Fair Distribution of Profit in Supply Chains

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Abstract

Most scientific publications on the subject of supply chain management only analyze which structures, processes and actions can contribute to value creation, often disregarding the distribution of profits collectively achieved in a network of autonomous actors. Distributive justice, or fair distribution of collectively created profits, is one of the most important means of securing the stability of networks. This paper therefore presents a proposal for an operationalization of the fairness term from an economic perspective. This proposal is specific to the distribution of profits in networks of autonomously acting companies. It is based on a cooperative game theory approach, the χ-value. A practical example is used to illustrate calculation of the χ-value.

Keywords: χ-value, cooperative game theory, distribution of profits, fairness; supply chain management

1. Scientific Problem

The basic idea behind supply chain management is that with the aid of goal-oriented management of cooperation, multiple companies can achieve special *profits* which cannot be realized without cooperation. These companies are generally assumed to be legally autonomous entities, and here it is also implied that cooperation is based on voluntary collaborations that are economically beneficial to each company involved. Such a form of inter-organizational cooperation can generally be described as a *network of autonomous actors*, for example a supply chain.

Most scientific publications on the subject of supply chain management *only* analyze which structures, processes and actions can contribute to the cooperative *creation* of profits. The question of how the *distribution* of collectively achieved profits can influence the stability of such a network is often *disregarded*.

This neglect of distribution aspects represents a significant *research gap*. The formation and the drifting apart of networks generally depend on the actors regarding the distribution of hoped for or already realized cooperation profits as fair. Distributive justice, or fair distribution of collectively created profits, is one of the most important approaches to secure the stability of networks in political, socio-scientific and behavior-economic publications on the subject of network theory.

This paper examines the *scientific problem* of how the cooperation gain can be distributed to the actors as network partners in such a way that all actors regard the *distribution outcome* as *fair*. To solve this problem, *scientific methods* from the area of *game theory* will be applied. Aspects informing the choice of methods are that the networks or supply chains considered here consist of legally independent companies (autonomous actors), who each pursue their own interests and do not have to comply with the instructions of their cooperation partners.

The *relevant literature* attempts to solve the problem of how profits that were collectively achieved in a network of autonomous actors can be distributed among the network partners in a fair way. Examples include the analysis of Fromen, 2004, