantly, an additional effect derives from the fact that the optimal exercise boundary of growth options increases in recession and decreases in boom. Intuitively, it is optimal to defer the exercise of an expansion option when the economy switches to recession, i.e., to wait for better times. Because the moneyness of growth options is regime-dependent, and because they represent levered claims, the continuation value of expansion options is more exposed to the macroeconomic state than the one of invested assets. Moreover, the changing moneyness causes expansion options to be less sensitive to the underlying development of the earnings process in recession than in boom, which reduces the value of the shareholders' option to defer default during bad times. Together, these effects amplify the counter-cyclicality of default thresholds for firms with a high portion of growth options. As marginal utility is high during bad times, the higher tendency to default in recession causes larger credit spreads under risk-neutral pricing for firms with expansion options than for those with only invested assets.

We then investigate the quantitative performance of the model in explaining empirically observed data. The literature suggests that an average BBB-rated firm has a 10 year credit spread in the range of $74 - 95$ basis points (bps). (This range is obtained by starting from the average bond yields reported in Davydenko and Strebulaev (2007) and Duffee (1998), and taking into account that around 35% of bond yields are due to non-default components). With our main set of parameters, a model without business cycle risk produces a mere 29 bps spread for an average firm. A standard macroeconomic model with optimal default thresholds in the spirit of Bhamra, Kuehn, and Strebulaev (2010b) or Chen (2010) implies a spread of 56 bps for average firms at issue which consist of only invested assets. Our estimate for the average BBB-rated US firm's asset composition is that total firm value is about 60% higher than the value of invested assets, which corresponds (approximately) to a Tobin's Q of 1.6.[1] For such a firm, we obtain a credit spread of about 66 bps when using optimal default thresholds, optimal expansion boundaries, and an earnings volatility such that the average asset volatility matches the one observed for BBB-rated firms. This spread is remarkably higher than the 39 bps our model implies for a firm with only invested assets. Note that the large difference arises even though leverage is kept constant; we only vary the characteristics of the assets themselves.

Empirical studies focus on aggregate data over cross-sections of firms, rather than on individual firm level data. Exploring this insight, Strebulaev (2007) is the first to show that it is crucial to consider the cross-sectional distribution of firms when relating the implications of capital structure models to empirical studies. Bhamra, Kuehn, and Strebulaev (2010b) extend this idea and investigate how the time-evolution of the cross-sectional distribution affects credit spreads and default probabilities. Following their approach, we characterize the aggregate dynamics by simulating over time a cross-section of average leverage ratios and asset composition ratios of individual firms which is structurally similar to the empirical distribution of BBB-rated firms. The average 10 and 20 year credit spreads of 81 and 103 basis points, respectively, from simulating this "true" cross-section in our model reflect their target credit spreads quite well. To solve the aggregate credit spread *puzzle*, a model needs to explain observed costs of debt while still matching historical default losses (given by the historical default probabilities and recovery rates), and asset volatilities. We consequently proceed by showing that the model-implied default rates and asset volatilities of BBB-rated firms

[1] Market values can be higher than book values also because of off-balance sheet assets, so there is, of course, a range for the asset composition of the "typical" firm.

are similar to the ones historically reported for realistic debt maturities. Hence, besides generating cross-sectional predictions for credit spreads, our model is also able to explain the credit spread puzzle.

The nature of assets, thus, has a powerful impact on costs of debt. Not surprisingly, it also affects the observed features of leverage. At initiation, we find that a firm with an average growth option optimally holds about $4-5\%$ lower leverage than one with only invested assets. Additionally, we obtain pro-cyclical optimal leverage decisions of firms, in line with Covas and Den Haan (2006) and Korteweg (2011). The reason is that the default risk is higher in recession than in boom. The negative relationship between growth options and leverage also maintains when simulating over time our model-implied cross-section of BBB-rated firms. In this simulation, however, firms deviate from their initially optimal leverage in a way such that the aggregate market leverage of the whole cross-section becomes counter-cyclical, consistent with Korajczyk and Levy (2003) and Bhamra, Kuehn, and Strebulaev (2010b).

We derive additional testable predictions when studying the aggregate dynamics of our model economy. Credit spreads and default rates are counter-cyclical, as reported in the literature. Next, aggregate investment patterns are strongly pro-cyclical, with investment spikes often occurring when the regime switches from recession to boom, reflecting the findings in the empirical investment literature (Barro, 1990; Cooper, Haltiwanger, and Power, 1999). Finally, our model makes specific cross-sectional predictions. For example, realized recovery rates are lower for growth firms.

Our paper contributes to several streams of previous research. First, the fact that growth options are empirically strongly associated with observed leverage has, of course, also prompted other explanations. The most prominent of these additional explanations, agency, comes in two primary forms: a shareholder-bondholder conflict and a manager-shareholder conflict. Appealing to the former, Smith and Watts (1992) and Rajan and Zingales (1995) suggest that debt costs associated with shareholder-bondholder conflicts typically increase with the number of growth options available to the firm due to underinvestment (Myers, 1977) and overinvestment by way of asset substitution (Jensen, 1986; see also Sundaresan and Wang, 2007).[2] According to Leland (1998), however, optimal leverage even increases when firms can engage in asset substitution. Similarly, Parrino and Weisbach (1999) conclude that stockholder-bondholder conflicts are too limited to explain the cross-sectional variation in capital structure. Childs, Mauer, and Ott (2005) show how short-term debt reduces agency costs. Hackbarth and Mauer (2010) demonstrate that the joint choice of debt priority structure and capital structure can virtually eliminate the suboptimal investment incentives of equityholders. Neither of the papers incorporates macroeconomic risk.

As for manager-shareholder conflicts, Morellec (2004) shows that agency costs of free cash flow can explain the low debt levels observed in practice, and the negative relationship between debt levels and the number of growth options; see also Barclay, Smith, and Morellec (2006). Morellec,

[2]See Lyandres and Zhdanov (2010) for an explanation for accelerated investment that does not rely on agency.

Nikolov, and Schürhoff (2009) conclude that even small costs of control challenges are sufficient to explain the low-leverage puzzle. It is still a matter of debate to what extent conflicts of interest between managers and stockholders cause the empirically observed patterns. Graham (2000), for example, tests a wide set of managerial entrenchment variables and finds only "weak evidence that managerial entrenchment permits debt conservatism" (p. 1931). In any case, our model is not inconsistent with either of these views. It offers a quantitatively important reason for the cross-sectional variation in leverage and credit spreads that derives solely from the nature of assets of firms.[3]

Second, at the core of our model is the idea from recent literature that macroeconomic (business cycle) risk matters in powerful ways for the costs of corporate debt and financial decisions, because firms are more likely to default when doing so is costly (see, e.g., Demchuk and Gibson, 2006; Almeida and Philippon, 2007; Bhamra, Kuehn, and Strebulaev, 2010b; Chen, 2010). What we add to this literature is the idea that the impact of business cycle risk depends on the asset base of a firm.

In contemporaneous and independent work, Chen and Manso (2010) set up a model similar to ours with expansion options. Their focus, however, is on the debt overhang problem, and not on explaining the credit spread puzzle or developing cross-sectional predictions – the central tasks of this paper.

Finally, our structural-equilibrium framework draws on insights from consumption-based asset pricing models (Lucas, 1978; Bansal and Yaron, 2004).

The paper proceeds as follows. In Section 2, we set up our valuation framework. We solve the model in Section 3. Section 4 discusses our parameter and firm sample choices, as well as the optimal default and expansion policies. Section 5 outlines qualitative properties of our model for the aggregate economy. We turn to the quantitative implications for BBB-rated firms in Section 6. Section 7 concludes.

2. The Model

We build a structural model with intertemporal macroeconomic risk, embedded inside a representative agent consumption-based asset pricing framework. The structural model is based on a standard continuous time model of capital structure decisions in the spirit of Mello and Parsons (1992), as extended by Hackbarth, Miao, and Morellec (2006) for business cycle fluctuations. Additionally, we explicitly model growth opportunities. Following Bhamra, Kuehn, and Strebulaev

[3]An alternative explanation for why low leverage may be optimal in the high-tech sectors is offered in Miao (2005). In his model, when a sector experiences technological growth, more competitors enter, leading to falling prices and possibly to a greater probability of default. Yet other explanations appeal to the fact that firms have the option to issue additional debt (Collin-Dufresne and Goldstein, 2001).

(2010b) or Chen (2010), embedding the model of capital structure into a consumption-based asset pricing model allows the valuation of corporate securities using the risk-neutral measure implied by the preferences of the representative agent.

The economy consists of N infinitely-lived firms with assets in place and possibly growth options, a large number of identical infinitely-lived households, and a government serving as a tax authority. We assume that there are two different macroeconomic states, namely boom (B) and recession (R). Formally, we define a time-homogeneous Markov chain $I_{t\geq 0}$ with state space $\{B, R\}$ and generator $Q := \begin{bmatrix} -\lambda_B & \lambda_B \\ \lambda_R & -\lambda_R \end{bmatrix}$, where $\lambda_i \in (0,1)$ denotes the rate of leaving state i. In the main analysis, we consider $\lambda_B < \lambda_R$, as in Hackbarth, Miao, and Morellec (2006).

The following properties hold: First, the probability that the chain stays in state i longer than some time $t \geq 0$ is given by $e^{-\lambda_i t}$. Second, the probability that the regime shifts from i to j during an infinitesimal time interval Δt is given by $\lambda_i \Delta t$. Third, the expected duration of regime i is $\frac{1}{\lambda_i}$, and the expected fraction of time spent in that regime is $\frac{\lambda_j}{\lambda_i+\lambda_j}$.

Aggregate output C_t follows a regime-switching geometric Brownian motion:

$$\frac{\mathrm{d}C_t}{C_t} = \theta_i \mathrm{d}t + \sigma_i^C \mathrm{d}W_t^C, \; i = B, R, \tag{1}$$

where W_t^C is a Brownian motion independent of the Markov chain, and θ_i, σ_i^C are the regime-dependent growth-rates and volatilities of the aggregate output. In equilibrium, aggregate consumption equals aggregate output. Hence, the above specification gives rise to uncertainty about the future moments of consumption growth.

To incorporate the impact of the intertemporal distribution of consumption risk on the representative household's utility, we assume the continuous-time analog of Epstein-Zin-Weil preferences (Epstein and Zin, 1989; Weil, 1990), which are of stochastic differential utility type (Duffie and Epstein, 1992a,b). Specifically, the utility index U_t over a consumption process C_s solves

$$U_t = \mathbb{E}^{\mathbb{P}}\left[\int_t^\infty \frac{\rho}{1-\delta} \frac{C_s^{1-\delta} - ((1-\gamma)\,U_s)^{\frac{1-\delta}{1-\gamma}}}{((1-\gamma)\,U_s)^{\frac{1-\delta}{1-\gamma}} - 1} \mathrm{d}s \,|\mathcal{F}_t\right], \tag{2}$$

where ρ is the rate of time preference, γ determines the coefficient of relative risk aversion for a timeless gamble, and $\Psi := \frac{1}{\delta}$ is the elasticity of intertemporal substitution for deterministic consumption paths.

The stochastic discount factor m_t then follows the dynamics

$$\frac{\mathrm{d}m_t}{m_t} = -r_i \mathrm{d}t - \eta_i \mathrm{d}W_t^C + (e^{\kappa_i} - 1)\,\mathrm{d}M_t, \tag{3}$$

with M_t being the compensated process associated with the Markov chain, and

$$r_i = -\rho \frac{(1-\gamma)}{1-\delta}\left(\frac{\delta-\gamma}{1-\gamma}h_i^{\delta-1}-1\right)+\gamma\theta_i-\frac{1}{2}\gamma(1+\gamma)\left(\sigma_i^C\right)^2-\lambda_i\left(e^{\kappa_i}-1\right) \tag{4}$$

$$\eta_i = \gamma\sigma_i^C \tag{5}$$

$$\kappa_i = (\delta-\gamma)\log\left(\frac{h_j}{h_i}\right), \tag{6}$$

see Chen (2010). h_B, h_R solve a non-linear system of equations given in the Appendix A.1, equation (A-5). r_i are the regime-dependent real risk-free interest rates, η_i the risk prices for systematic Brownian shocks affecting aggregate output, and κ_i is the relative jump size of the discount factor when the Markov chain leaves state i (and, consequently, $\kappa_j = \frac{1}{\kappa_i}$).

Credit spreads are based on nominal yields and taxes are collected on nominal earnings. To link nominal to real values such as the real interest rate introduced in the previous section, we specify a stochastic price index as

$$\frac{\mathrm{d}P_t}{P_t} = \pi\mathrm{d}t + \sigma^{P,C}\mathrm{d}W_t^C + \sigma^{P,id}\mathrm{d}W_t^P, \tag{7}$$

with W_t^P being a Brownian motion describing the idiosyncratic price index shock, independent of the consumption shock Brownian W_t^C and the Markov chain. π denotes the expected inflation rate, and $\sigma^{P,C} < 0, \sigma^{P,id} > 0$ are the volatilities of the stochastic price index associated with the consumption shock and the idiosyncratic price index shock, respectively. The nominal interest rate r_i^n is then given by

$$r_i^n = r_i + \pi - \sigma_P^2 - \sigma^{P,C}\eta_i, \tag{8}$$

with $\sigma_P := \sqrt{\left(\sigma^{P,C}\right)^2 + \left(\sigma^{P,id}\right)^2}$ being the total volatility of the stochastic price index.

At any point in time, the nominal earnings process of a firm follows

$$\frac{\mathrm{d}X_t}{X_t} = \mu_i\mathrm{d}t + \sigma_i^{X,C}\mathrm{d}W_t^C + \sigma^{P,id}\mathrm{d}W_t^P + \sigma^{X,id}\mathrm{d}W_t^X, \quad i = B, R, \tag{9}$$

where W_t^X is a Brownian motion describing an idiosyncratic shock, independent of the aggregate output shock W_t^C, the consumption price index shock W_t^P, and the Markov chain. μ_i are the regime-dependent drifts, $\sigma_i^{X,C} > 0$ the firm-specific regime-dependent volatilities associated with the aggregate output process, and $\sigma^{X,id} > 0$ the firm-specific volatility associated with the idiosyncratic Brownian shock. As suggested by the literature, we posit that $\sigma_B^{X,C} < \sigma_R^{X,C}$ (Ang and Bekaert, 2004).

Denote the risk-neutral measure by $\mathbb{Q}$. Following Chen (2010), the unlevered asset value is

$$V_t = X_t y_i \quad \text{for } I_t = i, \tag{10}$$

with y_i being the price-earnings ratio in state i:

$$y_i = \frac{r_j^n - \tilde{\mu}_j + \tilde{\lambda}_j + \tilde{\lambda}_i}{\left(r_i^n - \tilde{\mu}_i + \tilde{\lambda}_i\right)\left(r_j^n - \tilde{\mu}_j + \tilde{\lambda}_j\right) - \tilde{\lambda}_i \tilde{\lambda}_j}. \tag{11}$$

In Equation (11), $\tilde{\mu}_i$ are the expected growth rates of the firm's nominal earnings under the risk-neutral measure $\mathbb{Q}$

$$\tilde{\mu}_i := \mu_i - \sigma_i^{X,C}\left(\eta_i + \sigma^{P,C}\right) - \left(\sigma^{P,id}\right)^2, \tag{12}$$

and $\tilde{\lambda}_i$ are the risk-neutral transition intensities given by

$$\tilde{\lambda}_i = e^{\kappa_i}\lambda_i. \tag{13}$$

As in Bhamra, Kuehn, and Strebulaev (2010b), the price-earnings ratio in the main analysis is higher in boom, $y_B > y_R$. Finally, note that the total volatility of the earnings process is

$$\tilde{\sigma}_i = \sqrt{\left(\sigma_i^{X,C}\right)^2 + (\sigma^{P,id})^2 + (\sigma^{X,id})^2}. \tag{14}$$

The expansion option of the firm is modeled as an American call option on the earnings. Specifically, at any time $\bar{t}$, the firm can pay exercise costs K to achieve additional future earnings of sX_t for all $t \geq \bar{t}$ for some factor $s > 0$. We assume that if a firm exercises its expansion option, the option is converted into assets in place, such that the firm consists of only invested assets. The exercise of the growth option is assumed to be irreversible. At default, bondholders recover not only a fraction of the assets in place, but also a fraction of the option's value. Thus, the option can be exercised independently of the considered firm.

For the financing of investment, we present two variants. In the main analysis, we wish to abstract away from the effect of fund injections by debt- or equityholders to pay the exercise price, and instead to isolate the effect of growth options in the value of firms' assets on corporate securities. Therefore, we first assume that, at exercise, the firm pays the exercise costs K of the option by selling a part of its assets in place.[4] In detail, while exercising the option at time $\bar{t}$ entitles the firm to total future earnings of $(s+1)X_t$ for all $t \geq \bar{t}$, financing the exercise costs requires to sell a fraction $\frac{K}{X_{\bar{t}} y_{\bar{i}}}$ of these earnings, where $\bar{i}$ is the realized state of the economy at the time of exercise. Hence, the total earnings of the firm at any point in time after exercise correspond to $\left((s+1) - \frac{K}{X_{\bar{t}} y_{\bar{i}}}\right) X_t$. Second, we also consider equity financing of the exercise costs K.

The critical measure to capture the relative importance of a firm's expansion option in the value of its assets is the *asset composition ratio.* We define it as the value of the firm, divided by the

[4]Indirect financing by selling assets often occurs, e.g., when acquirers divest part of target companies' assets following takeovers (Bhagat, Shleifer, and Vishny, 1990; Kaplan and Weisbach, 1992). Of course, the model simplifies in that in reality, firms have different types of assets.

value of invested assets. Certainly, the value of the firm does not only contain the value of the invested assets and the expansion option, but also the value of the tax shield and bankruptcy costs. Nevertheless, we use this measure because the impact of the tax shield and bankruptcy costs on the ratio is relatively small, and, importantly, the direct empirical analog of the asset composition ratio is Tobin's Q.

Corporate taxes are paid at a constant rate τ, and full offsets of corporate losses are allowed. In our framework, firms are leveraged because debt allows it to shield part of its income from taxation. Once debt has been issued, a firm pays a total coupon c at each moment in time. Following the standard in the literature, we assume that firms finance coupons by injecting funds. At any point in time, shareholders have the option to default on their debt obligations, as well as the possibility to exercise an expansion option. Default is triggered when shareholders are no longer willing to inject additional equity capital to meet net debt service requirements (Leland, 1998). If default occurs, the firm is immediately liquidated and bondholders receive the unlevered asset value less default costs, reflecting the 'absolute priority' of debt claims. The default costs in regime i are assumed to be a fraction $1-\alpha_i$ of the unlevered asset value at default, with $\alpha_i \in (0, 1]$. We suppose that recovery rates are lower in recession, i.e., $\alpha_R < \alpha_B$ (Frye, 2000). The incentive to issue debt is limited due to the possibility of costly financial distress.

Equityholders face the following decisions: First, once debt has been issued, they select the default and expansion policies that maximize equity value. Hence, both expansion and default are chosen endogenously. Second, they determine the optimal capital structure by choosing the coupon level which maximizes the value of the firm. The model does not allow restructuring of debt neither when the option is exercised nor at endogenous restructuring points. The main reason is that expansion opportunities preclude a scaling feature of the model solution.[5]

The main text presents the model and its solution for infinite debt maturity. We also solve and use the case of finite debt maturity, in which we consider the stationary environment of Leland (1998): The firm issues debt with a constant principal p and a constant total coupon c paid at each moment in time. A fraction m of the total debt is continuously rolled over. In particular, the firm continuously retires outstanding debt principal at rate mp and replaces it with new debt vintages of identical coupon and principal. Finite maturity debt is, therefore, characterized by the tuple (c, m, p). This setup leads to a time-homogenous setting. Throughout the paper, it is assumed that debt is issued at par.

[5] The scaling property states that, conditional on the current regime of the economy, the optimal coupon, the optimal default thresholds, the investment boundaries, as well as the values of debt and equity at restructuring points are all homogeneous of degree one in earnings. The absence of a scaling property impedes not only closed-form results, but also the application of numerical solution methods with backward induction. Furthermore, even formulating an ansatz for the valuation of corporate securities with expansion opportunities and restructuring requires strong assumptions on the structure of the solution, in particular on the relation between restructuring and exercise boundaries.

3. Model solution

The model is solved by backward induction. We start by calculating the value of corporate securities of a firm consisting of only invested assets, taking the capital structure, default and expansion policies as given at this point. Next, the value of the growth option, also for given capital structure and policies, is derived. We then proceed with the value of corporate securities of a firm which consists not only of assets in place, but also holds an expansion option. Finally, we obtain the expansion and default policies which simultaneously maximize the value of equity, as well as the capital structure which maximizes the firm value.

As in Hackbarth, Miao, and Morellec (2006), we assume that the optimal strategies are of regime-dependent threshold type in X (for a formal proof in the case of expansion thresholds only, see Guo and Zhang, 2004). Precisely, suppose that $\hat{D}_i$ and D_i are the default thresholds in regime $i = B, R$ of a firm with only invested assets, and of a firm with both invested assets and a growth option, respectively. X_i denotes the exercise boundary of the growth option in regime $i = B, R$. In what follows, we present the case that $D_B < D_R$, $X_B < X_R$ and $\hat{D}_B < \hat{D}_R$, i.e., the boundaries are lower in boom for both expansion and default (before and after expansion).[6] Finally, we presume that $\max\left\{D_R, \hat{D}_R\right\} < X_B$, i.e., we are interested in firms which exercise their expansion option with a positive probability, and we exclude the possibility of immediate default after expansion. The optimal default and expansion policies for relevant parameter regions satisfy the assumed ordering.

3.1. Firms with only invested assets

Let $\hat{d}_i(X), \hat{t}_i(X), \hat{b}_i(X)$ and $\hat{f}_i(X)$ denote the value of corporate debt, taxes, bankruptcy cost, and total firm value, respectively, in regime $i = B, R$. Hackbarth, Miao, and Morellec (2006) show how to solve a similar structural model.[7] The solution for our case can be found in Appendix A.2.

3.2. The value of the growth option

In order to determine the value of corporate securities of firms with both assets in place and a growth option, we now need to calculate the value of a growth option under regime switches.[8]

[6] Note that we can assume without loss of generality that $D_B < D_R$ (if not, interchange the names of the regimes). The case $D_B < D_R, \hat{D}_B < \hat{D}_R$, and $X_B > X_R$, (i.e., the exercise boundary in recession is lower than the one in boom) can be solved by analogous techniques. A changing order of the default boundaries after exercising the option, i.e., the case that $D_B < D_R$ and $\hat{D}_B > \hat{D}_R$, is not considered. Finally, the case of the presence of only one regime is presented in Appendix A.3, Case 2.

[7] Even though they do not consider regime-dependence of volatility, the basic approach remains unchanged.

[8] We emphasize that the value of the option in the ultimate solution of the model indeed depends on the default policy of the firm. Equityholders choose default and expansion policies simultaneously. The resulting interdependence between the two policies affects the value of the growth option. This effect is not explicit in the presented calculations due to the backward solution method.

Denote the value functions of the growth option in regime B and R by $G_B(X)$ and $G_R(X)$, respectively. For each regime i, the option is exercised immediately whenever $X \geq X_i$ (option exercise region); otherwise it is optimal to wait (option continuation region). This structure results in the following system of ODEs for the value function:

For $0 \leq X < X_B$:

$$\begin{cases} r_B^n G_B(X) &= \tilde{\mu}_B X G_B'(X) + \frac{1}{2}\tilde{\sigma}_B^2 X^2 G_B''(X) + \tilde{\lambda}_B \left(G_R(X) - G_B(X)\right) \\ r_R^n G_R(X) &= \tilde{\mu}_R X G_R'(X) + \frac{1}{2}\tilde{\sigma}_R^2 X^2 G_R''(X) + \tilde{\lambda}_R \left(G_B(X) - G_R(X)\right) \end{cases} \tag{15}$$

For $X_B \leq X < X_R$:

$$\begin{cases} G_B(X) &= sXy_B - K \\ r_R^n G_R(X) &= \tilde{\mu}_R X G_R'(X) + \frac{1}{2}\tilde{\sigma}_R^2 X^2 G_R''(X) + \tilde{\lambda}_R \left(sXy_B - K - G_R(X)\right) \end{cases} \tag{16}$$

For $X \geq X_R$:

$$\begin{cases} G_B(X) &= sXy_B - K \\ G_R(X) &= sXy_R - K \end{cases} \tag{17}$$

Whenever the process X is in the option continuation region, which corresponds to system (15) and the second equation of (16), the required rate of return r_i^n (left-hand side) must be equal to the realized rate of return (right-hand side). The latter is obtained by Ito's lemma for regime switches (see, e.g., Yin, Song, and Zhang, 2004). Here, the last term accounts for a possible jump in the value of the growth option due to a regime switch. It is calculated as the probability of a regime shift, $\tilde{\lambda}_B$ or $\tilde{\lambda}_R$, times the associated change in the value of the option. The first equation of (16) and the system (17) state the payoff of the option at exercise, since the process is in the option exercise region in these cases.

The boundary conditions are:

$$\lim_{X \searrow 0} G_i(X) = 0, \quad i = B, R \tag{18}$$

$$\lim_{X \searrow X_B} G_R(X) = \lim_{X \nearrow X_B} G_R(X) \tag{19}$$

$$\lim_{X \searrow X_B} G_R'(X) = \lim_{X \nearrow X_B} G_R'(X) \tag{20}$$

$$\lim_{X \nearrow X_R} G_R(X) = sX_R y_R - K \tag{21}$$

$$\lim_{X \nearrow X_B} G_B(X) = sX_B y_B - K \tag{22}$$

Condition (18) ensures that the option value goes to zero as the earnings approach zero. Conditions (19) and (20) represent the value-matching and smooth-pasting conditions of the value function in recession at the exercise boundary in boom. The remaining conditions (21)-(22) are the value-matching conditions at the exercise boundaries in boom and recession, respectively. The solution of this system and its derivation are given in Appendix A.3.

We remark that similar to the occurrence of default, there are two possible ways of triggering the exercise of the expansion option: Either when the idiosyncratic shock X reaches the exercise boundary X_i in a given regime, or when the regime switches from recession to boom and X lies between X_B and X_R.

The above system of ODEs (15)-(17) subject to its boundary conditions (18)-(22) determines the value of the growth option for any given pair of exercise boundaries X_B and X_R. In the full model solution, we will need to derive option values for optimal exercise boundaries of equityholders in both levered and unlevered firms. In unlevered firms, the optimal exercise boundaries are denoted X_B^{unlev} and X_R^{unlev}, respectively. These optimal exercise boundaries solve the following additional boundary conditions:

$$\lim_{X \nearrow X_R^{unlev}} G'_R(X) = sy_R \tag{23}$$

$$\lim_{X \nearrow X_B^{unlev}} G'_B(X) = sy_B. \tag{24}$$

For ease of notation, we denote the unlevered value of the growth option by G_i^{unlev}, i.e., $G_i^{unlev}(X) = G_i\left(X \left| X_B^{unlev}, X_R^{unlev}\right.\right)$. Appendix A.3 discusses the solution.

3.3. Firms with invested assets and expansion options

Using the previous results, we finally derive the value of corporate securities of a general firm, as well as the default and expansion thresholds selected by shareholders.

In each regime, the firm faces three different regions depending on the value of X: Below the default threshold, the firm is in the default region where it defaults immediately, and debtholders receive a fraction α_i of the total asset value. The firm is in the continuation region if X is between the default threshold and the exercise boundary. Finally, the exercise region is reached if X is above the exercise boundary. After exercise, the firm consists of only invested assets, endowed with the initially determined optimal coupon level. As the default policy is an ex-post policy, the optimal default thresholds now correspond to the ones of a firm with only invested assets. That is, shareholders optimally adapt their default policy upon expansion. Debtholders anticipate this change.

3.3.1. The valuation of corporate debt

Let $d_i(X)$ denote the value of corporate debt in regime $i = B, R$. An investor holding corporate debt requires an instantaneous return equal to the nominal risk-free rate r_i^n. Again, an application of Ito's lemma with regime switches shows that debt satisfies the following system of ODEs:

For $X \leq D_B$:

$$\begin{cases} d_B(X) & = & \alpha_B \left(X y_B + G_B^{unlev}(X) \right) \\ d_R(X) & = & \alpha_R \left(X y_R + G_R^{unlev}(X) \right) \end{cases} \tag{25}$$

For $D_B < X \leq D_R$:

$$\begin{cases} r_B^n d_B(X) & = & c + \tilde{\mu}_B X d_B'(X) + \frac{1}{2}\tilde{\sigma}_B^2 X^2 d_B''(X) + \tilde{\lambda}_B \left(\alpha_R \left(X y_R + G_R^{unlev}(X) \right) - d_B(X) \right) \\ d_R(X) & = & \alpha_R \left(X y_R + G_R^{unlev}(X) \right) \end{cases} \tag{26}$$

For $D_R < X < X_B$:

$$\begin{cases} r_B^n d_B(X) & = & c + \tilde{\mu}_B X d_B'(X) + \frac{1}{2}\tilde{\sigma}_B^2 X^2 d_B''(X) + \tilde{\lambda}_B \left(d_R(X) - d_B(X) \right) \\ r_R^n d_R(X) & = & c + \tilde{\mu}_R X d_R'(X) + \frac{1}{2}\tilde{\sigma}_R^2 X^2 d_R''(X) + \tilde{\lambda}_R \left(d_B(X) - d_R(X) \right) \end{cases} \tag{27}$$

For $X_B \leq X < X_R$:

$$\begin{cases} d_B(X) & = & \hat{d}_B \left((s+1) X - \frac{K}{y_B} \right) \\ r_R^n d_R(X) & = & c + \tilde{\mu}_R X d_R'(X) + \frac{1}{2}\tilde{\sigma}_R^2 X^2 d_R''(X) + \tilde{\lambda}_R \left(\hat{d}_B \left((s+1) X - \frac{K}{y_B} \right) - d_R(X) \right) \end{cases} \tag{28}$$

And, finally, for $X \geq X_R$:

$$\begin{cases} d_B(X) & = & \hat{d}_B \left((s+1) X - \frac{K}{y_B} \right) \\ d_R(X) & = & \hat{d}_R \left((s+1) X - \frac{K}{y_R} \right) \end{cases} \tag{29}$$

In system (25), the firm is in the default region in both boom and recession. Here, debtholders receive $\alpha_i \left(X y_i + G_i^{unlev}(X) \right)$ at default. As the default boundary in boom is lower than the one in recession, system (26) corresponds to the firm being in the continuation region in boom, and in the default region in recession. For the continuation region in boom, the left-hand side of the first equation is the rate of return required by investors for holding corporate debt for one unit of time. The right-hand side is the realized rate of return, computed by Ito's lemma as the expected change in the value of debt plus the coupon payment c. The last term captures the possible jump in the value of debt in case of a regime switch, which triggers immediate default. Similarly, equations (27) describe the case that the firm is in the continuation region in both boom and recession. The next system, (28), deals with the case in which the firm is in the exercise region in boom, and in the continuation region in recession. After exercising the option, the firm owns total assets in place of $X y_i + s X y_i - K$, reflecting the notion that the exercise costs of the growth option are financed by selling assets. The value of debt must then be equal to the value of debt of a firm with only invested assets, i.e., $d_B(X) = \hat{d}_B((s+1)X - \frac{K}{y_B})$, which is the first equation in (28). The second equation is obtained by the same approach as in (27), where the last term captures the fact that a regime switch from recession to boom triggers immediate exercise of the expansion option in this case. Finally, equations (29) describe the case that the firm is in the exercise region in both boom and recession.

The system for finite debt maturity can be found in Appendix A.5.

The boundary conditions for debt are as follows:

$$\lim_{X \searrow D_R} d_B(X) = \lim_{X \nearrow D_R} d_B(X) \tag{30}$$

$$\lim_{X \searrow D_R} d'_B(X) = \lim_{X \nearrow D_R} d'_B(X) \tag{31}$$

$$\lim_{X \searrow D_B} d_B\left(X\right) = \alpha_B \left(D_B y_B + G_B^{unlev}(D_B)\right) \tag{32}$$

$$\lim_{X \searrow D_R} d_R\left(X\right) = \alpha_R \left(D_R y_R + G_R^{unlev}(D_R)\right) \tag{33}$$

$$\lim_{X \searrow X_B} d_R(X) = \lim_{X \nearrow X_B} d_R(X) \tag{34}$$

$$\lim_{X \searrow X_B} d'_R(X) = \lim_{X \nearrow X_B} d'_R(X) \tag{35}$$

$$\lim_{X \nearrow X_B} d_B\left(X\right) = \hat{d}_B \left(\left(s+1\right) X_B - \frac{K}{y_B}\right) \tag{36}$$

$$\lim_{X \nearrow X_R} d_R\left(X\right) = \hat{d}_R \left(\left(s+1\right) X_R - \frac{K}{y_R}\right) \tag{37}$$

(30) and (31) are the value-matching and smooth-pasting conditions for the debt value in boom at the default boundary in recession. Similarly, (34) and (35) are the corresponding conditions for the debt value in recession at the option exercise boundary in boom. (32) and (33) are the value-matching conditions at the default thresholds, and (36) and (37) are the value-matching conditions at the option exercise boundaries. The default thresholds and option exercise boundaries are chosen by shareholders, and, hence, we do not have the corresponding smooth-pasting conditions for debt.

The solution of this system is given in closed form in Appendix A.4.1.

3.3.2. The valuation of tax benefits

Let $t_i(X)$ denote the value of tax benefits in regime $i = B, R$. Debt coupon payments shield income from taxation. We assume full loss carry-forwards. Therefore, the value of tax benefits corresponds to the value of debt with recovery rates equal to zero and a coupon of $c\tau$. In detail, we obtain a system of equations akin to the system (25)-(29), and boundary conditions similar to (30) - (37). (32) - (33) translate into

$$\lim_{X \searrow D_i} t_i\left(X\right) = 0, \qquad i = B, R, \tag{38}$$

reflecting the loss of tax benefits at bankruptcy. At the option exercise boundary, we have that

$$\lim_{X \nearrow X_i} t_i(X) = \hat{t}_i \left((s+1) X_i - \frac{K}{y_i} \right), \qquad i = B, R, \tag{39}$$

corresponding to the conditions (36) - (37). In words, if the option is exercised, the value of the tax shield is equal to the one of a firm with only invested assets.

3.3.3. The valuation of default costs

Let $b_i(X)$ denote the value of default (or bankruptcy) costs in regime $i = B, R$. $b_i(X)$ can be calculated as the value of a debt contract with recovery rates $1 - \alpha_B$ and $1 - \alpha_R$, respectively, and a coupon of zero, as there are no continuous earnings associated with default costs.

The value-matching boundary conditions at default, (32) - (33), then correspond to

$$\lim_{X \searrow D_i} b_i(X) = (1 - \alpha_i) \left(D_i y_i + G_i^{unlev}(D_i) \right), \qquad i = B, R, \tag{40}$$

reflecting the fact that the value of default costs at the boundary must be 1 minus the recovery on the value of assets in place and the growth option. At option exercise, the value-matching conditions are given by

$$\lim_{X \nearrow X_i} b_i(X) = \hat{b}_i \left((s+1) X_i - \frac{K}{y_i} \right), \qquad i = B, R. \tag{41}$$

The intuition is that, at the exercise boundary of the option, default costs must be equal to the ones of a firm with only invested assets.

3.3.4. Firm value

Total firm value f_i in regime $i = B, R$ is given by the value of assets in place $y_i X$, plus the value of the expansion option $G_i(X)$ and the value of tax benefits from debt $t_i(X)$, less the value of default costs $b_i(X)$, i.e.,

$$f_i(X) = y_i X + G_i(X) + t_i(X) - b_i(X). \tag{42}$$

3.3.5. The valuation of equity

The levered firm value equals the sum of debt and equity values. Hence, the equity value $e_i(X)$, $i = B, R$, can be written in a closed form expression as

$$e_i(X) = f_i(X) - d_i(X) = y_i X + G_i(X) + t_i(X) - b_i(X) - d_i(X). \tag{43}$$

3.3.6. Default and expansion policies

Managers select the default and investment policies that maximize the value of equity ex-post. Denote these policies by D_i^* and X_i^*, respectively. Formally, the default policy which maximizes the equity value is determined by postulating that the first derivative of the equity value has to be zero at the default boundary in each regime. Simultaneously, optimality of the option exercise boundaries is achieved by equating the first derivative of the equity value at the exercise boundary with the first derivative of the equity value of a firm with only invested assets after expansion, evaluated at the corresponding earnings in both regimes. These four optimality conditions are smooth-pasting conditions for equity at the respective boundaries:

$$\begin{cases} e'_B(D_B^*) &= 0 \\ e'_R(D_R^*) &= 0 \\ e'_B(X_B^*) &= \hat{e}'_B(sX_B^* - \frac{K}{y_B}) \\ e'_R(X_R^*) &= \hat{e}'_R(sX_R^* - \frac{K}{y_R}). \end{cases} \quad (44)$$

We solve this system numerically.

3.3.7. Capital structure

For each coupon level c, debtholders evaluate debt at issuance anticipating the ex-post optimal default and expansion decisions of shareholders. As debt-issue proceeds accrue to shareholders, the latter do not only care about the value of equity, but also about the value of debt. Hence, the optimal capital structure is determined ex-ante by the coupon level c^* which maximizes the value of equity and debt, i.e., the value of the firm. Denote by $f_i^*(X)$ the firm value given optimal ex-post default and expansion thresholds as determined by the system (44). The ex-ante optimal coupon of this firm solves

$$c_i^* := \text{argmax}_c f_i^*(X). \quad (45)$$

As indicated in equation (45), the optimal initial capital structure depends on the current regime.

4. Results

This section summarizes the results of our model. We first describe our parameter choice in Section 4.1, and present the firm sample in Section 4.2. Next, Section 4.3 discusses the properties of the expansion option. Section 4.4, finally, analyzes the optimal default policies of individual firms with different portions of the expansion options' value in the overall value of assets. Economy-wide results are discussed in Sections 5 and 6.

4.1. Choice of parameters

Table 1 summarizes our parameter choice. Panel A shows the firm characteristics which are selected to roughly reflect a typical BBB-rated S&P 500 firm.[9] We start with an initial value of the idiosyncratic earnings X of 10. While this value is arbitrary, neither credit spreads nor optimal leverage ratios depend on this choice. As is standard in the literature, we set the tax advantage of debt to $\tau = 0.15$ (Hackbarth, Miao, and Morellec, 2006). Bhamra, Kuehn, and Strebulaev (2010b) estimate growth rates and systematic volatilities of nominal earnings in a two regime model. Their estimates are similar to those obtained by other authors who jointly estimate consumption and dividends with a state-dependent drift and volatility (Bonomo and Garcia, 1996). Hence, we choose the same growth rates and systematic volatilities of nominal earnings. The real earnings growth rates (μ_i) and volatilities ($\sigma_i^{X,C}$) correspond to their nominal counterparts net of inflation. Note that the relation $\sigma_B^{X,C} = 0.0869 < 0.1369 = \sigma_R^{X,C}$ captures the observation in Ang and Bekaert (2004) that asset volatilities are lower in boom than in recession.

Following Acharya, Bharath, and Srinivasan (2007), we assume that recovery rates fall during recession. They report that recovery in a distressed state of the industry is lower than the recovery in a healthy state of the industry by up to 20 cents on a dollar. The reason can be financial constraints that industry peers of defaulted firms face as proposed by the fire-sales or the industry-equilibrium theory of Shleifer and Vishny (1992), or time-varying market frictions such as adverse selection. We choose recovery rates as $\alpha_B = 0.7$ and $\alpha_R = 0.5$, respectively, which matches the 20 cents on a dollar difference in Acharya, Bharath, and Srinivasan (2007), and is close to the standard of 0.6 used in the literature (Hackbarth, Miao, and Morellec, 2006; Chen, 2010). Our qualitative results are insensitive to the choice of α_i as long as $\alpha_B > \alpha_R$.

Panel B shows the parameters we use to capture growth options. We select an exercise price of $K = 310$. The choice of a relatively high K is motivated by our intention to investigate firms which do not exercise their expansion option immediately. The scale parameter s for a typical firm is calibrated such that the asset composition ratio at initiation given optimal financing equals the average Tobin's Q of 1.6 in our sample of BBB-rated firms. In particular, s is set to $s = 1.89$ for firms initiated in boom, and to $s = 2.05$ for firms initiated in recession. To analyze growth firms with a larger (smaller) portion of option values in the overall value of their assets, we will later use higher (lower) scale parameters at initiation.

Panel C, finally, lists the variables describing the underlying economy. The regime-switching intensities (λ_i), the consumption growth rates (θ_i), and the consumption growth volatilities σ_i^C are estimated in Bhamra, Kuehn, and Strebulaev (2010b). We take the same values for comparability. In the described economy, the expected duration of regime B (R) corresponds to 3.68 (2.03) years, and the average fraction of time spent in regime B (R) is 0.64 (0.36). The inflation parameters are

[9] Our qualitative results do not depend on the ratings of firms.

estimated using the price index for personal consumption expenditures from the Bureau of Economic Analysis from 1947 to 2005. We obtain an expected inflation rate (π) of 0.0342, a volatility of the price index of 0.0137, and a correlation between the price index and real non-durables plus service consumption expenditures of -0.2575. These parameters imply a systematic price index volatility of $\sigma^{P,C} = -0.0035$ and an idiosyncratic price index volatility of $\sigma^{P,id} = 0.0132$.

The annualized rate of time preference, ρ, is 0.015, the relative risk aversion, γ, is equal to 10 and the elasticity of intertemporal substitution, Ψ, is set to 1.5. This parameter choice is commonly used in the literature (Bansal and Yaron, 2004; Chen, 2010).

Our calibration implies that real interest rates are $r_B = 0.0416$ and $r_R = 0.0227$ in the baseline specification. The relative decline in the value of invested assets following a shift from boom to recession is equal to 12.61%, which is similar to the one assumed in Hackbarth, Miao, and Morellec (2006).

INSERT TABLE 1 HERE

4.2. Firm Sample

Balance sheet and ratings data are collected over the period from 1995 to 2008 from Compustat. We use data for BBB-rated firms. We calculate the quasi-market leverage of a firm as the ratio of book debt to the sum of book debt and market value of equity. Tobin's Q is defined as total assets plus the market value of equity minus the book value of equity divided by total assets.[10] We delete financial and utility firms from the sample. For each firm, we calculate the average of the leverage ratios and Tobin's Qs over the observation period. Next, we cut extreme values of both average leverage and Tobin's Q at 1% to avoid that our results are driven by outliers. Our sample then consists of 717 distinct firms. Figure 1 plots the resulting data points. For the entire cross-section of BBB-rated firms, the mean leverage is 41.83%, and the mean Tobin's Q (asset composition ratio) is 1.59.

INSERT FIGURE 1 HERE

[10]In these definitions, we follow, e.g., Baker and Wurgler (2002), Fama and French (2002) and Daines, Gow, and Larcker (2010). Book debt is total assets (item 6, AT) minus book equity. Book equity is total assets minus total liabilities (item 181, LT) minus preferred stock (item 10, $PSTKL$, replaced by item 56 when missing, $PSTKRV$) plus deferred taxes (item 35, $TXDITC$) plus convertible debt (item 79, $DCVT$). The market value of equity is given by the closing price (item 24, $PRCC\_F$) times the number of common shares outstanding (item 25, $CSHO$).

4.3. Properties of the expansion option

To understand the implications of our model for credit spreads, it is instructive to first consider some properties of the expansion option.

Figure 2 depicts the equity value maximizing exercise policy of the expansion option in a typical firm initiated in boom. Recall that the expansion policy is simultaneously determined with the default policy.

INSERT FIGURE 2 HERE

The area above the dashed line is the exercise region in recession, and the area below the dashed line represents the continuation region. In boom, the regions are defined analogously with respect to the solid line. The graph is drawn for optimal leverage. Exercising the option at time $\bar{t}$ entitles the firm to total future earnings of $(s+1)X_t$ for all $t \geq \bar{t}$. As expected, the endogenous exercise boundaries decrease with s. For example, consider initiation in boom: With a scale parameter of $s = 1.89$ (which induces an asset composition ratio of 1.6 at initiation), the corresponding optimal option exercise boundaries are $X_B^* = 18.14$ and $X_R^* = 19.14$. Setting s to 2.73 creates a growth firm with an asset composition ratio of 2.2, and optimal option exercise boundaries of $X_B^* = 12.87$ and $X_R^* = 13.55$, respectively. Importantly, Figure 2 also shows that the expansion option is exercised at lower levels of the idiosyncratic earnings X in boom than in recession. Intuitively, the main reason is that the value of the real option of waiting is higher in recession due to the potential switch to boom with a higher valuation of earnings.[11] The same qualitative option value properties also hold at non-optimal leverage levels.

Figure 3 plots the value of the expansion option as a function of the earnings X, using jointly optimal expansion and default policies.

INSERT FIGURE 3 HERE

Obviously, the option's value is affected by the current regime. When the asset value jumps due to a regime switch, so does the value of the option. Critically, relative value changes of expansion options are higher than relative value changes of assets in place when the regime switches. The reasons are that options represent levered claims, and that the endogenous exercise boundary is higher in recession than in boom, as shown in Figure 2.[12]

[11] The regime dependent volatilities and default thresholds also affect optimal exercise boundaries in boom and recession. We find that the valuation of earnings is the dominating effect for reasonable parameter values.

[12] Relative value changes are determined in Appendix A.3. In untabulated results, we confirm numerically that the relative value changes are indeed higher for expansion options than for the underlying assets in place for plausible parameter values.

Additionally, Figure 3 shows that both option value functions are convex, but the value function in boom is steeper than the one in recession. Therefore, the expansion option's value is less sensitive to the underlying earnings in recession than in boom. Intuitively, the exercise boundary increases and the earnings' drift decreases in recession which drives options out-of-the money. As a consequence, an expansion option represents a less levered claim in bad times. While in recession the volatility of X is higher, the sensitivity of a growth option's value to changes in the earnings is lower. As discussed in the next section, this lower sensitivity attenuates the increase in the equityholder's default option due to a higher volatility of X during recession.

4.4. Optimal Default Policy

This section explains how the optimal default policy is affected by the presence of growth options in the value of firms' assets. In order to keep the intuition tractable, we do not comment on the (minor) impact of the exercise boundaries on default thresholds, which arises due to the simultaneous optimization of the expansion and default policy.

For all firms – those with and those without an expansion option – the optimal default policy is determined by recognizing that, at any point, shareholders can either make coupon payments and retain their claim together with the option to default, or forfeit the firm in exchange for the waiver of debt obligations. When the economy shifts from boom to recession, the present value of future earnings declines mainly because firm earnings have a lower drift, and because they become both more volatile and more correlated with the market. This present value decline reduces the continuation value (the expected value from keeping the firm alive) for equityholders, inducing them to default earlier (at higher earnings levels) in recession. We will refer to this effect as the *value effect*. On the other hand, a high earnings volatility in recession makes the option to default more valuable, which defers default in bad times. This is the *volatility effect*. As in the models for invested assets of Bhamra, Kuehn, and Strebulaev (2010b) and Chen (2010), the value effect usually dominates the volatility effect, generating higher default thresholds in recession, i.e., leading to counter-cyclical default thresholds. Counter-cyclical default thresholds together with a high volatility in bad times imply counter-cyclical default probabilities, consistent with empirical evidence (Chava and Jarrow, 2004; Vassalou and Xing, 2004). Additionally, default losses are empirically reported to be higher in recession because many firms experience poor performances during such times. Combined with higher marginal utilities in bad times, these mechanisms raise the present value of expected default losses for bondholders which leads to higher credit spreads and lower optimal leverage ratios than in standard contingent claim models.

Figure 4 draws the equity value maximizing default policy of levered firms initiated in boom. The graph shows default thresholds for a range of asset composition ratios. Leverage is held constant at 41.83%.[13] The solid line represents the default threshold in boom, and the dashed line the one in recession. For a firm with only invested assets the optimal default thresholds correspond to $D_B^* = 2.25$ and $D_R^* = 2.46$, for an average firm with an asset composition ratio of 1.6 to $D_B^* = 2.81$ and $D_R^* = 3.19$, and for a growth firm with an asset composition ratio of 2.2 to $D_B^* = 2.98$ and $D_R^* = 3.41$. In the no-default region above the line corresponding to a given regime, the continuation value for equityholders exceeds the default value and it is optimal for shareholders to keep injecting funds into the firm.

INSERT FIGURE 4 HERE

Two points from Figure 4 are particularly noteworthy. First, the optimal default thresholds increase as the asset composition ratio increases, inducing a higher default probability. This finding evolves from the observation that growth options represent levered claims which are relatively more sensitive than invested assets to a given decrease in X. Second, while all firms are more likely to default in recession than in boom, the increasing distance between D_B^* and D_R^* for larger asset composition ratios indicates that the counter-cyclicality of default boundaries is particularly pronounced for growth firms. The reason is that due to the higher relative value change of growth options upon a regime switch, the value effect is stronger for a firm with a high asset composition ratio. Additionally, because options represent less levered claims in recession than in boom, the increase in the equityholders' default option - due to the higher volatility of X when the regime switches to recession - is attenuated for growth firms. In other words, the volatility effect, which tends to decrease the distance between the default thresholds, is weaker for firms with larger expansion options.

5. Aggregate dynamics of leverage, asset composition, investment and defaults

In order to validate our structural-equilibrium framework with intertemporal macroeconomic risk and investment, we analyze the dynamic properties of our model-implied economy. In this section, we qualitatively compare the aggregate predictions for the entire economy to empirically reported capital structure, investment, and default patterns. In Section 6, we quantitatively explain observed credit spreads and leverage ratios of the subset of BBB-firms.

[13] When the scale parameter is changed but the coupon is left constant, default thresholds are not directly comparable. The reason is that the total asset value increases with s for every X. Taking constant leverage assures that the considered coupon changes consistently with the increase in the total asset value when we alter s.

5.1. Simulation

We generate a dynamic economy of average firms implied by our model. We consider 1,000 identical firms with infinite debt maturity. Initially, each firm's earnings are $X = 10$, and the option scale parameter is assumed to be $s = 1.89$ if the firm's initial regime is boom, and $s = 2.05$ otherwise. These choices of s imply an asset composition ratio of 1.6 in both states at initiation, given optimal leverage. Firms receive the same macroeconomic and inflation shocks, but experience different idiosyncratic shocks. Each firm observes its current earnings as well as the current regime on a monthly basis and behaves optimally: If the current earnings are below the corresponding regime-dependent default threshold, the firm defaults immediately; if the current earnings are above the corresponding regime-depending option exercise boundary, the firm exercises its expansion option; otherwise, the firm takes no action.

In our model, firms have a growth option which can only be exercised once. To maintain a balanced sample of firms, and to avoid that the average asset composition ratio is systematically trending towards the one of a firm with only invested assets when we simulate the economy over time, we exogenously introduce new firms. In particular, we substitute each defaulted or exercised firm by a new firm whose growth option is still intact. New firms have initial earnings of $X = 10$, and an option scale parameter s according to the current regime as described above.

To ensure convergence to the long-run steady state, we first simulate the economy for 100 years. The starting period for the reported results is the final period of the first 100 years of simulation. Next, we simulate the model for 200 years and present the aggregate dynamics.

5.2. Results

We start by discussing the cyclicality of leverage. Hackbarth, Miao, and Morellec (2006) generate counter-cyclical optimal leverage ratios in their macroeconomic model. As in our framework, the optimal coupon rate of debt initiated in boom exceeds the one in recession. At the same time, the value of assets is greater in boom. The second effect dominates the first, generating the counter-cyclicality in optimal leverage. We additionally incorporate the empirical fact that asset volatility is regime-dependent. Because the latter decreases in boom and increases in recession, our optimal coupon rate varies more than in Hackbarth, Miao, and Morellec (2006) when the regime changes. With this extension, the change in the value of optimal debt dominates the change in the value of assets, generating pro-cyclical optimal leverage ratios for realistic parameter values, in line with Covas and Den Haan (2006) and Korteweg (2011). Figure 5 plots the simulated market leverage in the economy. Shaded areas represent recessions. Even though our optimal initial leverage ratios are pro-cyclical, the simulated time series shows that actual aggregated market leverage is counter-cyclical. The reason is that when firms are stuck with the debt issued at initiation, the equity value declines more than the debt value during recessions which tends to increase leverage in bad times. This prediction conforms to Korajczyk and Levy (2003) who show that unconstrained firms' leverage ratios vary counter-cyclically.

INSERT FIGURE 5 HERE

Figure 6 shows the time series of the aggregate asset composition ratio in the simulated economy. As expansion options are more sensitive to the underlying stochastic processes than invested assets, the ratio behaves pro-cyclically, as reported in the literature.

INSERT FIGURE 6 HERE

We investigate aggregate default rates in Figure 7. Simulated default rates are counter-cyclical, consistent with the empirical fact that most defaults occur during economic recessions. Additionally, the graph shows several spikes in default rates that occur right at the time when the economy enters into a recession, consistent with the empirical evidence in Duffie, Saita, and Wang (2007) and Das, Duffie, Kapadia, and Saita (2007) (see, e.g., around years 50 and 90). Recall that defaults can occur because either the idiosyncratic earnings reach the default threshold in a given regime, or due to a change of the macroeconomic regime from boom to recession. The clustered default waves occur due to an increase in firms' default thresholds upon such a regime change. All firms between the two thresholds default simultaneously when the regime switches to recession, even though their earnings do not exhibit instantaneous regime-induced changes. After such waves of default, the default frequency tends to remain high during recessions.

As a refinement of this general result, we expect that the tendency to default during recession should be particularly pronounced for firms with high expansion options. This prediction is suggested by the fact that the degree of counter-cyclicality of default thresholds is positively related to the initial asset composition ratio. We investigate the propensity to default during recession in a dynamic, simulation-based setting by counting default rates of two separate aggregate economies. The first one is designed as above, consisting of firms with both assets in place and growth options, such that the asset composition ratio at initiation is 1.6. The second setting consists of firms with only invested assets. To construct a number of cross-sectional distributions of firms, we first simulate 20 dynamic economies for 10 years. Using each economy obtained at the end of the first 10 years, the default rates in both regimes are observed for 50 subsequent simulations of the following 20 years, resulting in a total of 1,000 simulations. The average percentage of defaults which occurs during recession is then calculated.[14] We find that in the first economy, on average,

[14]The distance to default in the aggregate economy of firms with only invested assets is trending over time. The reason is that firms which default are replaced, but there are no option exercises after which well performing firms could be replaced. Consequently, we do not compare absolute default rates of the two economies, but rather the fraction of defaults occurring in each regime.

75.41%, 76.79%, and 77.66% of total defaults of firms with assets in place and growth opportunities occur during recession over 5, 10, and 20 years, respectively. In the economy where firms only have invested assets, the corresponding numbers are considerably smaller at 66.40%, 71.66%, and 73.71%, respectively.

This finding is also related to the observation that, on average, growth firms have lower recovery rates than value firms (Cantor and Varma, 2005). The standard argument offered by Shleifer and Vishny (1992) is that growth firms as potential buyers of growth assets have little cash relative to the value of assets. Hence, they are likely to be themselves credit constrained when other growth firms sell their assets upon default, which lowers recovery rates. Our model delivers an alternative explanation: We show that growth options in the value of firms' assets create a propensity to default during recession, when recovery rates are low.

INSERT FIGURE 7 HERE

A significant literature suggests that business cycle shocks common to all firms play a crucial role in explaining aggregate investment. In particular, there is evidence that aggregate investment is characterized by both episodes of very intense investment activity and periods of very low investment activity (Doms and Dunne, 1998; Oivind and Schiantarelli, 2003). Moreover, aggregate investment and the probability of investment spikes are strongly pro-cyclical (Barro, 1990; Cooper, Haltiwanger, and Power, 1999). Our model reflects these features. First, when the regime switches from recession to boom, firms in the region between the two investment boundaries exercise their expansion option simultaneously by investing K. Figure 8 shows that investment spikes often occur upon such regime switches (see for example around year 35, or year 60). After these spikes, simulated investment rates tend to remain high during boom due to the positive drift of the earnings. Hence, we observe pro-cyclical investment spikes followed by higher investment activity during booms. At the other end, investment activity often dries out when the economy switches from boom to recession, because the optimal exercise boundary jumps up and the expected earnings' drift turns negative. Our model also predicts that observed investment waves should be mainly driven by firms with high expansion options.

INSERT FIGURE 8 HERE

Finally, we plot simulated average credit spreads in Figure 9. Credit spreads are calculated as $(c/d_i(X))-(c/RF)$, where RF is the value of a risk free bond with an identical coupon.[15] Consistent with the empirical literature (Fama and French, 1989), we find counter-cyclical credit spreads.

[15] RF is given by

$$RF = \frac{c\left(r_j^n + \tilde{\lambda}_i + \tilde{\lambda}_j\right)}{r_i^n r_j^n + r_j^n \tilde{\lambda}_i + r_i^n \tilde{\lambda}_j}. \tag{46}$$

When the economy stays in boom, credit spreads tend to decline as distances-to-default increase due to the positive expected drift of the earnings and the lower default threshold. Conversely, in recession, credit spreads rise as distances-to-default tend to decline and the volatility increases.

INSERT FIGURE 9 HERE

6. Quantitative implications and empirical predictions

In this section, we discuss the quantitative implications and empirical predictions of our model. The attention is restricted to BBB-rated firms since it has been argued that the pricing of very high-grade investment firms is dominated by factors other than credit risk such as liquidity risk or a tax component (Longstaff, Mithal, and Neis, 2005; De Jong and Joost, 2006). We start by determining target observed average credit spreads. Duffee (1998) estimates an average yield spread in the industrial sector between BBB-rated bonds and Treasury yields of 198, 148, and 149 bps for bonds with a mean maturity of 21 years (long), 8.9 years (medium), and 4.7 years (short), respectively. Davydenko and Strebulaev (2007) report somewhat lower spreads of 143 bps for bonds with $15-30$ years (long), 115 bps for $7-15$ years (medium), and 115 bps for $1-7$ years maturity, respectively.[16] From these spreads, we subtract 35.5% to reflect the results in Longstaff, Mithal, and Neis (2005) and Han and Zhou (2011) who find non-default components in BBB bond yields of 29% and 42%, respectively. We arrive at a plausible target range of around 92 to 128 bps for long maturities, 74 to 95 bps for medium maturities, and 74 to 96 bps for short maturities.[17] Panel A in Table 2 tabulates these target credit spread ranges. In Panel B, we also report empirical default rates of BBB-rated debt over 5, 10, and 20 years from Moody's (2010).

INSERT TABLE 2 HERE

We discuss the implications of our model for credit spreads and leverage along two dimensions. First, we follow the traditional way of investigating a typical individual firm. Second, we implement an approach similar to the one proposed by Bhamra, Kuehn, and Strebulaev (2010b) where credit spreads and leverage ratios are calculated as cross-sectional averages based on a simulation of the empirical distribution of BBB-rated individual firms.

[16] The estimates of short and medium maturities in Huang and Huang (2003) are higher because of the embedded call options in the corporate bond sample and the inclusion of two recessions with high spreads.

[17] We recalculate target ranges by subtracting the absolute non-default component for BBB firms of 61.8 bps reported in Han and Zhou (2011), or by subtracting the 29% reported in Longstaff, Mithal, and Neis (2005) for an earlier sample period. Our model's performance does not depend on the exact definition of targets.

6.1. Credit spreads

6.1.1. Typical firm with endogenous default boundary

Credit spreads for various models on newly issued corporate debt are calculated in Table 3 for 5 (short), 10 (medium), and 20 (long) years maturity.[18] We follow the standard approach in structural models by calibrating the idiosyncratic earnings volatility such that the total asset volatility is approximately 25% in each model, the average asset volatility of firms with outstanding rated corporate debt (Schaefer and Strebulaev, 2008). Additionally, we fix leverage at the average ratio of 41.83% in our BBB-firm sample.

Importantly, the default boundaries and expansion thresholds are assumed to be chosen optimally by equityholders, as we are interested in whether our model can generate both realistic prices of corporate claims and realistic endogenous default and expansion rates. Specifying default boundaries exogenously such that a model's actual default probabilities match the data (as done in Chen, Collin-Dufresne, and Goldstein (2009) or Huang and Huang (2003)) would not only substantially dilute the value of the option to default, but would also distort the value of the expansion option because the latter depends on the default policy.

It is well-known that structural models of default typically generate credit spreads which are too low compared to their empirical counterpart. To illustrate this point, we first analyze the model without business cycle risk in Panel A of Table 3. The expected drifts and systematic volatilities of earnings and consumption are set equal to their unconditional means. Panel A shows credit spreads for different maturities of the standard structural model of Leland (1998). The empirical target credit spreads in Table 2 are about 5 times larger for the short maturity, and about 3 times larger for the medium and long term than those predicted by the structural model.

INSERT TABLE 3 HERE

Bhamra, Kuehn, and Strebulaev (2010b) and Chen (2010) derive structural multi-regime models for typical firms which consist of only invested assets. We closely replicate their approach for an average firm within a two-regime model. To match the asset volatility of 25%, the idiosyncratic earnings volatility is set to $\sigma^{X,id} = 0.21$. Panel B reports unconditional credit spreads, calculated as a weighted average of the state-dependent credit spreads, where the weights correspond to the long-run distribution of the Markov chain. For comparability to our setting with expansion options, the results without debt restructuring are presented. While the credit spreads for typical firms of 35, 56, and 78 bps for 5, 10, and 20 years maturity, respectively, are clearly higher than in the one regime case, they are still considerably below their targets.[19]

[18] For the value of a finite maturity risk-free bond see Appendix A.5, formula (A-117).

[19] Bhamra, Kuehn, and Strebulaev (2010b) use higher recovery rates, lower leverage and do not model the impact of principal repayments on default thresholds, which results in marginally lower credit spreads in their static case.

Next, we investigate our model with expansion options for a typical BBB-rated firm. Note that for a given idiosyncratic earnings volatility, firms with different asset composition ratios have different total asset volatilities due the inherent leverage of their expansion option. Moreover, a firm's asset volatility is not constant over time, as its option's moneyness changes when X moves towards or away from the exercise boundary. To obtain the average volatility for a certain rating class, the standard approach in the literature is to average the calculated asset volatilities over all firms with the same rating (Vassalou and Xing, 2004; Duan, 1994; Schaefer and Strebulaev, 2008). We calibrate the idiosyncratic volatility $\sigma^{X,id}$ to the empirically reported average asset volatility of 0.25: Given an idiosyncratic volatility $\sigma^{X,id}$, we simulate model-implied samples of BBB-rated firms over 10 years, and calculate the resulting average asset volatility. (Details on the simulation can be found in Appendix A.6.1.) The calibration yields $\sigma^{X,id} = 0.168$ which ensures that the average asset volatility of our simulated BBB-rated firms with expansion options corresponds to its empirical counterpart.[20]

Panel C of Table 3 shows the resulting credit spreads for typical firms. Several aspects are noteworthy about these results. Our model increases the unconditional credit spreads of an average firm for 5, 10, and 20 years from 18 bps to 44 bps (+144%), from 29 bps to 66 bps (+128%), and from 41 bps to 81 bps (+98%), respectively, compared to the one regime model in Panel A. To understand this large effect, recall first that macroeconomic models generate larger credit spreads than one regime models because recessions are times of high marginal utility, so that default losses that occur during these times will affect investors more. An important economic implication is that the average duration of bad times in the risk-neutral world is longer than in the actual world. Since the representative agent uses risk-neutral and not actual probabilities to account for risk and to compute prices, credit spreads are larger and the agent behaves more conservatively than historical default losses imply. Second, if firms have a higher tendency to default in recession, this discrepancy will increase due to the higher risk premium. Our model shows that because of the strong sensitivity of option values to regime switches, and because they are less sensitive to the underlying earnings during recession, the counter-cyclicality of default thresholds is more pronounced for firms with larger growth options. The resulting stronger counter-cyclicality of the default probability of growth firms thus drives up their credit spreads. As can be seen in row 2 of Panel C, the credit spreads for an average firm, consisting of both invested assets and growth options, are 44 bps, 66 bps, and 81 bps for debt maturities of 5, 10, and 20 years, respectively.

Chen (2010) obtains larger 10 year credit spreads in a model with 9 states and a dynamic capital structure, but uses higher leverage, and a cash flow volatility which induces a much higher asset volatility than empirically observed.

[20] We also repeat this exercise with different specifications, such as alternative simulation length and debt maturity. The resulting idiosyncratic volatilities are fairly insensitive to these variations. An alternative approach is to calibrate the idiosyncratic volatility to the cumulative default probability of BBB-rated firms (Chen, 2010). This procedure, however, usually leads to asset volatilities which are higher than the ones empirically observed.

This is, respectively, 26%, 18%, and 5% higher than the credit spreads of an average firm in the standard macroeconomic model with only invested assets.[21]

Besides the fact that they generate too low credit spreads, another problem of existing structural models is that the implied term structure of credit spreads at initiation is much steeper than its empirical counterpart for a typical firm. The reason is that the implied spreads are particularly low at the short end. Most existing studies in the macroeconomic model literature use the default thresholds of infinite maturity debt (that is, debt without principal repayments) to numerically calculate the risk-neutral default probability for each maturity. As the credit risk literature identifies firms' debt maturity as an important determinant of credit risk (Gopalan, Song, and Yerramilli, 2010; He and Xiong, 2011), we endogenously derive optimal default thresholds also for less than infinite debt maturity following the approach of Leland (1998). Due to the continuous principal repayments, these thresholds are considerably higher for short maturities than for infinite debt, resulting in larger credit spreads at the short end. The resulting term structure of credit spreads for an average firm in Panel A, B, and C is consequently flatter and, hence, closer to the shape observed in target spreads than when using default thresholds of infinite maturity debt.[22]

The rows in Panel C of Table 3 identify the cross-sectional relationship between the asset composition ratio and credit risk. To tease out the effect of growth options on credit spreads, we vary the asset composition ratio by altering s. As raising s increases the value of the expansion option, we simultaneously adapt the coupon to maintain a constant leverage of 41.83%.[23] This exercise shows that the asset base of the firm is an important driver of credit risk, implying a positive relationship between the portion of growth options in the value of a firm's assets and the costs of debt. In particular, altering the asset composition ratio of a firm from 1 to 2.2 increases credit spreads by about 55% to 95%, depending on the debt maturity. This effect is remarkable given that we solely vary the assets' characteristics. It arises for two reasons in our model. First, because options are levered, and due to the endogenous investment boundary, expansion options are more sensitive to the underlying uncertainty, and, hence, more volatile. This higher volatility drives up the default probability of growth firms. Second, a higher portion of the expansion option's value in the overall asset value of a firm induces a higher counter-cyclicality of the default probability,

[21] We cannot directly compare the results for invested assets in Panel C to the ones for average firms in Panel B, even though the latter consist of only invested assets. The reason is that in our model, the idiosyncratic volatility is calibrated such that the asset volatility of the entire sample of BBB-rated firms matches 0.25, whereas in Panel B, $\sigma^{X,id}$ is chosen such that firms with only invested assets have an asset volatility of 0.25.

[22] We use default boundaries for the appropriate debt maturities in both Panels B and C in order to highlight the pure effect of expansion options on credit spreads.

[23] Alternatively, changing both s and K to alter the asset composition qualitatively retains the aggregate and cross-sectional predictions. Holding s constant while only varying K implies large decreases in the option exercise boundaries for relatively small increases of the asset composition ratio. In the extreme, a firm with a very low K will exercise its expansion option almost immediately; in essence, credit spreads then virtually mirror those of a firm with only invested assets, diluting the model's cross-sectional predictions. Note also that any variation in K changes the costs of investment. By only varying s, we instead avoid that our results are driven by different sizes of the expected financing in case of equity-financed investment costs.

which raises expected default costs. The higher default probability and larger default costs both increase the costs of debt for growth firms.

Note that while firms with growth options generally have a higher credit spread than firms with only invested assets (ceteris paribus), credit risk is concave in the asset composition ratio. This concavity occurs because firms with a larger asset composition ratio are closer to their exercise boundary, where credit spreads also reflect that the asset volatility and the counter-cyclicality of the default thresholds will decrease when a firm exercises its expansion option.

Our predictions are qualitatively consistent with empirical findings. For example, Davydenko and Strebulaev (2007) find that market-to-book asset values, the ratio of research and development expenses to total investment expenditure, and one minus the ratio of net property, plant, and equipment to total assets are all significantly and positively related to credit spreads (Table VI on p. 2652). Similarly, Molina (2005) documents that firms with a higher ratio of fixed assets to total assets have lower bond yield spreads and higher ratings (Table II on p. 1438). This evidence implies that, empirically, even after controlling for most factors relevant to credit risk in standard structural models, credit spreads are higher for growth firms. Hence, while an average firm with valuable growth options exhibits, for example, a different tax advantage of debt or payout ratio than a firm which only consists of invested assets, simple variation of such input parameters would not explain these findings. What is needed to address the aggregate puzzle and the mentioned cross-sectional evidence is a model which generates higher explained credit risk than standard models for a *given* level of input parameters. Our model delivers this result.

6.1.2. True cross-section

The previous section calculates credit spreads of a typical individual firm which is consistent with the historically observed average input parameters of firms in the same rating class of which the individual firm is representative.

In this section, inspired by the work of Bhamra, Kuehn, and Strebulaev (2010b), we employ a simulation approach to capture the dynamics of the cross-sectional distribution of leverage and asset compositions of BBB-rated firms. The central insight of this approach is that BBB-rated firms are very different with respect to their firm characteristics such as the asset composition ratio and leverage, and that credit spreads and default rates are highly non-linear in these characteristics. Moreover, the previous section considers credit spreads solely at debt issuance points, when the principal corresponds to the market value of debt. The majority of empirically reported spreads are, however, based on observations made at times when debt is not being issued. To capture the impact of these issues, it is important to calculate credit spreads and default rates for a simulated sample of firms which matches the observed empirical distribution, i.e., the true observed cross-section of BBB-rated companies. The resulting average of simulated credit spreads can then be compared to the empirical average credit spread. Simultaneously, the approach allows us to verify whether the default probabilities implied by our model correspond to the reported historical default probabilities of BBB-rated firms.

To obtain the implications of the true cross-section of BBB-rated firms, we start by generating a distribution of firms implied by the model. In particular, we set up a grid of optimally leveraged firms with scale parameters s from 0 up to the largest possible value such that the option is not exercised immediately. The step size is 0.05, and 50 identical firms are considered for each value of the option scale parameter. Earnings paths of all firms are then simulated forward over 10 years, resulting in a model-implied economy populated by more than 3000 firms. This economy has a broad range of leverage ratios and asset composition ratios.

In a second step, we match our historical distribution of BBB-rated firms with its model-implied counterpart. For each observation in the average empirical cross-section, we select the firm in our model-implied economy with the minimum distance regarding the percentage deviation from the target average market leverage and asset composition ratio. The matching is generally very accurate. Considering a debt maturity of 10 years yields an average Euclidean distance of 0.0648, with the 85%-quantile being 0.0865.[24] That is, on average, only 15% of the firms are matched with the root of the sum of the squared percentage deviations being larger than 8.65%.[25] Note that while our initial model-implied economy potentially contains firms with different ratings, the described matching procedure allows us to construct a cross-sectional distribution of model-implied firms which closely reflects its empirical BBB-rated counterpart.

Next, earnings paths of the 717 matched BBB-model-firms are simulated forward for 20 years on a monthly basis. This simulation is repeated 50 times.

The outcome of both the matching and the forward simulation of the matched sample also depends on the particular realizations of the idiosyncratic shocks and the states of the economy in the first simulation step. Hence, to explore the distributional properties of our results, the entire procedure is conducted 20 times, which results in a total of $1,000$ simulations. Details on the simulation are given in Appendix A.6.2.

Panel D of Table 3 summarizes the results. The average credit spreads, calculated during 5 years after the matching, are 60 bps for 5 years, 81 bps for 10 years, and 103 bps for 20 years.[26]

[24]Other debt maturities yield virtually identical results for the matching accuracy.

[25]The market leverage is matched with an average distance of 0.0248. The average percentage distance of the asset composition ratio of 0.0549 is larger. This number is driven by a few firms with unusually high asset composition ratios. As they would optimally exercise their expansion option immediately in our model, these firms are matched with model firms with a somewhat lower asset composition ratio. We expect a minor impact of this limitation on our results, because firms with unusually high asset composition ratios also have very low leverages, and, hence, are not driving our average credit spreads.

[26]We follow Bhamra, Kuehn, and Strebulaev (2010b) in measuring average credit spreads over a 5 year period. During longer periods, many firms could deviate substantially from the initial average distribution, and would, therefore, not be BBB-rated anymore.

Hence, our model closely matches the historical levels reported in Table 2 for 10 and 20 years. 5 year credit spreads are somewhat lower than their target. We also measure the cyclicality of credit spreads. Average 10 year credit spreads, for example, are 58 basis points during boom, and 112 during recession. As expected, they are strongly counter-cyclical.

Importantly, average credit spreads for the simulated true cross-section are considerably higher than the ones of a typical firm at initiation. There are two reasons for this result. First, some firms will be near default, and credit spreads are convex in the distance to default. Second, the market value of debt corresponds to the principal at initiation. In practice, however, firms are not at initiation most of the time. The actual market value of debt will, therefore, often underestimate the burden from the principal repayments, and especially so for firms approaching their default boundary. The reason is that the market value can hardly go beyond the principal as it is bounded above by the value of riskless debt, but can easily reach values below the principal when earnings deteriorate. Our simulation of the true cross-section captures these asymmetric deviations over time, resulting in higher average credit spreads than those of firms observed at initiation. Compared to Bhamra, Kuehn, and Strebulaev (2010b), the additional credit spreads generated from simulating the true cross-section are lower, because we do not incorporate debt restructuring.

To verify whether our model generates default rates corresponding to the empirically reported default frequencies for realistic debt maturities, we also count cumulative default rates in the simulated true cross-section. The model-implied average and median cumulative default rates over several years are reported in each Panel of Table 4. Panel A presents default rates over 5, 10, and 20 years from simulations with firms issuing infinite maturity debt. Panels B, C, and D show default rates from simulations with firms issuing finite maturity debt. Due to the principal repayments, default thresholds of firms with finite maturity debt are considerably higher than those of firms with infinite maturity debt. Note that simulated credit spreads are consistent with a range of realized ex-post default rates, as observed default rates vary depending on a particular realization of good and bad states. Therefore, we also report the 25% and 75% percentiles of the distribution.

Empirically, Datta, Iskandar-Datta, and Patel (2000) report a mean maturity of IPO bonds of 12 years, Guedes and Opler (1996) document an average maturity of 12.2 years for seasoned debt offers, and Davydenko and Strebulaev (2007) measure a mean time to maturity of BBB-bonds in the industrial sector of 9.51 years. Panel C of Table 4 shows that when assuming that firms have a debt maturity of 10 years, our model-implied median default rates over 5, 10, and 20 years are very close to the historical default probabilities observed from 1920 to 2009 reported in Table 2. Hence, for a realistic debt maturity, our median economy is consistent with historical default frequencies of BBB-rated firms. The average default rates are somewhat larger than their targets due to a few realizations with long sojourn times in recession, resulting in high default rates.[27] Panels A and

[27]The standard deviation of the sojourn times generated by Markov chains is quite large. In our model, long sojourn times in recession cause high default rates for some sample paths. As default rates are non-linear in the distance to default, long sojourn times in boom do not counterbalance these high rates.

D show that while the generated rates tend to be too low in Panel A, but too large in Panel D, historical default frequencies still fall within the 25% to 75% range of model-implied median default rates for most years.

The large difference between Panel A and D in both average and median default rates illustrates that debt maturities and the associated default thresholds have an important effect on model-implied default rates. It is, therefore, important to incorporate a realistic debt maturity when calibrating models with endogenous default thresholds.

INSERT TABLE 4 HERE

In sum, our results demonstrate that the average credit spreads implied by our model for the true cross-section are simultaneously consistent with historically observed average asset volatilities, and, especially for typical debt maturities, with default rates reported for BBB-rated firms.

6.2. Leverage

This section analyzes the features of leverage ratios resulting from our model. We first investigate how growth options affect the initial choice of optimal leverage in our model. At initiation, a firm consisting of only invested assets has an optimal leverage which is between 4 and 5 percentage points higher than the one of a typical firm with an asset composition ratio of 1.6 for all debt maturities.[28] The reason is that a higher asset composition ratio increases the default probability, particularly so in recessions where default losses are larger and harder to bear. Due to the resulting higher costs of debt, firms with growth options optimally select lower initial leverage.

As argued by Bhamra, Kuehn, and Strebulaev (2010a), however, it can be misleading to make quantitative statements simply based on optimal leverage at issue. Hence, we investigate the leverage ratios of our cross-section of BBB-rated firms simulated over 5 years after matching. For the main analysis, the debt maturity is assumed to be 10 years.

INSERT TABLE 5 HERE

Panel A in Table 5 shows that the average leverage is 40.89%, which is, naturally, close to the average of 41.83% of our BBB-rated firm sample used for the matching. (The average leverage is 40.57%, 40.93%, and 41.45% for 5 years, 20 years, and infinite debt maturity, respectively.)

[28]The difference depends on the initial regime and the debt maturity. For example, with infinite debt maturity, the difference in optimal initial leverage between a firm with only invested assets and a firm with an asset composition ratio of 1.6 is 4% if the firms are initiated in boom. (The optimal leverage ratios in this case are 45.4% and 41.4%, respectively.) For firms initiated in recession, the difference is 4.4% (= 44.2% minus 39.8%).

In Panel B, we compare leverage ratios in boom and recession. While optimal leverage is pro-cyclical at initiation, it is counter-cyclical over time for the cross-section of BBB-rated firms. In particular, the average leverage is 36.94% in boom, and 46.20% in recession. The reason is that the market value of equity is more sensitive to regime switches than the market value of debt, making leverage counter-cyclical. This mechanism dominates the optimally pro-cyclical leverage choice at initiation for our typical firms. The result mirrors the property we previously established for the aggregate economy, and confirms that it holds also when matching to real empirical samples.

Finally, Panel C investigates the relationship between growth options and market leverage. Regressing the average leverage of each firm on its average asset composition ratio in our empirical BBB-rated firm sample yields a coefficient of -0.165. We conduct the same regressions with the averages of asset composition ratios and leverage ratios from each of the 1000 simulations of the true cross-section. The average coefficient from this regression is -0.184, close to its empirical counterpart. Hence, the observed magnitude of the negative relationship between growth options and market leverage is preserved during the simulation.

Our qualitative finding for the cross-sectional relationship between growth options and leverage is widely accepted (Bradley, Jarrell, and Kim, 1984; Barclay, Smith, and Morellec, 2006; Johnson, 2003; Rajan and Zingales, 1995). Consistent with the literature, the coefficient is robustly negative. Moreover, its quantitative size, implied by the 25% and 75% quantiles, is comparable to the one in empirical studies. Fama and French (2002), for example, obtain a coefficient of -0.096 in their regression of market leverage on a similar ratio of asset composition after controlling for standard controls, and Johnson (2003) finds that increasing the asset composition ratio by one decreases leverage by around 7.8 percentage points in a pooled regression.

6.3. Robustness

In this section, we discuss the robustness of the results to variations in the critical input parameters. Additionally, we also show how our predictions are affected if we assume that the expansion is financed by issuing equity instead of selling assets.

To analyze the impact of preferences on our results, we show 10 year credit spreads and the simulated average leverage for $\gamma = 7.5$ in the second column of Table 6, a value which is also sometimes used in the literature (Bansal and Yaron, 2004; Chen, 2010). All other parameters are kept constant at their baseline levels from Table 1. The debt maturity is assumed to be 10 years.

INSERT TABLE 6 HERE

Lower risk aversion induces a smaller demand for precautionary savings, which increases the real risk-free rate. At the same time, it raises the risk-neutral earnings drift, because the risk prices for systematic Brownian shocks (η_i) decrease. Both mechanisms reduce the default probability, leading to the lower credit spreads and slightly lower leverage.

In column 3 of Table 6, we investigate the impact of the exercise costs on credit spreads and leverage. As we are mainly interested in firms with intact expansion options, we present the results for K equal to 350, i.e., a higher K than in the baseline case. (Lowering K induces many growth firms to exercise their expansion option almost immediately.) Generally, credit spreads and the average leverage are very similar to the ones of our baseline specification. For large asset composition ratios, such as with 2.2, credit spreads at initiation slightly increase because a higher K induces a larger distance to the optimal exercise boundary compared to the baseline specification. This increase in credit spreads from the larger distance arises because close to the exercise boundary, credit spreads reflect the fact that the firm will imminently be converted into a firm with only invested assets, and, hence, with lower credit risk.

Finally, we also analyze in Column 4 of Table 6 the case where the exercise price of the expansion option (K) is financed by issuing additional equity instead of selling assets. Appendix A.7.1 presents the resulting system of ODEs for corporate debt. New equity decreases the leverage after exercise and, hence, lowers credit risk. As firms with a high asset composition ratio are close to the endogenous exercise boundary where new equity-financing occurs, credit spreads are strongly reduced for typical growth firms compared to the benchmark model. In the simulation of the true cross-section, however, the effect is small because most firms have a large distance to the exercise boundary.[29] Additionally, Panel B shows that the average leverage is only marginally affected.

The result for typical growth firms in column 4 shows that close to firms' exercise boundaries, credit spreads are driven by the expected new financing upon investment, and do not primarily reflect the nature of assets. This insight validates our focus on asset-financing rather than on equity-financing of growth option exercises to analyze the isolated impact of the asset composition on credit risk and corporate policy choices.

We conclude that while alternative specifications and settings can have an impact on the quantitative results, our qualitative aggregate and cross-sectional predictions are robust.

7. Conclusion

It is now well-accepted that macroeconomic risk is central for understanding credit risk and capital structure choices. Specifically, defaults are more likely in recession, when they are particularly costly and harder to bear. This counter-cyclicality increases the costs of debt for all firms. But to explain the cross-sectional variation in apparently excessive costs of debt, we need variation inside

[29] In fact, those firms which contribute the most to the average credit spread, i.e., distressed firms, are particularly far away from the exercise boundary.

the firm. This paper formalizes the role of one particularly important aspect of this heterogeneity, the asset composition of firms. It is not surprising that in principle the asset composition can be important for optimal capital structure. After all, economists have devoted much effort to understanding the difference between value and growth firms in terms of their financial structure, starting with Myers (1977) and Jensen (1986). Little was known, however, about the quantitative importance of this factor and its relation with macroeconomic risk.

The present structural equilibrium model allows us to jointly analyze a firm's expansion policy and financial leverage in the presence of macroeconomic risk. We demonstrate that, in fact, incorporating the combination of these factors goes a long way towards explaining average credit spread levels, and the cross-sectional variation in both costs of debt and leverage without the need to appeal to factors such as agency costs. Our model implies that companies with a high portion of expansion options tend to be riskier in general, and, at the same time, particularly sensitive to macroeconomic risk. They are not only more volatile (because growth options represent levered claims), but also have a higher propensity to default in bad times than firms with a low portion of expansion options. Thus, the default probability and its counter-cyclicality are higher the greater the ratio of expansion options to total assets. Together with higher marginal utility of the representative agent in recession, this relation (exacerbated by costly liquidation in recession) implies higher costs of debt and more important endogenous shadow costs of leverage for firms with growth options than for those with only invested assets. Thus, our findings explain why the credit spread puzzle is empirically more pronounced for growth firms, and why growth firms hold less debt even after controlling for standard determinants of credit risk. Moreover, because the economy is made up of a cross-sectional mix of firms, the model accounts, in quantitatively fairly accurate ways, for the average credit spread puzzle.

We have studied one type of (arguably important) real options of firms, namely, growth options. However, firms have a wide and varying range of options, including abandonment and shut-down options. A model incorporating these options could, therefore, yield further cross-sectional predictions.

While recent research has made important progress in enhancing our understanding of average credit risk, the cross-section of credit risk has not received sufficient attention. Analyzing it empirically is, fortunately, quite feasible. Liquid credit default swap quotes are now widely available on a firm-by-firm basis, allowing researchers to investigate specific relationships between firm-specific characteristics such as growth options and credit spreads. Our paper also provides a theoretical basis that can guide empirical research in this direction.

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8. Figures

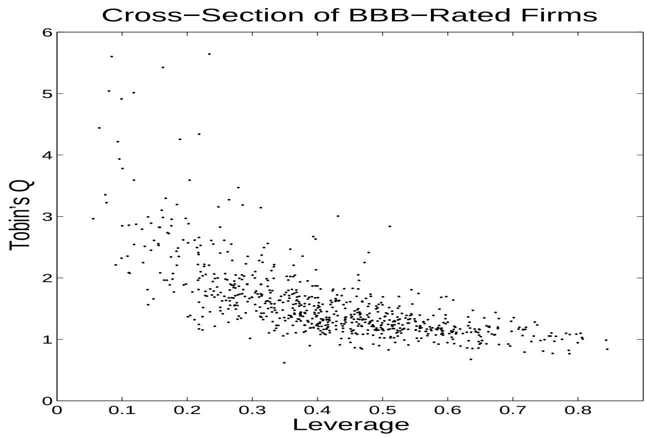

Figure 1. This scatterplot shows the average leverage and Tobin's Q for each observed BBB-rated firm over the period from 1995 to 2008.

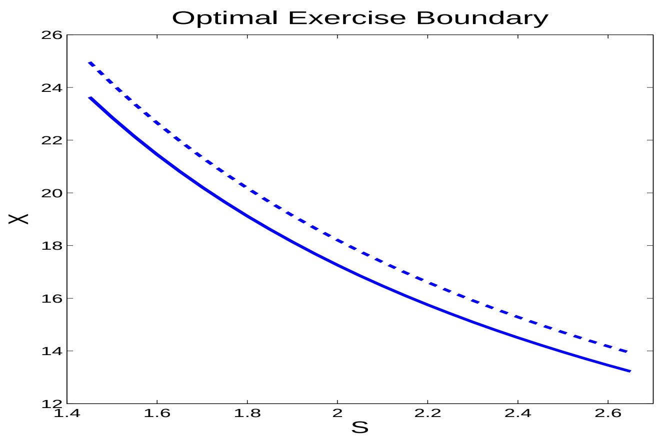

Figure 2. The solid line shows the optimal exercise boundary in boom for a range of scale parameters s. The dashed line represents the corresponding exercise boundary in recession. The graph is drawn for optimal leverage with infinite debt maturity. The baseline parameter specification from Table 1 is used.

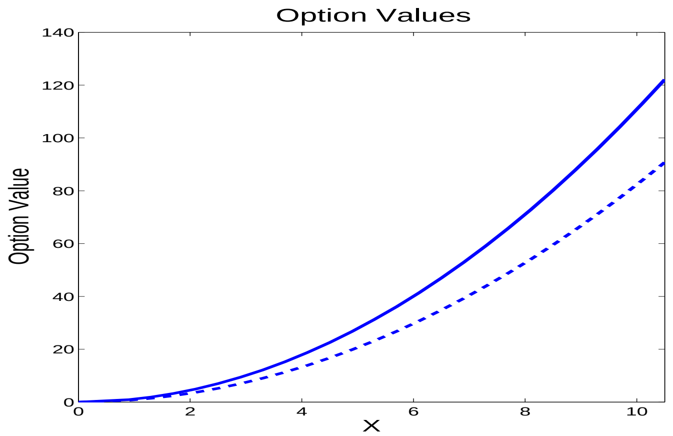

Figure 3. The solid line represents the value of the expansion option in boom for a range of starting earnings between 0 and 10. The dashed line shows the corresponding values of the same option in recession. The graph is drawn for optimal leverage with infinite debt maturity. The baseline parameter specification from Table 1 is used.

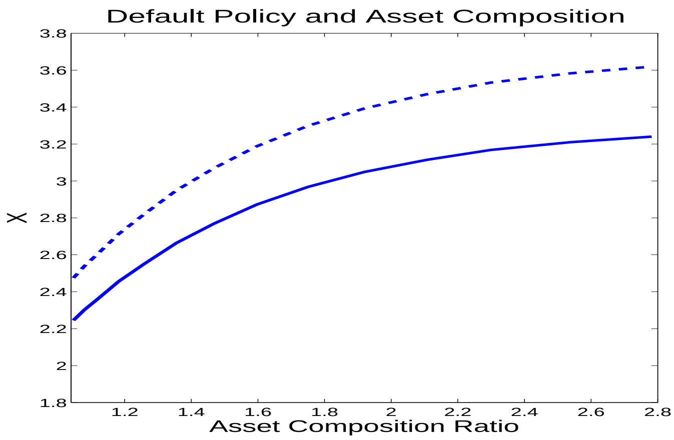

Figure 4. The solid line represents the default threshold in boom for a range of asset composition ratios. The dashed line shows the default threshold in recession. The graph is drawn for constant leverage (41.83%) at each point. Debt maturity is assumed to be infinite. The baseline parameter specification from Table 1 is used, with s being varied to generate the desired asset composition ratio.

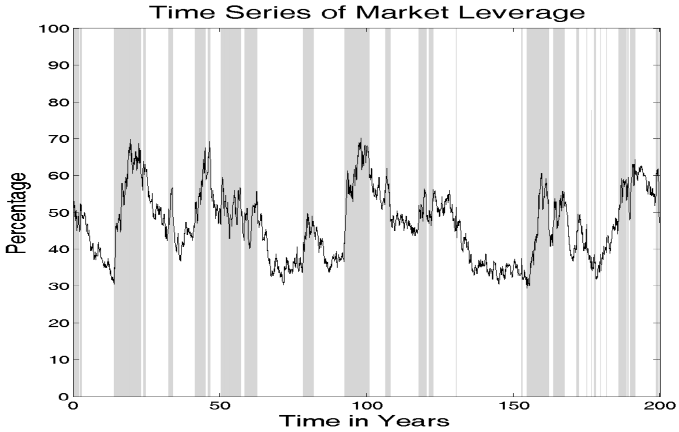

Figure 5. The solid line represents the aggregate market leverage of the simulated economy. The shaded areas represent times of recession. Standard parameters from Table 1 are used. Debt maturity is assumed to be infinite.

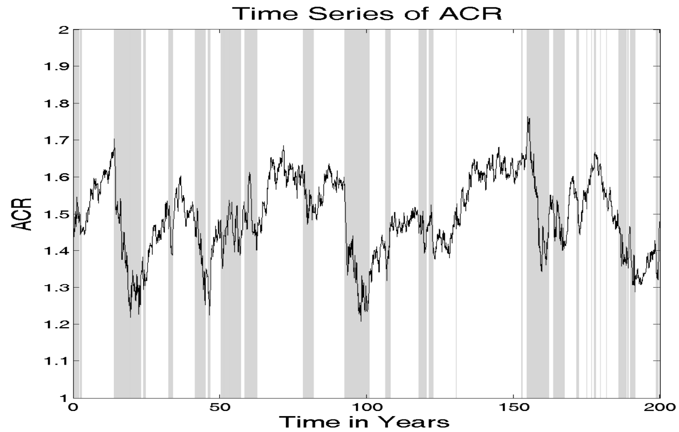

Figure 6. The solid line represents the aggregate asset composition ratio of the simulated economy. The shaded areas represent times of recession. Standard parameters from Table 1 are used. Debt maturity is assumed to be infinite.

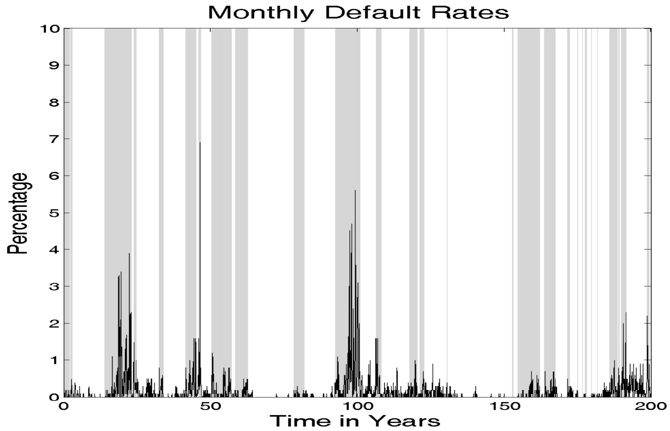

Figure 7. The solid line represents the percentage of firms which default during a given month in the simulated economy. The shaded areas represent times of recession. Standard parameters from Table 1 are used. Debt maturity is assumed to be infinite.

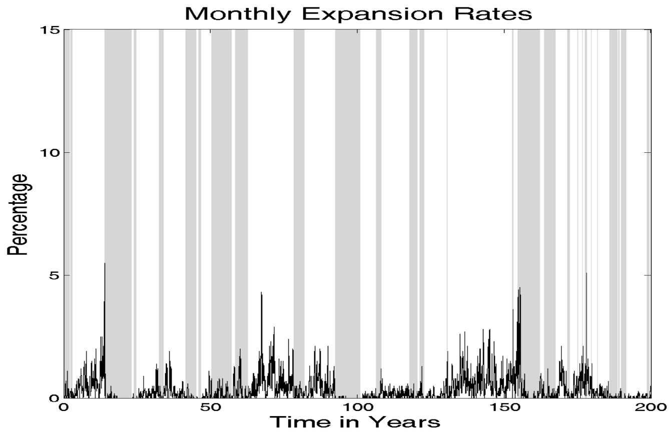

Figure 8. The solid line represents the percentage of firms which exercise their expansion options during a given month in the simulated economy. The shaded areas represent times of recession. Standard parameters from Table 1 are used. Debt maturity is assumed to be infinite.

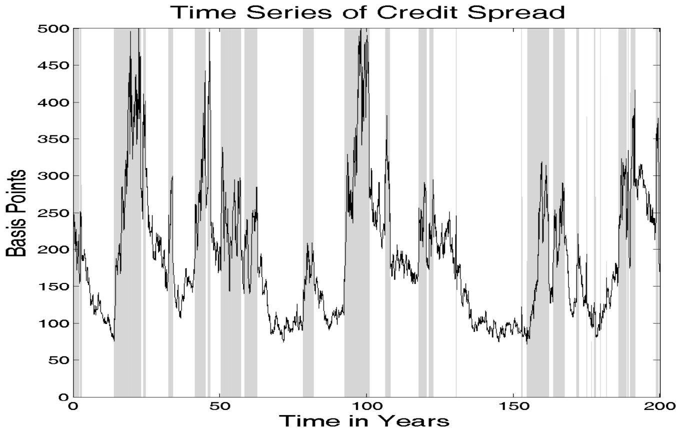

Figure 9. The solid line represents the average credit spread of the simulated economy. The shaded areas represent times of recession. Standard parameters from Table 1 are used. Debt maturity is assumed to be infinite.

9. Tables

Table 1
Baseline Parameter Choice

This table describes our baseline scenario. Panel A contains the calibrated parameters of a typical BBB-rated S&P 500 firm. Panels B and C show our parameter choice for the expansion option and our workhorse macro economy, respectively. The asset composition ratio (ACR) is the value of the firm, divided by the value of the invested assets.

Parameter	Boom	Recession
Panel A. Firm Characteristics		
Initial Value of Idiosyncratic Earnings (X)	10	10
Tax Advantage of Debt (τ)	0.15	0.15
Real Earnings Growth Rate (μ_i)	0.044	-0.0743
Systematic Earnings Volatility ($\sigma_i^{X,C}$)	0.0869	0.1369
Recovery Rate (α_i)	0.7	0.5
Panel B. Expansion Option Parameters of a Typical Firm (ACR=1.6)		
Exercise Price (K)	310	310
Scale Parameter if Initiated in Boom (s)	1.89	
Scale parameter if Initiated in Recession (s)	2.05	
Panel C. Economy		
Regime Switching Intensity (λ)	0.2718	0.4928
Consumption Growth Rate (θ_i)	0.042	0.0141
Consumption Growth Volatility (σ_i^C)	0.0094	0.0114
Expected Inflation Rate (π)	0.0342	0.0342
Systematic Price Index Volatility ($\sigma^{P,C}$)	-0.0035	-0.0035
Idiosyncratic Price Index Volatility ($\sigma^{P,id}$)	0.0132	0.0132
Rate of time preference (ρ)	0.015	0.015
Relative Risk Aversion (γ)	10	10
Elasticity of Intertemporal Substitution (Ψ)	1.5	1.5

Table 2

Target Credit Spreads and Default Probabilities

This table lists our target credit spreads and default probabilities. Panel A reports target average credit spreads for various debt maturities. They are calculated as the BBB-rated bond minus treasury yields of Davydenko and Strebulaev (2007) and Duffee (1998), net of a 35.5% non-default component. Credit spreads are quoted in basis points. Panel B reports average cumulative issuer-weighted default rates in percent for BBB-debt over 5, 10, and 20 years for US firms (Moody's, 2010).

Panel A: Target Credit Spreads (in basis points)			
Debt Maturity	Short	Medium	Long
Davydenko and Strebulaev (2007)	74	74	92
Duffee (1998)	96	95	128
Panel B: Historical BBB Default Probabilities (in percent)			
Years	5	10	20
1920-2009	3.136	7.213	13.684
1970-2009	1.926	4.851	12.327

Table 3
Implications for Credit Spreads

This table demonstrates the implications of our model for credit spreads of BBB-rated firms. The asset composition ratio (ACR) is defined as firm value, divided by the value of the invested assets. Parameters are taken from Table 1, and the leverage is set equal to 41.83%. In the one regime model, parameters are chosen to match their unconditional mean. The standard two regime model is adapted from Bhamra, Kuehn, and Strebulaev (2010b). Credit spreads for various debt maturities are calculated as the coupon divided by the debt value, minus the yield on an otherwise identical riskfree bond. They are quoted in basis points. Credit spreads of typical firms in Panels B and C are obtained by weighting the credit spreads in boom and recession by the average expected times spent in each regime, respectively. Panel D contains the average credit spreads of our simulated true cross-section of BBB-rated firms.

Debt Maturity (Years)	5	10	20
Panel A: One Regime Model			
Average Firm	18	29	41
Panel B: Standard Two Regime Model With Only Invested Assets			
Average Firm	35	56	78
Panel C: Two Regime Model With Expansion Option			
Invested Assets (ACR=1)	24	39	55
Average Firm (ACR=1.6)	44	66	81
Growth Firm (ACR=2.2)	47	70	85
Panel D: Two Regime Model With True Cross-Section			
Average Credit Spread	60	81	103

Table 4

Implications for Default Rates

This table shows the simulated cumulative default rates in percent of our true cross-section of BBB-rated firms. Panels A to D vary the underlying debt maturity used to calculate the default thresholds in our model.

Years	5	10	20
Panel A: Infinite Debt Maturity			
Average Default Rates	2.51	6.94	13.48
Median Default Rates	0.98	3.35	9.21
25% Quantile of Default Rates	0.35	1.26	3.63
75% Quantile of Default Rates	2.65	8.79	18.69
Panel B: 20 Years Debt Maturity			
Average Default Rates	4.51	10.58	18.61
Median Default Rates	1.95	5.44	13.11
25% Quantile of Default Rates	0.70	2.09	5.44
75% Quantile of Default Rates	5.30	13.95	26.08
Panel C: 10 Years Debt Maturity			
Average Default Rates	5.85	12.38	20.54
Median Default Rates	2.65	6.83	14.37
25% Quantile of Default Rates	0.98	2.93	6.83
75% Quantile of Default Rates	6.83	16.88	30.40
Panel D: 5 Years Debt Maturity			
Average Default Rates	8.64	16.91	25.96
Median Default Rates	4.74	11.92	20.36
25% Quantile of Default Rates	1.81	5.02	10.18
75% Quantile of Default Rates	11.02	24.13	38.49

Table 5

Implications for Leverage

This table demonstrates the implications of our model for the leverage features of the true cross-section of BBB-rated firms. Leverage ratios (given in percent) are calculated as the market value of debt divided by the market value of the firm. The asset composition ratio (ACR) is defined as firm value, divided by the value of the invested assets. Parameters are taken from Table 1. The debt maturity is assumed to be 10 years.

Panel A: Unconditional Leverage		
Average Leverage	40.89	
Panel B: Conditional Leverage		
Regime	Boom	Recession
Average Leverage	36.94	46.20
Median Leverage	34.36	44.19
25% Quantile	22.49	29.88
75% Quantile	48.51	60.39
Panel C: Regression of Leverage on ACR		
Average Coefficient	-0.184	
Median Coefficient	-0.184	
25% Quantile	-0.268	
75% Quantile	-0.096	

Table 6

Credit Spreads and Leverage for Alternative Specifications

This table shows 10 year credit spreads and simulated average leverage ratios of BBB-rated firms for alternative specifications of our basic model. The asset composition ratio (ACR) is defined as firm value, divided by the value of the invested assets. Credit spreads are calculated as the coupon divided by the debt value, minus the yield on an otherwise identical riskfree bond. They are quoted in basis points. The altered parameter is indicated in the first line, all other parameters are taken from Table 1. Credit spreads in the first 3 lines of Panel A for typical firms at issue are obtained by weighting the credit spreads in boom and recession by the expected times spent in each regime, respectively. The leverage is set equal to 41.83% to generate the credit spreads of typical firms. The last row in Panel A contains average credit spreads of our simulated true cross-section of BBB-rated firms. Panel B shows simulated average leverage ratios for BBB-rated firms. The debt maturity is assumed to be 10 years.

Specification	$\gamma = 7.5$	$K = 350$	Equity Financing
Panel A: 10 Year Credit Spreads			
Invested Assets (ACR=1)	33	39	39
Average Firm (ACR=1.6)	53	67	65
Growth Firm (ACR=2.2)	56	72	58
True Cross-Section	70	83	80
Panel B: Unconditional Leverage			
Average Leverage	41.10	41.22	41.14

A. Appendix

The full Appendix can be made available on a website upon publication.

A.1. The stochastic discount factor

Case 1: The general case with 2 regimes. Solving the associated Bellman equation (see Chen, 2010), it can be shown that the stochastic discount factor m_t follows the dynamics

$$\frac{\mathrm{d}m_t}{m_t} = -r_i \mathrm{d}t - \eta_i \mathrm{d}W_t^C + \left(e^{\kappa_i} - 1\right) \mathrm{d}M_t^i, \tag{A-1}$$

with M_t being the compensated process associated with the Markov chain, and

$$\begin{aligned} r_i &= -\rho \frac{(1-\gamma)}{1-\delta} \left(\frac{\delta - \gamma}{1-\gamma} h_i^{\delta - 1} - 1 \right) + \gamma \theta_i - \frac{1}{2} \gamma (1+\gamma) \left(\sigma_i^C \right)^2 - \lambda_i \left(e^{\kappa_i} - 1 \right) & \text{(A-2)} \\ \eta_i &= \gamma \sigma_i^C & \text{(A-3)} \\ \kappa_i &= (\delta - \gamma) \log \left(\frac{h_j}{h_i} \right), & \text{(A-4)} \end{aligned}$$

and h_B, h_R solve

$$0 = \rho \frac{1-\gamma}{1-\delta} h_i^{\delta - \gamma} + \left((1-\gamma) \theta_i - \frac{1}{2} \gamma (1-\gamma) \left(\sigma_i^C \right)^2 - \rho \frac{1-\gamma}{1-\delta} \right) h_i^{1-\gamma} + \lambda_i \left(h_j^{1-\gamma} - h_i^{1-\gamma} \right). \tag{A-5}$$

Case 2: Only 1 regime. In order to disentangle the effect of business cycle risk, we also consider the case of the presence of only one economic regime. We omit regime indeces. The dynamics of the stochastic discount factor then read

$$\frac{\mathrm{d}m_t}{m_t} = -r \mathrm{d}t - \eta \mathrm{d}W_t^C. \tag{A-6}$$

The real interest rate r and the risk price η are given by

$$\begin{aligned} r &= -\frac{\rho (1-\gamma)}{1-\delta} \left(\frac{\delta - \gamma}{1-\gamma} h^{\delta - 1} - 1 \right) + \gamma \theta - \frac{1}{2} \gamma (1+\gamma) \left(\sigma^C \right)^2, & \text{(A-7)} \\ \eta &= \gamma \sigma^C, & \text{(A-8)} \end{aligned}$$

with

$$h = \left(-\frac{\rho}{(1-\delta)\theta - \frac{1}{2}\gamma (1-\delta) \left(\sigma^C\right)^2 - \rho} \right)^{\frac{1}{1-\delta}}. \tag{A-9}$$

As before, the nominal interest rate is calculated as

$$r^n = r + \pi - \sigma_P^2 - \sigma^{P,C} \eta, \tag{A-10}$$

and the expected growth rate is given by

$$\tilde{\mu} = \mu - \sigma^{X,C} \left(\eta + \sigma^{P,C} \right) - \left(\sigma^{P,id} \right)^2. \tag{A-11}$$

The price-earnings ratio simplifies to

$$y = \frac{1}{r^n - \tilde{\mu}}, \tag{A-12}$$

and the total earnings volatility is

$$\tilde{\sigma} = \sqrt{\left(\sigma^{X,C}\right)^2 + \left(\sigma^{P,id}\right)^2 + \left(\sigma^{X,id}\right)^2}. \tag{A-13}$$

A.2. Firms with only invested assets

A.2.1. The valuation of corporate debt

Case V1: $\hat{D}_B < \hat{D}_R$.[30] We use the notation '^' to indicate that a parameter or function refers to a firm with only invested assets (e.g. the default boundaries $\hat{D}_i$). An investor holding corporate debt requires an instantaneous return equal to the risk-free rate r_i^n. Once the firm defaults, debtholders receive a fraction α_i of the asset value Xy_i. The required rate of return on debt must be equal to the realized rate of return plus the proceeds of debt. Therefore, an application of Ito's lemma with regime switches shows that debt satisfies the following system of ODEs:
For $0 \leq X \leq \hat{D}_B$:

$$\begin{cases} \hat{d}_B(X) &= \alpha_B X y_B \\ \hat{d}_R(X) &= \alpha_R X y_R. \end{cases} \tag{A-14}$$

For $\hat{D}_B < X \leq \hat{D}_R$:

$$\begin{cases} r_B^n \hat{d}_B(X) &= c + \tilde{\mu}_B X \hat{d}'_B(X) + \frac{1}{2}\tilde{\sigma}_B^2 X^2 \hat{d}''_B(X) + \tilde{\lambda}_B \left(\alpha_R X y_R - \hat{d}_B(X)\right) \\ \hat{d}_R(X) &= \alpha_R X y_R. \end{cases} \tag{A-15}$$

For $X > \hat{D}_R$:

$$\begin{cases} r_B^n \hat{d}_B(X) &= c + \tilde{\mu}_B X \hat{d}'_B(X) + \frac{1}{2}\tilde{\sigma}_B^2 X^2 \hat{d}''_B(X) + \tilde{\lambda}_B \left(\hat{d}_R(X) - \hat{d}_B(X)\right) \\ r_R^n \hat{d}_R(X) &= c + \tilde{\mu}_R X \hat{d}'_R(X) + \frac{1}{2}\tilde{\sigma}_R^2 X^2 \hat{d}''_R(X) + \tilde{\lambda}_R \left(\hat{d}_B(X) - \hat{d}_R(X)\right). \end{cases} \tag{A-16}$$

The functional form of the solution is

$$\hat{d}_i(X) = \begin{cases} \alpha_i X y_i & X \leq \hat{D}_i & i = B, R \\ \hat{C}_1 X^{\beta_1^B} + \hat{C}_2 X^{\beta_2^B} + C_3 X + C_4 & \hat{D}_B < X \leq \hat{D}_R, & i = B \\ \hat{A}_{i1} X^{\gamma_1} + \hat{A}_{i2} X^{\gamma_2} + A_{i5} & X > \hat{D}_R, & i = B, R, \end{cases} \tag{A-17}$$

where $\hat{A}_{B1}, \hat{A}_{B2}, \hat{A}_{R1}, \hat{A}_{R2}, A_5, \hat{C}_1, \hat{C}_2, C_3, C_4, \gamma_1, \gamma_2, \beta_1^B$, and β_2^B are real-valued parameters to be determined. We first consider the region $X > \hat{D}_R$, and use the standard approach of plugging in the functional

[30] The solution of the case $\hat{D}_B > \hat{D}_R$ can be found by the according change in notation.

form $\hat{d}_i(X) = \hat{A}_{i1}X^{\gamma_1} + \hat{A}_{i2}X^{\gamma_2} + A_{i5}$ into both equations of (A-16). Comparing coefficients and solving the resulting 2-dimensional system of equations for A_{i5}, we find that

$$A_{i5} = \frac{c\left(r_j^n + \tilde{\lambda}_i + \tilde{\lambda}_j\right)}{r_i^n r_j^n + r_j^n \tilde{\lambda}_i + r_i^n \tilde{\lambda}_j}. \tag{A-18}$$

Next, $\hat{A}_{Rk}$ is always a multiple of $\hat{A}_{Bk}$, $k = 1, 2$, with the factor $l_k := \frac{1}{\tilde{\lambda}_B}(r_B^n + \tilde{\lambda}_B - \tilde{\mu}_B\gamma_k - \frac{1}{2}\tilde{\sigma}_B^2\gamma_k(\gamma_k - 1))$, i.e., $A_{Bk} = l_k A_{Rk}$.

Using these results and comparing coefficients again, we find that γ_1 and γ_2 correspond to the negative roots of the quartic equation

$$\left(\tilde{\mu}_R\gamma + \frac{1}{2}\tilde{\sigma}_R^2\gamma(\gamma - 1) - \tilde{\lambda}_R - r_R^n\right)\left(\tilde{\mu}_B\gamma + \frac{1}{2}\tilde{\sigma}_B^2\gamma(\gamma - 1) - \tilde{\lambda}_B - r_B^n\right) = \tilde{\lambda}_R\tilde{\lambda}_B, \tag{A-19}$$

with the reason for taking the negative roots being the no-bubbles condition for debt stated below. By arguments of Guo (2001), this quartic equation always has four distinct real roots, two of them being negative, and two of them positive.

Next, we consider the region $\hat{D}_B \leq X \leq \hat{D}_R$, i.e., the realized state of the Markov chain is boom (if not, the solution is already known by the second equation of system (A-15)). Again, plugging in the functional form $d_B(X) = \hat{C}_1 X^{\beta_1^B} + \hat{C}_2 X^{\beta_2^B} + C_3 X + C_4$ into the first equation of (A-15), we find by comparison of coefficients that

$$\begin{aligned}
\beta_{1,2}^B &= \frac{1}{2} - \frac{\tilde{\mu}_B}{\tilde{\sigma}_B^2} \pm \sqrt{\left(\frac{1}{2} - \frac{\tilde{\mu}_B}{\tilde{\sigma}_B^2}\right)^2 + \frac{2\left(r_B^n + \tilde{\lambda}_B\right)}{\tilde{\sigma}_B^2}} \\
C_3 &= \frac{\tilde{\lambda}_B\alpha_R y_R}{r_B^n + \tilde{\lambda}_B - \tilde{\mu}_B} \\
C_4 &= \frac{c}{r_B^n + \tilde{\lambda}_B},
\end{aligned} \tag{A-20}$$

for $r_B^n + \tilde{\lambda}_B - \tilde{\mu}_B \neq 0$. The unknown parameters are now $\hat{A}_{B1}, \hat{A}_{B2}, \hat{C}_1$ and $\hat{C}_2$. The boundary conditions read

$$\lim_{X\to\infty} \frac{\hat{d}_i(X)}{X} < \infty, \quad i = B, R \tag{A-21}$$

$$\lim_{X\searrow\hat{D}_R} \hat{d}_B(X) = \lim_{X\nearrow\hat{D}_R} \hat{d}_B(X) \tag{A-22}$$

$$\lim_{X\searrow\hat{D}_R} \hat{d}'_B(X) = \lim_{X\nearrow\hat{D}_R} \hat{d}'_B(X) \tag{A-23}$$

$$\lim_{X\searrow\hat{D}_B} \hat{d}_B(X) = \alpha_B D_B y_B \tag{A-24}$$

$$\lim_{X\searrow\hat{D}_B} \hat{d}_R(X) = \alpha_R D_R y_R. \tag{A-25}$$

Condition (A-21) is the no-bubbles condition used above in determining the appropriate roots of equation (A-19). The default thresholds $\hat{D}_R$ and $\hat{D}_B$ are chosen by the equityholders, and are taken as given by the debtholders. The boundary conditions are, hence, the value-matching conditions (A-22), (A-24), and (A-25), and the smooth-pasting condition at the higher default threshold $\hat{D}_B$ for the debt function in recession $\hat{d}_R(\cdot)$, equation (A-23). As the default thresholds are not related to an optimality concept from the point of view of the debtholders, there are no smooth-pasting conditions at default to consider.

We plug in the functional form (A-17) into conditions (A-22)-(A-25), and obtain a four-dimensional linear system in the four unknowns $\hat{A}_{B1}, \hat{A}_{B2}, \hat{C}_1$ and $\hat{C}_2$:

$$\begin{aligned} \hat{A}_{B1}\hat{D}_R^{\gamma_1} + \hat{A}_{B2}\hat{D}_R^{\gamma_2} + A_5 &= \hat{C}_1\hat{D}_R^{\beta_1^B} + \hat{C}_2\hat{D}_R^{\beta_2^B} + C_3\hat{D}_R + C_4 \\ \hat{A}_{B1}\gamma_1\hat{D}_R^{\gamma_1} + \hat{A}_{B2}\gamma_2\hat{D}_R^{\gamma_2} &= \hat{C}_1\beta_1^B\hat{D}_R^{\beta_1^B} + \hat{C}_2\beta_2^B\hat{D}_R^{\beta_2^B} + C_3\hat{D}_R \\ \alpha_B\hat{D}_B y_B &= \hat{C}_1\hat{D}_R^{\beta_1^B} + \hat{C}_2\hat{D}_R^{\beta_2^B} + C_3\hat{D}_R + C_4 \\ l_1\hat{A}_{B1}\hat{D}_R^{\gamma_1} + l_2\hat{A}_{B2}\hat{D}_R^{\gamma_2} + A_5 &= \alpha_R\hat{D}_R y_R. \end{aligned} \tag{A-26}$$

We define the matrices

$$\begin{aligned} \hat{M} &:= \begin{bmatrix} \hat{D}_R^{\gamma_1} & \hat{D}_R^{\gamma_2} & -\hat{D}_R^{\beta_1^B} & -\hat{D}_R^{\beta_2^B} \\ \gamma_1\hat{D}_R^{\gamma_1} & \gamma_2\hat{D}_R^{\gamma_2} & -\beta_1^B\hat{D}_R^{\beta_1^B} & -\beta_2^B\hat{D}_R^{\beta_2^B} \\ 0 & 0 & \hat{D}_R^{\beta_1^B} & \hat{D}_R^{\beta_2^B} \\ l_1\hat{D}_R^{\gamma_1} & l_2\hat{D}_R^{\gamma_2} & 0 & 0 \end{bmatrix} \\ \hat{b} &:= \begin{bmatrix} C_3\hat{D}_R + C_4 - A_{B5} \\ C_3\hat{D}_R \\ \alpha_B\hat{D}_B y_B - C_3\hat{D}_R - C_4 \\ \alpha_R\hat{D}_R y_R - A_{R5} \end{bmatrix}, \end{aligned}$$

such that $\hat{M}\begin{bmatrix}\hat{A}_{B1} & \hat{A}_{B2} & \hat{C}_1 & \hat{C}_2\end{bmatrix}^T = \hat{b}$. Hence the solution of the unknowns left is given by

$$\begin{bmatrix}\hat{A}_{B1} & \hat{A}_{B2} & \hat{C}_1 & \hat{C}_2\end{bmatrix}^T = \hat{M}^{-1}\hat{b}. \tag{A-27}$$

Case 2: Only 1 regime. Omitting the regime-index, we define all parameters and functions as in Case V1, and let $\hat{D}_1$ be the default threshold. Note that for $\mathbb{P} = \mathbb{Q}$ a.e. and $y = 1$, this case corresponds to the model of Leland (1994). Equations (A-10)-(A-13) provide all the parameters needed in the setup and solution of the model in the 1-regime case. Using that the required return must be equal to the expected realized return plus the proceeds from debt, we find the following system to solve:

$$\begin{aligned} r^n\hat{d}(X) &= c + \tilde{\mu}X\hat{d}'(X) + \tfrac{\tilde{\sigma}^2}{2}X^2\hat{d}''(X) \quad X > \hat{D} \\ \hat{d}(X) &= \alpha X y \quad\quad\quad\quad\quad\quad\quad\quad\quad\; X \leq \hat{D}. \end{aligned} \tag{A-28}$$

The boundary conditions are the no bubbles condition, as well as value-matching at default:

$$\begin{aligned} \lim_{X\to\infty}\frac{\hat{d}(X)}{X} &< \infty \\ \lim_{X\searrow\hat{D}}\hat{d}(X) &= \alpha y\hat{D}. \end{aligned} \tag{A-29}$$

The functional form of the solution is

$$\hat{d}(X) = \begin{cases} \alpha y X & X < \hat{D} \\ \hat{B} X^{\beta_2} + \frac{c}{r} & X \geq \hat{D}, \end{cases} \tag{A-30}$$

where $\hat{B}$ and β_2 are real-valued parameters. It is straightforward to show that

$$\beta_2 = \frac{1}{2} - \frac{\tilde{\mu}}{\tilde{\sigma}^2} - \sqrt{\left(\frac{1}{2} - \frac{\tilde{\mu}}{\tilde{\sigma}^2}\right)^2 + \frac{2r^n}{\tilde{\sigma}^2}} \tag{A-31}$$

$$\hat{B} = \left(\alpha y \hat{D} - \frac{c}{r^n}\right) \hat{D}^{-\beta_2}. \tag{A-32}$$

A.2.2. The valuation of tax benefits

The value of tax benefits $\hat{t}_i(X)$ corresponds to the value of debt with recovery rates equal to zero, and a coupon of $c\tau$ (and analogously for Case 2).

A.2.3. The valuation of default costs

As there are no continuous earnings associated with default costs, value function of default costs $\hat{b}_i(X)$ can be calculated as the value of a debt contract with recovery rates $1 - \alpha_B$ and $1 - \alpha_R$, respectively, and a coupon of zero. Case 2 can be treated analogously.

A.2.4. Firm value

Total firm value $\hat{f}_i$ in regime $i = B, R$ corresponds to the value of assets $y_i X$, plus the value of tax benefits from debt $\hat{t}_i(X)$, less the value of potential default costs $\hat{b}_i(X)$, i.e.,

$$\hat{f}_i(X) = X y_i + \hat{t}_i(X) - \hat{b}_i(X).$$

Analogously, for Case 2, we have

$$\hat{f}(X) = X y + \hat{t}(X) - \hat{b}(X).$$

A.2.5. The valuation of equity

The levered firm value equals the sum of debt and equity values. Hence, equity value $\hat{e}_i(X)$, $i = B, R$, may be written as

$$\hat{e}_i(X) = \hat{f}_i(X) - \hat{d}_i(X) = X y_i + \hat{t}_i(X) - \hat{b}_i(X) - \hat{d}_i(X), \tag{A-33}$$

or, for the Case 2,

$$\hat{e}(X) = \hat{f}(X) - \hat{d}(X) = X y + \hat{t}(X) - \hat{b}(X) - \hat{d}(X). \tag{A-34}$$

This is the closed-form expression for equity.

A.2.6. Default policy

Once debt has been issued, managers select the ex-post default policy that maximizes the value of equity. Formally, the default policy is determined by postulating that the derivative of the equity value has to be zero at the according default boundary. It is straightforward to calculate the first derivative of equity in closed form, using the derivative of the functional forms of the value of debt, default costs, and tax shield. The system to solve is for Case V1:

$$\begin{cases} \hat{e}'_B(\hat{D}^*_B) &= 0 \\ \hat{e}'_R(\hat{D}^*_R) &= 0. \end{cases} \tag{A-35}$$

We solve this problem numerically. For Case 2, the system is

$$\hat{e}'(D^*) = 0, \tag{A-36}$$

which is solvable in closed form.

Note that for a given coupon, all value functions can be calculated by following the approach up to system (A-35) or system (A-36), depending on the case. This fact will be used later for the calculation of the value of corporate securities of a firm consisting of both assets in place and an expansion option.

A.2.7. Capital structure

Denote by $\hat{f}^*_i(X)$ the firm value of a firm with only invested assets, given optimal ex-post default thresholds. The ex-ante optimal coupon of a firm solves
in Case V1

$$\hat{c}^* := \text{argmax}_{\hat{c}} \hat{f}^*_i(X), \tag{A-37}$$

and in Case 2

$$\hat{c}^* := \text{argmax}_{\hat{c}} \hat{f}^*(X). \tag{A-38}$$

A.3. The value of the growth option

Case G1: $X_R > X_B$. Recall that the system to solve is:

For $0 \leq X < X_B$:

$$\begin{cases} r^n_B G_B(X) &= \tilde{\mu}_B X G'_B(X) + \frac{1}{2}\tilde{\sigma}^2_B X^2 G''_B(X) + \tilde{\lambda}_B \left(G_R(X) - G_B(X)\right) \\ r^n_R G_R(X) &= \tilde{\mu}_R X G'_R(X) + \frac{1}{2}\tilde{\sigma}^2_R X^2 G''_R(X) + \tilde{\lambda}_R \left(G_B(X) - G_R(X)\right) \end{cases} \tag{A-39}$$

For $X_B \leq X < X_R$:

$$\begin{cases} G_B(X) &= sXy_B - K \\ r^n_R G_R(X) &= \tilde{\mu}_R X G'_R(X) + \frac{1}{2}\tilde{\sigma}^2_R X^2 G''_R(X) + \tilde{\lambda}_R \left(sXy_B - K - G_R(X)\right) \end{cases} \tag{A-40}$$

For $X \geq X_R$:

$$\begin{cases} G_B(X) &= sXy_B - K \\ G_R(X) &= sXy_R - K, \end{cases} \tag{A-41}$$

subject to the boundary conditions:

$$\lim_{X \searrow 0} G_i(X) = 0, \quad i = B, R \tag{A-42}$$

$$\lim_{X \searrow X_B} G_R(X) = \lim_{X \nearrow X_B} G_R(X) \tag{A-43}$$

$$\lim_{X \searrow X_B} G'_R(X) = \lim_{X \nearrow X_B} G'_R(X) \tag{A-44}$$

$$\lim_{X \nearrow X_R} G_R(X) = sX_R y_R - K \tag{A-45}$$

$$\lim_{X \nearrow X_B} G_B(X) = sX_B y_B - K \tag{A-46}$$

The functional form of the solution is given by

$$G_i(X) = \begin{cases} \bar{A}_{i3} X^{\gamma_3} + \bar{A}_{i4} X^{\gamma_4} & X < X_B, & i = B, R \\ \bar{C}_1 X^{\beta_1^R} + \bar{C}_2 X^{\beta_2^R} + \bar{C}_3 X + \bar{C}_4 & X_B \leq X < X_R, & i = R \\ sXy_i - K & X \geq X_i & i = B, R, \end{cases} \tag{A-47}$$

where $\bar{A}_{B3}, \bar{A}_{B4}, \bar{A}_{R1}, \bar{A}_{R2}, \bar{C}_1, \bar{C}_2, \bar{C}_3, \bar{C}_4, \gamma_3, \gamma_4, \beta_1^R$, and β_2^R are real-valued parameters to be determined. The notation$^-$ indicates that a parameter refers to the value of the growth option (and only to the value of the growth option). We first consider the region $X < X_B$, and use the standard approach of plugging in the functional form $G_i(X) = \bar{A}_{i3} X^{\gamma_3} + \bar{A}_{i4} X^{\gamma_4}$ into both equations of (A-39). Comparison of coefficients yields that $\bar{A}_{Rk}$ is always a multiple of $\bar{A}_{Bk}$, $k = 3, 4$, with the factor $\bar{l}_k := \frac{1}{\tilde{\lambda}_B}(r_B^n + \tilde{\lambda}_B - \tilde{\mu}_B \gamma_k - \frac{1}{2}\tilde{\sigma}_B^2 \gamma_k(\gamma_k - 1))$, i.e., $\bar{A}_{Bk} = \bar{l}_k \bar{A}_{Rk}$. Note that even though the factor $\bar{l}_k$ is of similar structure as the one found in the calculation of the value of debt of a firm with only invested assets, their values differ due to the different roots γ_i in the formulae. Using this relationship and comparing coefficients again, we find that γ_3 and γ_4 correspond to the positive roots of the quartic equation

$$\left(\tilde{\mu}_R \gamma + \frac{1}{2}\tilde{\sigma}_R^2 \gamma(\gamma - 1) - \tilde{\lambda}_R - r_R^n\right)\left(\tilde{\mu}_B \gamma + \frac{1}{2}\tilde{\sigma}_B^2 \gamma(\gamma - 1) - \tilde{\lambda}_B - r_B^n\right) = \tilde{\lambda}_R \tilde{\lambda}_B. \tag{A-48}$$

The reason for taking the positive roots being that the option value has to approach zero as the earnings approaches zero.

Next, we consider the region $X_B \leq X < X_R$. Note that in the case of interest the Markov chain is in recession (otherwise, the solution is already known). Again, plugging in the functional form $G_R(X) = \bar{C}_1 X^{\beta_1} + \bar{C}_2 X^{\beta_2} + \bar{C}_3 X + \bar{C}_4$ into the second equation of (A-40), we find by comparison of coefficients that

$$\begin{aligned} \beta_{1,2}^R &= \frac{1}{2} - \frac{\tilde{\mu}_R}{\tilde{\sigma}_R^2} \pm \sqrt{\left(\frac{1}{2} - \frac{\tilde{\mu}_R}{\tilde{\sigma}_R^2}\right)^2 + \frac{2\left(r_R^n + \tilde{\lambda}_R\right)}{\tilde{\sigma}_R^2}} \\ \bar{C}_3 &= \frac{s\tilde{\lambda}_R y_B}{r_R^n - \tilde{\mu}_R + \tilde{\lambda}_R} \\ \bar{C}_4 &= -\frac{K\tilde{\lambda}_R}{r_R^n + \tilde{\lambda}_R}. \end{aligned} \tag{A-49}$$

It is left to solve for the unknown parameters $\bar{A}_{B3}, \bar{A}_{B4}, \bar{C}_1$ and $\bar{C}_2$. Plugging in the functional form (A-47) into conditions (A-43)-(A-46) yields

$$\begin{aligned}
\bar{C}_1 X_B^{\beta_1^R} + \bar{C}_2 X_B^{\beta_2^R} + \bar{C}_3 X_B + \bar{C}_4 &= \bar{l}_1 \bar{A}_{B3} X_B^{\gamma_3} + \bar{l}_2 \bar{A}_{B4} X_B^{\gamma_4} && \text{(A-50)}\\
\bar{C}_1 \beta_1^R X_B^{\beta_1^R} + \bar{C}_2 \beta_2^R X_B^{\beta_2^R} + \bar{C}_3 X_B &= \bar{l}_1 \bar{A}_{B3} \gamma_3 X_B^{\gamma_3} + \bar{l}_2 \gamma_4 \bar{A}_{B4} X_B^{\gamma_4} && \text{(A-51)}\\
\bar{C}_1 X_R^{\beta_1^R} + \bar{C}_2 X_R^{\beta_2^R} + \bar{C}_3 X_R + \bar{C}_4 &= s y_R X_R - K && \text{(A-52)}\\
\bar{A}_{B3} X_B^{\gamma_3} + \bar{A}_{B4} X_B^{\gamma_4} &= s y_B X_B - K && \text{(A-53)}
\end{aligned}$$

This four-dimensional system is linear in its four unknowns $\bar{A}_{B3}, \bar{A}_{B4}, \bar{C}_1$ and $\bar{C}_2$. We define the matrices

$$\begin{aligned}
\bar{M} &:= \begin{bmatrix} \bar{l}_1 X_B^{\gamma_3} & \bar{l}_2 X_B^{\gamma_4} & -X_B^{\beta_1^R} & -X_B^{\beta_2^R} \\ \bar{l}_1 \gamma_3 X_B^{\gamma_3} & \bar{l}_2 \gamma_4 X_B^{\gamma_4} & -\beta_1^R X_B^{\beta_1^R} & -\beta_2^R X_B^{\beta_2^R} \\ 0 & 0 & X_R^{\beta_1^R} & X_R^{\beta_2^R} \\ X_B^{\gamma_3} & X_B^{\gamma_4} & 0 & 0 \end{bmatrix} \\
\bar{b} &:= \begin{bmatrix} \bar{C}_3 X_B + \bar{C}_4 \\ \bar{C}_3 X_B \\ -\bar{C}_3 X_R - \bar{C}_4 + s y_R X_R - K \\ s y_B X_B - K \end{bmatrix},
\end{aligned}$$

such that $\bar{M} \begin{bmatrix} \bar{A}_{B3} & \bar{A}_{B4} & \bar{C}_1 & \bar{C}_2 \end{bmatrix}^T = \bar{b}$. Hence the solution to the remaining unknowns is given by

$$\begin{bmatrix} \bar{A}_{B3} & \bar{A}_{B4} & \bar{C}_1 & \bar{C}_2 \end{bmatrix}^T = \bar{M}^{-1} \bar{b}. \qquad \text{(A-54)}$$

Note that the relative price change sensitivity is

$$\frac{G_i'(X)}{G_i(X)} = \begin{cases} \frac{\gamma_3 \bar{A}_{i3} X^{\gamma_3 - 1} + \bar{A}_{i4} \gamma_4 X^{\gamma_4 - 1}}{\bar{A}_{i3} X^{\gamma_3} + \bar{A}_{i4} X^{\gamma_4}} & X < X_B, \quad i = B, R \\ \frac{\bar{C}_1 \beta_1 X^{\beta_1 - 1} + \bar{C}_2 \beta_2 X^{\beta_2 - 1} + \bar{C}_3}{\bar{C}_1 X^{\beta_1} + \bar{C}_2 X^{\beta_2} + \bar{C}_3 X + \bar{C}_4} & X_B \le X < X_R, \quad i = R \\ \frac{s y_i}{s y_i X - K} & X \ge X_i \quad i = B, R. \end{cases} \qquad \text{(A-55)}$$

Finally, consider the unlevered value of the growth option, whose optimal exercise boundaries are determined by the additional boundary conditions (23)-(24):

$$\begin{aligned}
\lim_{X \nearrow X_R^{unlev}} G_R'(X) &= s y_R && \text{(A-56)}\\
\lim_{X \nearrow X_B^{unlev}} G_B'(X) &= s y_B. && \text{(A-57)}
\end{aligned}$$

The calculations are the same up to system (A-49). System (A-50)-(A-53) is augmented by the two equations corresponding to the additional boundary conditions:

$$\begin{aligned}
\bar{C}_1^{unlev} \beta_1^R \left(X_R^{unlev}\right)^{\beta_1^R - 1} + \bar{C}_2^{unlev} \beta_2^R \left(X_R^{unlev}\right)^{\beta_2^R - 1} + \bar{C}_3 &= s y_R && \text{(A-58)}\\
\bar{A}_{B3}^{unlev} \gamma_3 \left(X_B^{unlev}\right)^{\gamma_3 - 1} + \bar{A}_{B4}^{unlev} \gamma_4 \left(X_B^{unlev}\right)^{\gamma_4 - 1} &= s y_B. && \text{(A-59)}
\end{aligned}$$

The full system is six-dimensional with the six unknowns $\bar{A}_{B3}^{unlev}$, $\bar{A}_{B4}^{unlev}$, $\bar{C}_1^{unlev}$, $\bar{C}_2^{unlev}$ X_B^{unlev}, and X_R^{unlev}, linear in the first four unknowns, and non-linear in the last two unknowns. It is solved numerically, using relation (A-54) for any given pair of exercise boundaries in the numerical solution algorithm.

Case 2: Only 1 regime. Denote the investment boundary by X_1. We find that the system to solve is given by:

$$\begin{array}{rcll} rG(X) & = & \tilde{\mu}XG'(X) + \frac{\tilde{\sigma}^2}{2}X^2G''(X) & X < X_1 \\ G(X) & = & sXy - K & X \geq X_1 \end{array} \tag{A-60}$$

The boundary conditions are given by a value matching condition and the fact that he option must become worthless as the asset value approaches zero:

$$\lim_{X \nearrow X_1} G(X) = syX_1 - K \tag{A-61}$$

$$\lim_{X \searrow 0} G(X) = 0 \tag{A-62}$$

The functional form of the solution is

$$G(X) = \begin{cases} \bar{A}X^{\beta_1} & X < X_1 \\ sXy - K & X \geq X_1, \end{cases} \tag{A-63}$$

where $\bar{A}$ and β_1 are real-valued parameters to be determined. It is then straightforward to show that

$$\beta_1 = \frac{1}{2} - \frac{\tilde{\mu}}{\tilde{\sigma}^2} + \sqrt{\left(\frac{1}{2} - \frac{\tilde{\mu}}{\tilde{\sigma}^2}\right)^2 + \frac{2r}{\tilde{\sigma}^2}} \tag{A-64}$$

$$\bar{A} = (syX_1 - K)\, X_1^{-\beta_1}, \tag{A-65}$$

which is the solution for the option. The relative price change sensitivity of the option is

$$\frac{G'(X)}{G(X)} = \begin{cases} \frac{\beta_1}{X} & X < X_1 \\ \frac{sy}{syX - K} & X \geq X_1. \end{cases} \tag{A-66}$$

A.4. Firms with invested assets and expansion options

A.4.1. The valuation of corporate debt

Case A1: $D_B < D_R$, $\hat{D}_B < \hat{D}_R$, and $X_R > X_B$. This case constitutes the one presented in the main text. For brevity of notation, define $\bar{s} := s + 1$. Recall that the system to solve is:
For $0 \leq X \leq D_B$:

$$\begin{cases} d_B(X) = \alpha_B\left(Xy_B + G_B^{unlev}(X)\right) \\ d_R(X) = \alpha_R\left(Xy_R + G_R^{unlev}(X)\right) \end{cases} \tag{A-67}$$

For $D_B < X \leq D_R$:

$$\begin{cases} r_B^n d_B(X) = c + \tilde{\mu}_B X d_B'(X) + \frac{1}{2}\tilde{\sigma}_B^2 X^2 d_B''(X) + \tilde{\lambda}_B\left(\alpha_R\left(Xy_R + G_R^{unlev}(X)\right) - d_B(X)\right) \\ d_R(X) = \alpha_R\left(Xy_R + G_R^{unlev}(X)\right) \end{cases} \tag{A-68}$$

For $D_R < X < X_B$:

$$\begin{cases} r_B^n d_B(X) &= c + \tilde{\mu}_B X d_B'(X) + \frac{1}{2}\tilde{\sigma}_B^2 X^2 d_B''(X) + \tilde{\lambda}_B (d_R(X) - d_B(X)) \\ r_R^n d_R(X) &= c + \tilde{\mu}_R X d_R'(X) + \frac{1}{2}\tilde{\sigma}_R^2 X^2 d_R''(X) + \tilde{\lambda}_R (d_B(X) - d_R(X)) \end{cases} \tag{A-69}$$

For $X_B \leq X < X_R$:

$$\begin{cases} d_B(X) &= \hat{d}_B\left(\bar{s}X - \frac{K}{y_B}\right) \\ r d_R(X) &= c + \tilde{\mu}_R X d_R'(X) + \frac{1}{2}\tilde{\sigma}_R^2 X^2 d_R''(X) + \tilde{\lambda}_R \left(\hat{d}_B\left(\bar{s}X - \frac{K}{y_B}\right) - d_R(X)\right) \end{cases} \tag{A-70}$$

For $X \geq X_R$:

$$\begin{cases} d_B(X) &= \hat{d}_B\left(\bar{s}X - \frac{K}{y_B}\right) \\ d_R(X) &= \hat{d}_R\left(\bar{s}X - \frac{K}{y_R}\right). \end{cases} \tag{A-71}$$

The system is subject to the following boundary conditions:

$$\begin{aligned} \lim_{X\searrow D_R} d_B(X) &= \lim_{X\nearrow D_R} d_B(X) \\ \lim_{X\searrow D_R} d_B'(X) &= \lim_{X\nearrow D_R} d_B'(X) \\ \lim_{X\searrow D_B} d_B(X) &= \alpha_B\left(D_B y_B + G_B^{unlev}(D_B)\right) \\ \lim_{X\searrow D_R} d_R(X) &= \alpha_R\left(D_R y_R + G_R^{unlev}(D_R)\right) \\ \lim_{X\searrow X_B} d_R(X) &= \lim_{X\nearrow X_B} d_R(X) \\ \lim_{X\searrow X_B} d_R'(X) &= \lim_{X\nearrow X_B} d_R'(X) \\ \lim_{X\nearrow X_B} d_B(X) &= \hat{d}_B\left(\bar{s}X_B - \frac{K}{y_B}\right) \\ \lim_{X\nearrow X_R} d_R(X_R) &= \hat{d}_R\left(\bar{s}X_R - \frac{K}{y_R}\right). \end{aligned} \tag{A-72}$$

In order to solve this system, we start with the functional form of the solution:

$$d_i(X) = \begin{cases} \alpha_i\left(X y_i + G_i^{unlev}(X)\right) & X \leq D_i & i = B, R, \\ C_1 X^{\beta_1^B} + C_2 X^{\beta_2^B} + C_3 X + C_4 + C_5 X^{\gamma_3} + C_6 X^{\gamma_4} & D_B < X \leq D_R, & i = B \\ A_{i1} X^{\gamma_1} + A_{i2} X^{\gamma_2} + A_{i3} X^{\gamma_3} + A_{i4} X^{\gamma_4} + A_5 & D_R < X \leq X_B, & i = B, R \\ B_1 X^{\beta_1^R} + B_2 X^{\beta_2^R} + Z(X) & X_B < X \leq X_R, & i = R \\ \hat{d}_i\left(\bar{s}X - \frac{K}{y_i}\right) & X > X_i, & i = B, R, \end{cases} \tag{A-73}$$

where $A_{B1}, A_{B2}, A_{R1}, A_{R2}, C_1, C_2, C_3, C_4, C_5, C_6, B_1, B_2, \beta_1^B, \beta_2^B, \beta_1^R, \beta_2^R, \gamma_3$, and γ_4 are real-valued parameters to be determined (or to be confirmed). The function $Z(X)$ as stated in the sixth line of (A-73) is of closed form. It will be given explicitly in the following calculations.

We first consider the region $X_B \geq X > D_R$. Using the standard approach of plugging in the functional form $d_i(X) = A_{i1}X^{\gamma_1} + A_{i2}X^{\gamma_2} + A_{i3}X^{\gamma_3} + A_{i4}X^{\gamma_4} + A_{i5}$ into both equations of (A-69), comparing coefficients, and solving, we confirm that

$$A_{i5} = \frac{c\left(r_j^n + \tilde{\lambda}_i + \tilde{\lambda}_j\right)}{r_i^n r_j^n + r_j^n \tilde{\lambda}_i + r_i^n \tilde{\lambda}_j}, \tag{A-74}$$

and we find again that A_{Rk} is always a multiple of A_{Bk}, $k = 1, \ldots, 4$, with the factor $l_k := \frac{1}{\tilde{\lambda}_B}(r_B^n + \tilde{\lambda}_B - \tilde{\mu}_B\gamma_k - \frac{1}{2}\tilde{\sigma}_B^2\gamma_k(\gamma_k - 1))$, i.e., $A_{Bk} = l_k A_{Rk}$. Using this relationship and comparing coefficients, we find that $\gamma_1, \gamma_2, \gamma_3$, and γ_4 correspond to the roots of the quartic equation (A-19), which is:

$$\left(\tilde{\mu}_R\gamma + \frac{1}{2}\tilde{\sigma}_R^2\gamma(\gamma-1) - \tilde{\lambda}_R - r_R^n\right)\left(\tilde{\mu}_B\gamma + \frac{1}{2}\tilde{\sigma}_B^2\gamma(\gamma-1) - \tilde{\lambda}_B - r_B^n\right) = \tilde{\lambda}_R\tilde{\lambda}_B. \tag{A-75}$$

Recall that by arguments of Guo (2001), we know that this quartic equation always has four distinct real roots, two of them being negative, and two positive. The value of debt in both regimes will be subject to boundary conditions from both below (default) and above (exercise of expansion option). In order to meet all boundary conditions, we need four terms with the according factors A_{ik} as well as exponents γ_k, which requires usage of all four roots of (A-75). The no-bubbles condition is already implemented in the value function of a firm with only invested assets $\hat{d}_i$, and, hence, does not need to be imposed again. The unknown parameters left for this region are $A_{Bk},\ k = 1, \ldots, 4$.

Next, we consider the region $D_B \leq X \leq D_R$, i.e., the realized state of the Markov chain is boom (in recession, the solution is already known by the second equation of system (A-68)). Plugging in the functional form $d_B(X) = C_1X^{\beta_1^B} + C_2X^{\beta_2^B} + C_3X + C_4 + C_5X^{\gamma_3} + C_6X^{\gamma_4}$ into the second equation of (A-68), we find by comparison of coefficients that

$$\beta_{1,2}^B = \frac{1}{2} - \frac{\tilde{\mu}_B}{\tilde{\sigma}_B^2} \pm \sqrt{\left(\frac{1}{2} - \frac{\tilde{\mu}_B}{\tilde{\sigma}_B^2}\right)^2 + \frac{2\left(r_B^n + \tilde{\lambda}_B\right)}{\tilde{\sigma}_B^2}} \tag{A-76}$$

$$C_3 = \frac{\tilde{\lambda}_B\alpha_R y_R}{r_B^n + \tilde{\lambda}_B - \tilde{\mu}_B} \tag{A-77}$$

$$C_4 = \frac{c}{r_B^n + \tilde{\lambda}_B} \tag{A-78}$$

$$C_5 = \frac{\tilde{\lambda}_B\alpha_R\bar{l}_1\bar{A}_{B3}^{unlev}}{r_B^n - \tilde{\mu}_B\gamma_3 - \frac{1}{2}\tilde{\sigma}_B^2\gamma_3(\gamma_3 - 1) + \tilde{\lambda}_B} \tag{A-79}$$

$$C_6 = \frac{\tilde{\lambda}_B\alpha_R\bar{l}_2\bar{A}_{B4}^{unlev}}{r_B^n - \tilde{\mu}_B\gamma_4 - \frac{1}{2}\tilde{\sigma}_B^2\gamma_4(\gamma_4 - 1) + \tilde{\lambda}_B}. \tag{A-80}$$

We require again that $r_B^n + \tilde{\lambda}_B - \tilde{\mu}_B \neq 0$. Note that the denominators of C_5 and C_6 are different from zero as long as the Markov chain I is recurrent, i.e., if $\tilde{\lambda}_i > 0$, $i = B, R$ (see equation (A-75)). The parameters $\beta_{1,2}^B, C_3$, and C_4 are the same as for a firm with only invested assets, cf. Appendix A.2, equations (A-20). C_5 and C_6 are influenced by the parameters in the solution of the growth option, $\bar{l}_1, \bar{l}_2, \bar{A}_{B3}^{unlev}$, and $\bar{A}_{B4}^{unlev}$ (see Appendix A.3). The two additional terms of the solution for this region, $C_5X^{\gamma_3}$ and $C_6X^{\gamma_4}$, reflect the fact that the firm does not only consist of assets in place, but also of the growth option. As debtholders get also a fraction of the growth option's value at regime-switching induced default, the value of the option directly influences the solution in this region. This influence explains the occurrence of the growth option parameters in C_5 and C_6, as well as the use of the same exponents as in the calculation of the value of the option, γ_3 and γ_4. Note that the approach and the intuition regarding the exponents γ_3 and γ_4 for this region are completely different than for the previously discussed region $X_B \geq X > D_R$, where these exponents occur only due to the valuation of debt itself, independent of the growth option, and must be calculated as a part of the solution. The unknown parameters left for this region are C_1 and C_2.

Finally, consider the region $X_B < X \leq X_R$ for $i = R$. The corresponding differential equation is (see (A-70)):

$$r_R^n d_R(X) = c + \tilde{\mu}_R X d_R'(X) + \frac{1}{2}\tilde{\sigma}_R^2 X^2 d_R''(X) + \tilde{\lambda}_R \hat{d}_B(\bar{s}X - \frac{K}{y_B}). \tag{A-81}$$

In order to solve this inhomogeneous differential equation, we use a standard approach by first finding a fundamental system of solutions of the homogenous differential equation, and then calculating the solution of the inhomogeneous equation as the sum of the solutions of the homogenous equation and a particular solution of the nonhomogeneous equation. A reference for this approach is Polyanin and Zaitsev (2003), pages 21-23.[31]

(A-81) is equivalent to

$$X^2 d_R''(X) + \frac{2\tilde{\mu}_R}{\tilde{\sigma}_R^2} X d_R'(X) - \frac{2\left(r_R^n + \tilde{\lambda}_R\right)}{\tilde{\sigma}_R^2} d_R(X) = -\frac{2c}{\tilde{\sigma}_R^2} - \frac{2\tilde{\lambda}_R}{\tilde{\sigma}_R^2}\hat{d}_B(\bar{s}X - \frac{K}{y_B}). \tag{A-82}$$

Therefore, the according homogenous differential equation is

$$X^2 d_R''(X) + \frac{2\tilde{\mu}_R}{\tilde{\sigma}_R^2} X d_R'(X) - \frac{2\left(r_R^n + \tilde{\lambda}_R\right)}{\tilde{\sigma}_R^2} d_R(X) = 0. \tag{A-83}$$

A fundamental system of solutions is given by $\{z_1, z_2\}$, with

$$\begin{aligned} z_1 &:= X^{\beta_1^R}, \\ z_2 &:= X^{\beta_2^R}, \end{aligned}$$

and

$$\beta_{1,2}^R = \frac{1}{2} - \frac{\tilde{\mu}_R}{\tilde{\sigma}_R^2} \pm \sqrt{\left(\frac{1}{2} - \frac{\tilde{\mu}_R}{\tilde{\sigma}_R^2}\right)^2 + \frac{2\left(r_R^n + \tilde{\lambda}_R\right)}{\tilde{\sigma}_R^2}}. \tag{A-84}$$

These solutions can be calculated by plugging the functional form into the homogenous ODE (A-83), and solving for $\beta_{1,2}^R$.

[31] Technically, the above explained two-step procedure for the solution of the ODE is required due to the assumption that the exercise of the option is financed by selling a part of the assets in place, resulting in the fact that the function $\hat{d}$ in the ODE is not evaluated at X, but at $\bar{s}X - \frac{K}{y_B}$. Under the alternative assumption that the exercise of the option is equity-financed, the function $\hat{d}$ is evaluated at a multiple of X instead. In this case, we can exploit the additive nature of the ODE, and calculate the solution as a weighted sum of solutions, including the value function of debt of a firm with only invested assets. The functional form for the region $X_B < X \leq X_R$ is then comparable to the one for the region $D_B \leq X \leq D_R$.

For notational convenience, we now define $f_2 := X^2$, $f_1 := \frac{2\tilde{\mu}_R}{\tilde{\sigma}_R^2}X$, $f_0 := -\frac{2\left(r_R^n+\tilde{\lambda}_R\right)}{\tilde{\sigma}_R^2}$, and

$$g\left(X\right) := -\frac{2c}{\tilde{\sigma}_R^2} - \frac{2\tilde{\lambda}_R}{\tilde{\sigma}_R^2}\hat{d}_B(\bar{s}X - \frac{K}{y_B}). \tag{A-85}$$

These notations allow to write the ODE (A-82) as:

$$f_2 d_R''(X) + f_1 d_R'(X) + f_0 d_R(X) = g(X). \tag{A-86}$$

The general solution of this inhomogeneous ODE is given by

$$d_R\left(X\right) = B_1 z_1 + B_2 z_2 + z_2 \underbrace{\int z_1 \frac{g}{f_2}\frac{\mathrm{d}X}{W}}_{=:I_1(X)} - z_1 \underbrace{\int z_2 \frac{g}{f_2}\frac{\mathrm{d}X}{W}}_{=:I_2(X)}, \tag{A-87}$$

where $W = z_1 z_2' - z_2 z_1'$ is the Wronskian determinant, and B_1 and B_2 are coefficients (see e.g. Polyanin and Zaitsev (2003), page 22, (7)). The first two terms are a linear combination of the solutions of the homogenous ODE, and the last two terms are a particular solution of the inhomogeneous ODE.

We start by calculating the Wronskian determinant

$$\begin{aligned} W &= z_1 z_2' - z_2 z_1' \\ &= \beta_2^R X^{\beta_1^R} X^{\beta_2^R - 1} - \beta_1^R X^{\beta_1^R - 1} X^{\beta_2^R} \\ &= \left(\beta_2^R - \beta_1^R\right) X^{\beta_1^R + \beta_2^R - 1}. \end{aligned} \tag{A-88}$$

The integral $I_1(X)$ is, hence:

$$\begin{aligned}
I_1(X) &= \int z_1 \frac{g}{f_2}\frac{\mathrm{d}X}{W} \\
&= \int X^{\beta_1^R} X^{-2} \frac{1}{\beta_2^R - \beta_1^R} X^{1-\beta_1^R-\beta_2^R} g(X)\mathrm{d}X \\
&= \frac{1}{\beta_2^R - \beta_1^R} \int x^{-1-\beta_2^R} g(X)\mathrm{d}X \qquad \text{(A-89)} \\
&= \frac{1}{\beta_2^R - \beta_1^R} \int x^{-1-\beta_2^R} \left(-\frac{2c}{\tilde{\sigma}_R^2} - \frac{2\tilde{\lambda}_R}{\tilde{\sigma}_R^2} \hat{d}_B \left(\bar{s}X - \frac{K}{y_B} \right) \right) \mathrm{d}X \\
&= \frac{1}{\beta_2^R - \beta_1^R} \int x^{-1-\beta_2^R} \left(-\frac{2c}{\tilde{\sigma}_R^2} \right. \\
&\quad \left. -\frac{2\tilde{\lambda}_R}{\tilde{\sigma}_R^2} \left\{ \hat{A}_{B1} \left(\bar{s}X - \frac{K}{y_B} \right)^{\gamma_1} + \hat{A}_{B2} \left(\bar{s}X - \frac{K}{y_B} \right)^{\gamma_2} + \frac{c}{r} \right\} \right) \mathrm{d}X \\
&= -\frac{2\tilde{\lambda}_R \hat{A}_{B1}}{\left(\beta_2^R - \beta_1^R\right)\tilde{\sigma}_R^2} \underbrace{\int X^{-1-\beta_2^R} \left(\bar{s}X - \frac{K}{y_B} \right)^{\gamma_1} \mathrm{d}X}_{=:I_{11}(X)} \\
&\quad -\frac{2\tilde{\lambda}_R \hat{A}_{B2}}{\left(\beta_2^R - \beta_1^R\right)\tilde{\sigma}_R^2} \underbrace{\int X^{-1-\beta_2^R} \left(\bar{s}X - \frac{K}{y_B} \right)^{\gamma_2} \mathrm{d}X}_{=:I_{12}(X)} \qquad \text{(A-90)} \\
&\quad + \frac{2c\left(\tilde{\lambda}_R + r_R^n\right)}{\left(\beta_2^R - \beta_1^R\right) r_R^n \beta_2^R \tilde{\sigma}_R^2} X^{-\beta_2^R}.
\end{aligned}$$

We use the definition of the function $g(X)$, see (A-85), and the solution of the debt value of a firm with only invested assets $\hat{d}_R(\cdot)$, see Appendix A.2, (A-17).

The integrals $I_{11}(X)$ and $I_{12}(X)$ can be evaluated immediately with standard computer algebra packages. Alternatively, using the integral representation of Gauss' hypergeometric function ${}_2F_1(\cdot,\cdot,\cdot;\cdot)$, we can write the closed-form solution of the integrals as

$$I_{11}(X) = \frac{1}{\gamma_1 - \beta_2^R} \bar{s}^{\gamma_1} X^{\gamma_1 - \beta_2^R} {}_2F_1\left(-\gamma_1, \beta_2^R - \gamma_1, \beta_2^R - \gamma_1 + 1; -\frac{K}{\bar{s}Xy_B} \right), \qquad \text{(A-91)}$$

$$I_{12}(X) = \frac{1}{\gamma_2 - \beta_2^R} \bar{s}^{\gamma_2} X^{\gamma_2 - \beta_2^R} {}_2F_1\left(-\gamma_2, \beta_2^R - \gamma_2, \beta_2^R - \gamma_2 + 1; -\frac{K}{\bar{s}Xy_B} \right). \qquad \text{(A-92)}$$

Plugging the solutions (A-91) and (A-92) back into the expression for the integral I_1, (A-90) yields

$$\begin{aligned}
I_1(X) &= -\frac{2\tilde{\lambda}_R \hat{A}_{B1}}{\left(\beta_2^R - \beta_1^R\right)\tilde{\sigma}_R^2} \frac{1}{\gamma_1 - \beta_2^R} \bar{s}^{\gamma_1} X^{\gamma_1 - \beta_2^R} {}_2F_1\left(-\gamma_1, \beta_2^R - \gamma_1, \beta_2^R - \gamma_1 + 1; -\frac{K}{\bar{s}Xy_B} \right) \\
&\quad -\frac{2\tilde{\lambda}_R \hat{A}_{B2}}{\left(\beta_2^R - \beta_1^R\right)\tilde{\sigma}_R^2} \frac{1}{\gamma_2 - \beta_2^R} \bar{s}^{\gamma_2} X^{\gamma_2 - \beta_2^R} {}_2F_1\left(-\gamma_2, \beta_2^R - \gamma_2, \beta_2^R - \gamma_2 + 1; -\frac{K}{\bar{s}Xy_B} \right) \qquad \text{(A-93)} \\
&\quad + \frac{2c\left(\tilde{\lambda}_R + r\right)}{\left(\beta_2^R - \beta_1^R\right) r \beta_2^R \tilde{\sigma}_R^2} X^{-\beta_2^R}.
\end{aligned}$$

Similarly, we find for the second integral $I_2(X)$:

$$\begin{aligned} I_2(X) &= -\frac{2\tilde{\lambda}_R \hat{A}_{B1}}{\left(\beta_2^R-\beta_1^R\right)\tilde{\sigma}_R^2}\frac{1}{\gamma_1-\beta_1^R}\bar{s}^{\gamma_1}X^{\gamma_1-\beta_1^R}{}_2F_1\left(-\gamma_1,\beta_1^R-\gamma_1,\beta_2^R-\gamma_1+1;-\frac{K}{\bar{s}Xy_B}\right) \\ &\quad -\frac{2\tilde{\lambda}_R \hat{A}_{B2}}{\left(\beta_2^R-\beta_1^R\right)\tilde{\sigma}_R^2}\frac{1}{\gamma_2-\beta_1^R}\bar{s}^{\gamma_2}X^{\gamma_2-\beta_1^R}{}_2F_1\left(-\gamma_2,\beta_1^R-\gamma_2,\beta_2^R-\gamma_2+1;-\frac{K}{\bar{s}Xy_B}\right) \\ &\quad +\frac{2c\left(\tilde{\lambda}_R+r\right)}{\left(\beta_2^R-\beta_1^R\right)r\beta_1^R\tilde{\sigma}_R^2}X^{-\beta_1^R}. \end{aligned} \tag{A-94}$$

Plugging (A-93) and (A-94) into (A-87), we finally obtain the solution

$$d_R(X) = B_1X^{\beta_1^R} + B_2X^{\beta_2^R} + Z(X), \tag{A-95}$$

with

$$\begin{aligned} Z(X) &= \frac{2}{\beta_1\beta_2\tilde{\sigma}_R^2}\frac{c}{r}\left(\tilde{\lambda}_R+r\right) \\ &\quad + \sum_{i,k=1,2}\frac{2(-1)^{i+1}\bar{s}^{\gamma_k}\hat{A}_{Bk}}{\tilde{\sigma}_R^2\left(\beta_2^R-\beta_1^R\right)\left(\gamma_k-\beta_i^R\right)}X^{\gamma_k}{}_2F_1\left(-\gamma_k,\beta_i^R,\beta_i^R-\gamma_k+1;-\frac{K}{\bar{s}Xy_B}\right), \end{aligned} \tag{A-96}$$

for some parameters B_1 and B_2 determined by the boundary conditions.

In order to treat the boundary conditions, we also need the first derivative of Z:

$$\begin{aligned} Z'(X) &= \frac{\mathrm{d}}{\mathrm{d}X}Z(X) \\ &= \frac{\mathrm{d}}{\mathrm{d}X}\left(X^{\beta_2^R}I_1(X) - X^{\beta_2^R}I_2(X)\right) \\ &= \beta_2^R X^{\beta_2^R}I_1(X) + \frac{1}{\beta_2^R-\beta_1^R}X^{\beta_2^R}X^{-1-\beta_1^R}g(X) \\ &\quad -\beta_1^R X^{\beta_1^R}I_2(X) - \frac{1}{\beta_2^R-\beta_1^R}X^{\beta_1^R}X^{-1-\beta_1^R}g(X) \\ &= \beta_2^R X^{\beta_2^R}I_1(X) - \beta_1^R X^{\beta_1^R}I_2(X) \\ &= \sum_{i,k=1,2}\frac{2(-1)^{i+1}\bar{s}^{\gamma_k}\hat{A}_{Bk}\beta_i^R}{\tilde{\sigma}_R^2\left(\beta_2^R-\beta_1^R\right)\left(\gamma_k-\beta_i^R\right)}X^{\gamma_k}{}_2F_1\left(-\gamma_k,\beta_i^R,\beta_i^R-\gamma_k+1;-\frac{K}{\bar{s}Xy_B}\right). \end{aligned} \tag{A-97}$$

To solve for the unknown parameters $A_{B1}, A_{B2}, A_{B3}, A_{B4}, C_1, C_2, B_1$ and B_2, we plug the functional form (A-73) into the system of boundary conditions (A-72):

$$
\begin{aligned}
\sum_{k=1}^{4} A_{Bk} D_R^{\gamma_k} + A_5 &= C_1 D_R^{\beta_1^B} + C_2 D_R^{\beta_2^B} + C_3 X + C_4 + C_5 X^{\gamma_3} + C_6 X^{\gamma_4} \\
\sum_{k=1}^{4} A_{Bk} \gamma_k D_R^{\gamma_k} &= C_1 \beta_1^B D_R^{\beta_1^B} + C_2 \beta_2^B D_R^{\beta_2^B} + C_3 X + C_5 \gamma_3 X^{\gamma_3} + C_6 \gamma_4 X^{\gamma_4} \\
\alpha_B \left(D_B y_B + G_B^{unlev}(D_B) \right) &= C_1 D_B^{\beta_1^B} + C_2 D_B^{\beta_2^B} + C_3 D_B + C_4 + C_5 D_B^{\gamma_3} + C_6 D_B^{\gamma_4} \\
\sum_{k=1}^{4} l_k A_{Bk} D_R^{\gamma_k} + A_5 &= \alpha_R \left(D_R y_R + G_R^{unlev}(D_R) \right) \\
\sum_{k=1}^{4} l_k A_{Bk} X_B^{\gamma_k} + A_5 &= B_1 X_B^{\beta_1^R} + B_2 X_B^{\beta_2^R} + Z(X_B) \\
\sum_{k=1}^{4} l_k A_{Bk} \gamma_k X_B^{\gamma_k} &= B_1 \beta_1^R X_B^{\beta_1^R} + B_2 \beta_2^R X_B^{\beta_2^R} + X_B Z'(X_B) \\
\sum_{k=1}^{4} A_{Bk} X_B^{\gamma_k} + A_5 &= \hat{d}_B \left(\bar{s} X_B - \frac{K}{y_B} \right) \\
B_1 X_R^{\beta_1^R} + B_2 X_R^{\beta_2^R} + Z(X_R) &= \hat{d}_R \left(\bar{s} X_R - \frac{K}{y_R} \right).
\end{aligned}
\tag{A-98}
$$

Using matrix notation, we can write

$$
M := \begin{bmatrix}
D_R^{\gamma_1} & D_R^{\gamma_2} & D_R^{\gamma_3} & D_R^{\gamma_4} & -D_R^{\beta_1^B} & -D_R^{\beta_2^B} & 0 & 0 \\
\gamma_1 D_R^{\gamma_1} & \gamma_2 D_R^{\gamma_2} & \gamma_3 D_R^{\gamma_3} & \gamma_4 D_R^{\gamma_4} & -\beta_1^B D_R^{\beta_1^B} & -\beta_2^B D_R^{\beta_2^B} & 0 & 0 \\
0 & 0 & 0 & 0 & D_B^{\beta_1^B} & D_B^{\beta_2^B} & 0 & 0 \\
l_1 D_R^{\gamma_1} & l_2 D_R^{\gamma_2} & l_3 D_R^{\gamma_3} & l_4 D_R^{\gamma_4} & 0 & 0 & 0 & 0 \\
l_1 X_B^{\gamma_1} & l_2 X_B^{\gamma_2} & l_3 X_B^{\gamma_3} & l_4 X_B^{\gamma_4} & 0 & 0 & -X_B^{\beta_1^R} & -X_B^{\beta_2^R} \\
l_1 \gamma_1 X_B^{\gamma_1} & l_2 \gamma_2 X_B^{\gamma_2} & l_3 \gamma_3 X_B^{\gamma_3} & l_4 \gamma_4 X_B^{\gamma_4} & 0 & 0 & -\beta_1^R X_B^{\beta_1^R} & -\beta_2^R X_B^{\beta_2^R} \\
X_B^{\gamma_1} & X_B^{\gamma_2} & X_B^{\gamma_3} & X_B^{\gamma_4} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & X_R^{\beta_1^R} & X_R^{\beta_2^R}
\end{bmatrix}
$$

$$
b := \begin{bmatrix}
-A_{B5} + C_3 D_R + C_4 + C_5 D_R^{\gamma_1} + C_6 D_R^{\gamma_2} \\
C_3 D_R + \gamma_1 C_5 D_R^{\gamma_1} + C_6 \gamma_2 D_R^{\gamma_2} \\
-C_3 D_B - C_4 - C_5 D_B^{\gamma_3} - C_6 D_B^{\gamma_4} + \alpha_B \left(D_B y_B + G_B^{unlev}(D_B) \right) \\
-A_{R5} + \alpha_R \left(D_R y_R + G_R^{unlev}(D_R) \right) \\
-A_{R5} + Z\left(X_B\right) \\
X_B Z'\left(X_B\right) \\
-A_{B5} + \hat{d}_B \left(\bar{s} X_B - \frac{K}{y_B} \right) \\
-Z\left(X_R\right) + \hat{d}_R \left(\bar{s} X_R - \frac{K}{y_R} \right)
\end{bmatrix}.
$$

Thus, the solution to the remaining unknowns is given by

$$
\begin{bmatrix} A_{B1} & A_{B2} & A_{B3} & A_{B4} & C_1 & C_2 & B_1 & B_2 \end{bmatrix}^T = M^{-1} b. \tag{A-99}
$$

Case 2: Only 1 regime. Denote the default boundary by D_1, and recall that X_1 is the firm's investment boundary, while $\hat{D}_1$ denotes the default boundary of a firm with only invested assets. Postulating that in the continuation region the required return must be equal to the expected realized return plus the proceeds from debt, we find that the system to solve is:

$$\begin{aligned} d(X) &= \alpha\left(yX + G^{unlev}(X)\right) && X \leq D_1 \\ rd(X) &= c + \tilde{\mu} X d'(X) + \tfrac{\tilde{\sigma}^2}{2} X^2 d''(X) && D_1 < X < X_1 \\ d(X) &= \hat{d}\left(\bar{s}X - \tfrac{K}{y}\right) && X \geq X_1. \end{aligned} \tag{A-100}$$

The first and second equations are analogous to the two regime case. In the third equation, we postulate that above the exercise boundary X the debt value of the firm must be equal to the one of a firm with only invested assets. As in the two regime case, the conversion of the growth option into assets in place is arranged such that the total value of the firm's assets remains unchanged at the exercise of the option. The boundary conditions are the value-matching conditions at default and exercise:

$$\lim_{X \searrow D_1} d(X) = \alpha\left(yD_1 + G^{unlev}(D_1)\right) \tag{A-101}$$

$$\lim_{X \nearrow X_1} d(X) = \hat{d}\left(\bar{s}X_1 - \frac{K}{y}\right). \tag{A-102}$$

Note that for $X > X_1$ the value of debt is equal to the one of a firm with only invested assets. As the latter is calculated using a no-bubbles condition, we do not have to postulate this condition for the function $d(X)$ again.

The functional form of the solution is

$$d(X) = \begin{cases} \alpha\left(yX + G^{unlev}(X)\right) & X \leq D_1 \\ B_3 X^{\beta_1} + B_4 X^{\beta_2} + A_5 & D_1 < X < X_1 \\ \hat{d}\left(\bar{s}X - \frac{K}{y}\right) & X \geq X_1, \end{cases} \tag{A-103}$$

where B_3, B_4, A_5, β_1, and β_2 are real-valued parameters to be determined (or to be confirmed). The only region left to solve for is $D_1 < X < X_1$. By plugging the functional form (A-103) into the differential equation (A-100) and comparing coefficients, we find that

$$\beta_{1,2} = \frac{1}{2} - \frac{\tilde{\mu}}{\tilde{\sigma}^2} - \sqrt{\left(\frac{1}{2} - \frac{\tilde{\mu}}{\tilde{\sigma}^2}\right)^2 + \frac{2r}{\tilde{\sigma}^2}} \tag{A-104}$$

$$A_5 = \frac{c}{r}. \tag{A-105}$$

Finally, B_3 and B_4 are determined by the two-dimensional linear system defined by the above boundary conditions:

$$B_3 D_1^{\beta_1} + B_4 D_1^{\beta_2} + \frac{c}{r} = \alpha\left(yD_1 + G^{unlev}(D_1)\right) \tag{A-106}$$

$$B_3 X_1^{\beta_1} + B_4 X_1^{\beta_2} + \frac{c}{r} = \hat{d}\left(\bar{s}X_1 - \frac{K}{y}\right). \tag{A-107}$$

Using matrix notation, and

$$\begin{aligned}
M_1 &:= \begin{bmatrix} D_1^{\beta_1} & D_1^{\beta_2} \\ X_1^{\beta_1} & X_1^{\beta_2} \end{bmatrix}, \\
b_1 &:= \begin{bmatrix} \alpha\left(yD_1 + G^{unlev}\left(D_1\right)\right) - \frac{c}{r} \\ \hat{d}\left(\bar{s}X_1 - \frac{K}{y}\right) - \frac{c}{r} \end{bmatrix},
\end{aligned}$$

we find that

$$\begin{bmatrix} B_3 & B_4 \end{bmatrix}^T = M_1^{-1} b_1 \tag{A-108}$$

$$= \frac{1}{D_1^{\beta_1} X_1^{\beta_2} - D_1^{\beta_2} X_1^{\beta_1}} \begin{bmatrix} X_1^{\beta_2} & -D_1^{\beta_2} \\ -X_1^{\beta_1} & D_1^{\beta_1} \end{bmatrix} \begin{bmatrix} \alpha\left(yD_1 + G^{unlev}\left(D_1\right)\right) - \frac{c}{r} \\ \hat{d}\left(\bar{s}X_1 - \frac{K}{y}\right) - \frac{c}{r} \end{bmatrix}, \tag{A-109}$$

which completes the calculation of the solution.

A.4.2. The valuation of tax benefits

For Case 1, see the main text. Analogously, the value of tax benefits $t(X)$ in Case 2 can be calculated as the value of debt with a recovery rate of zero and a coupon equal to $c\tau$.

A.4.3. The valuation of default costs

Case 1 can be found in the main text. Analogously, the value of default costs $b(X)$ in Case 2 corresponds to the value of debt with a recovery rate of $1-\alpha$ and a coupon of zero.

A.4.4. Firm value

The main text states the firm value in Case 1. Analogously, for Case 2, the firm value $f(X)$ is

$$f(X) = Xy + G(X) + t(X) - b(X). \tag{A-110}$$

A.4.5. The valuation of equity

Case 1 is given in the main text. For Case 2, the value of equity $e(X)$ is given by

$$e(X) = f(X) - d(X) = Xy + G(X) + t(X) - b(X) - d(X). \tag{A-111}$$

A.4.6. Default policy

Following similar arguments as in Case 1 (main text), the optimal default and investment policies in Case 2, D^* and X^*, are determined by the conditions

$$\begin{cases} e'(D^*) &= 0 \\ e'(X^*) &= \hat{e}'\left(sX^* - \frac{K}{y}\right) \end{cases} \tag{A-112}$$

A.4.7. Capital structure

Analogously to Case 1 in the main text, denote, for Case 2, by $f^*(X)$ the firm value given ex-post optimal default and expansion thresholds as determined by the system (A-112). The optimal coupon of this firm then solves

$$c^* := \text{argmax}_c f^*(X). \tag{A-113}$$

A.5. The Value of Finite Maturity Debt

We only present the case of the presence of two regimes. The setup and solution for the case of one regime can be derived analogously.

A.5.1. Firms with invested assets only

Hackbarth, Miao, and Morellec (2006) present the solution of a similar model for firms with only invested assets. We consider the standard case that the default boundary in boom is lower than the one in recession, i.e., $\hat{D}_B < \hat{D}_R$. Given debt characteristics (c, m, p), the ODE for the value of debt writes:

For $0 \leq X \leq \hat{D}_B$:

$$\begin{cases} \hat{d}_B(X) &= \alpha_B X y_B \\ \hat{d}_R(X) &= \alpha_R X y_R. \end{cases} \tag{A-114}$$

For $\hat{D}_B < X \leq \hat{D}_R$:

$$\begin{cases} (r_B^n + m)\,\hat{d}_B(X) &= c + mp + \tilde{\mu}_B X \hat{d}'_B(X) + \frac{1}{2}\tilde{\sigma}_B^2 X^2 \hat{d}''_B(X) + \tilde{\lambda}_B \left(\alpha_R X y_R - \hat{d}_B(X)\right) \\ \hat{d}_R(X) &= \alpha_R X y_R. \end{cases} \tag{A-115}$$

For $X > \hat{D}_R$:

$$\begin{cases} (r_B^n + m)\,\hat{d}_B(X) &= c + mp + \tilde{\mu}_B X \hat{d}'_B(X) + \frac{1}{2}\tilde{\sigma}_B^2 X^2 \hat{d}''_B(X) + \tilde{\lambda}_B \left(\hat{d}_R(X) - \hat{d}_B(X)\right) \\ (r_R^n + m)\,\hat{d}_R(X) &= c + mp + \tilde{\mu}_R X \hat{d}'_R(X) + \frac{1}{2}\tilde{\sigma}_R^2 X^2 \hat{d}''_R(X) + \tilde{\lambda}_R \left(\hat{d}_B(X) - \hat{d}_R(X)\right). \end{cases} \tag{A-116}$$

The boundary conditions are the same as in the infinite maturity case, see (A-21)-(A-25). The solution can be found analogously.

Note that for finite maturities, the value of the risk free bond with a given coupon can be calculated as

$$RF = \frac{(c+mp)\left(r_j^n + m + \tilde{\lambda}_i + \tilde{\lambda}_j\right)}{\left(r_i^n + m\right)\left(r_j^n + m\right) + \left(r_j^n + m\right)\tilde{\lambda}_i + \left(r_i^n + m\right)\tilde{\lambda}_j}. \tag{A-117}$$

A.5.2. Firms with invested assets and expansion options

In our framework, debt characteristics (c, m, p) are chosen at initiation and are then constant over time. This setting allows us to calculate the solution for firms with both invested assets and growth options in closed-form, even for finite maturity debt. The standard case with $D_B < D_R$, $\hat{D}_B < \hat{D}_R$, and $X_R > X_B$ is presented. For given debt characteristics (c, m, p), the value of finite maturity corporate debt satisfies the following ODE:

For $0 \leq X \leq D_B$:

$$\begin{cases} d_B(X) &= \alpha_B\left(Xy_B + G_B^{unlev}(X)\right) \\ d_R(X) &= \alpha_R\left(Xy_R + G_R^{unlev}(X)\right) \end{cases} \tag{A-118}$$

For $D_B < X \leq D_R$:

$$\begin{cases} \left(r_B^n + m\right) d_B(X) &= c + mp + \tilde{\mu}_B X d_B'(X) + \frac{1}{2}\tilde{\sigma}_B^2 X^2 d_B''(X) \\ & \quad + \tilde{\lambda}_B\left(\alpha_R\left(Xy_R + G_R^{unlev}(X)\right) - d_B(X)\right) \\ d_R(X) &= \alpha_R\left(Xy_R + G_R^{unlev}(X)\right) \end{cases} \tag{A-119}$$

For $D_R < X < X_B$:

$$\begin{cases} \left(r_B^n + m\right) d_B(X) &= c + mp + \tilde{\mu}_B X d_B'(X) + \frac{1}{2}\tilde{\sigma}_B^2 X^2 d_B''(X) + \tilde{\lambda}_B\left(d_R(X) - d_B(X)\right) \\ \left(r_R^n + m\right) d_R(X) &= c + mp + \tilde{\mu}_R X d_R'(X) + \frac{1}{2}\tilde{\sigma}_R^2 X^2 d_R''(X) + \tilde{\lambda}_R\left(d_B(X) - d_R(X)\right) \end{cases} \tag{A-120}$$

For $X_B \leq X < X_R$:

$$\begin{cases} d_B(X) &= \hat{d}_B\left(\bar{s}X - \frac{K}{y_B}\right) \\ \left(r_R^n + m\right) d_R(X) &= c + mp + \tilde{\mu}_R X d_R'(X) + \frac{1}{2}\tilde{\sigma}_R^2 X^2 d_R''(X) \\ & \quad + \tilde{\lambda}_R\left(\hat{d}_B\left(\bar{s}X - \frac{K}{y_B}\right) - d_R(X)\right) \end{cases} \tag{A-121}$$

For $X \geq X_R$:

$$\begin{cases} d_B(X) &= \hat{d}_B\left(\bar{s}X - \frac{K}{y_B}\right) \\ d_R(X) &= \hat{d}_R\left(\bar{s}X - \frac{K}{y_R}\right). \end{cases} \tag{A-122}$$

Here, $\hat{d}_i(\cdot)$ denotes the value of debt with the same principal, coupon, and debt maturity of a firm with only invested assets, see Appendix A.5.1. The boundary conditions are the same as in the case of infinite maturity debt, see (A-72)-(A-99).

The solution can be derived analogously to the case of infinite maturity debt. Technically, for given debt characteristics (c, m, p), the value of finite maturity debt corresponds to the value of infinite maturity debt with a coupon $c + mp$ and nominal interest rates of $r_i^n + m$.

The assumption that debt is issued at par requires that

$$p = d_i(X), \tag{A-123}$$

where i denotes the regime at initiation. This equation is solved numerically.

A.6. Details on the simulations

A.6.1. Calibration of the idiosyncratic volatility

We calibrate the firm-level idiosyncratic volatility of our BBB sample to the empirically observed total asset volatility of 0.25. The procedure starts by simulating a model-implied economy for 10 years (pre-matching simulation). Next, we match the model-implied distribution after 10 years with the empirical cross-section of BBB-rated firms, and finally simulate the obtained matched sample for another 10 years (post-matching simulation). The average asset volatility of the post-matching simulation is then calculated. The details of this procedure are as follows.

We consider infinite maturity debt in the pre-matching simulation for all debt maturities in the post-matching simulation. We do so to abstract away from the impact of different initial principals on the results, allowing us to analyze the pure effect of debt maturities on credit spreads in the post-matching simulation. Additionally, starting with infinite maturity debt yields initial leverage ratios (principals) close to the ones empirically reported.[32] The model-implied economy is generated as follows. Starting with a value firm ($s = 0$), we generate a range of firms by increasing the option scale parameter s by steps of 0.05, up to the largest possible value of s such that the option is not exercised immediately. At initiation, the capital structure is chosen optimally for all firms. For each option scale parameter s, 50 firms are considered, resulting in an initial sample of more than 3,000 firms. During the 10-year pre-matching simulation, firms default and expand optimally. Defaulted firms are not replaced, and exercised firms continue as firms with only invested assets. At the end of the pre-matching simulation, we calculate the model-implied leverage and asset composition ratio for each firm, using the assumed debt maturity and the corresponding optimal boundaries. We obtain a model-implied distribution of firms covering a broad range of both asset composition ratios and leverage ratios.

In the second step, we match our average historical distribution of BBB-rated firms with its model-implied counterpart. For each observation in the average historical distribution, we select the firm in our model-implied economy at the final period of the pre-matching simulation which exhibits the minimum distance regarding the percentage deviation from the target market leverage and asset composition ratio. That is, the empirical observation of a firm with leverage lev_{emp} and asset composition ratio acr_{emp} is matched with the model-implied firm with leverage lev_{mi} and asset composition ratio acr_{mi} if - given the set of all model-implied firms - it minimizes the Euclidean distance

$$\sqrt{\left(\frac{lev_{emp} - lev_{mi}}{lev_{emp}}\right)^2 + \left(\frac{acr_{emp} - acr_{mi}}{acr_{emp}}\right)^2}. \tag{A-124}$$

[32] A robustness analysis confirms that starting with finite maturity debt in the pre-matching simulation yields slightly lower credit spreads in the post-matching simulation, as the initial principals are smaller.

The final step conducts a post-matching simulation with the obtained sample of model implied BBB-firms over 10 years. For each simulation, we obtain the realized asset volatility for each firm, and calculate the resulting average asset volatility over firms. When measuring and averaging asset volatilities, we incorporate the entire initially matched BBB-sample, including the evolution of the assets of firms which default during the 10-year post-matching simulation. This approach avoids a weighting bias when averaging over simulations towards firms with lower leverage and asset volatility which have a smaller tendency to default during the post-matching simulation.

The pre-matching simulation and the subsequent matching is conducted 20 times. The initial regime is chosen according to the stationary distribution of the states, i.e., the pre-matching simulation starts in boom $100\frac{\lambda_R}{\lambda_B+\lambda_R}\%$ of the total number of simulations. This approach also guarantees convergence to the steady-state distribution of regimes at the time of matching. For each matched sample of firms, the post-matching simulation is run 50 times. These numbers result in a total of 1,000 simulations. The procedure is conducted for different post-matching debt maturities.

A.6.2. Simulation of the true cross-section

To ensure consistency, the simulation of the true cross-section is done analogously to the one performed to calibrate the idiosyncratic volatility: We first simulate a model-implied distribution of firms for 10 years (pre-matching simulation), and then match the model-implied distribution with the average empirical cross-section (for details, see above). The final step consists of simulating the matched sample for 20 years (post-matching simulation). We assume that firms default and exercise optimally. Defaulted firms are immediately deleted, whereas exercised firms are maintained in the sample, and continue as firms with only invested assets. Credit spreads and leverage ratios are measured during 5 years after the matching: For each firm in the sample, we calculate the actual credit spread and leverage every month, and then report the average over all firms and all simulations. Default rates are observed for 5, 10, and 20 years. In order to incorporate the impact of the realized regimes at initiation and at the time of matching, we present quantiles of post-matching average rates. As in the calibration of the volatility, the initial state is chosen according to the stationary distribution. The pre-matching simulation is run 20 times, and the post-matching simulation is conducted 50 times, resulting in a total of 1,000 simulations.

A.7. Robustness tests

A.7.1. Financing the exercise of the growth option by issuing additional equity

We consider the case that the exercise price λ of the growth option is financed by issuing additional equity. The corresponding system of ODEs for corporate debt is:

For $0 \leq X \leq D_B$:

$$\begin{cases} d_B(X) &= \alpha_B \left(X y_B + G_B^{unlev}(X) \right) \\ d_R(X) &= \alpha_R \left(X y_R + G_R^{unlev}(X) \right) \end{cases} \tag{A-125}$$

For $D_B < X \leq D_R$:

$$\begin{cases} r_B^n d_B(X) & = & c + \tilde{\mu}_B X d_B'(X) + \frac{1}{2}\tilde{\sigma}_B^2 X^2 d_B''(X) + \tilde{\lambda}_B \left(\alpha_R \left(X y_R + G_R^{unlev}(X)\right) - d_B(X)\right) \\ d_R(X) & = & \alpha_R \left(X y_R + G_R^{unlev}(X)\right) \end{cases} \quad \text{(A-126)}$$

For $D_R < X < X_B$:

$$\begin{cases} r_B^n d_B(X) & = & c + \tilde{\mu}_B X d_B'(X) + \frac{1}{2}\tilde{\sigma}_B^2 X^2 d_B''(X) + \tilde{\lambda}_B \left(d_R(X) - d_B(X)\right) \\ r_R^n d_R(X) & = & c + \tilde{\mu}_R X d_R'(X) + \frac{1}{2}\tilde{\sigma}_R^2 X^2 d_R''(X) + \tilde{\lambda}_R \left(d_B(X) - d_R(X)\right) \end{cases} \quad \text{(A-127)}$$

For $X_B \leq X < X_R$:

$$\begin{cases} d_B(X) & = & \hat{d}_B(\bar{s}X) \\ r_R^n d_R(X) & = & c + \tilde{\mu}_R X d_R'(X) + \frac{1}{2}\tilde{\sigma}_R^2 X^2 d_R''(X) + \tilde{\lambda}_R \left(\hat{d}_B(\bar{s}X) - d_R(X)\right) \end{cases} \quad \text{(A-128)}$$

For $X \geq X_R$:

$$\begin{cases} d_B(X) & = & \hat{d}_B(\bar{s}X) \\ d_R(X) & = & \hat{d}_R(\bar{s}X)\,. \end{cases} \quad \text{(A-129)}$$

The boundary conditions now read:

$$\begin{aligned} \lim_{X \searrow D_R} d_B(X) &= \lim_{X \nearrow D_R} d_B(X) \\ \lim_{X \searrow D_R} d_B'(X) &= \lim_{X \nearrow D_R} d_B'(X) \\ \lim_{X \searrow D_B} d_B(X) &= \alpha_B \left(D_B y_B + G_B^{unlev}(D_B)\right) \\ \lim_{X \searrow D_R} d_R(X) &= \alpha_R \left(D_R y_R + G_R^{unlev}(D_R)\right) \\ \lim_{X \searrow X_B} d_R(X) &= \lim_{X \nearrow X_B} d_R(X) \\ \lim_{X \searrow X_B} d_R'(X) &= \lim_{X \nearrow X_B} d_R'(X) \\ \lim_{X \nearrow X_B} d_B(X) &= \hat{d}_B(\bar{s}X_B) \\ \lim_{X \nearrow X_R} d_R(X_R) &= \hat{d}_R(\bar{s}X_R)\,. \end{aligned} \quad \text{(A-130)}$$

The solution to this system follows by standard arguments from the theory of differential equations. Technically, this modification constitutes a simplification of the presented main case: The functional form is straightforward and does not need to be determined as the solution of an inhomogeneous ODE using the fundamental system of solutions of the homogenous ODE (cf. footnote 31). Therefore, we do not present the solution here.

swiss:finance:institute

c/o University of Geneva
40 bd du Pont d'Arve
1211 Geneva 4
Switzerland

T +41 22 379 84 71
F +41 22 379 82 77
RPS@sfi.ch
www.SwissFinanceInstitute.ch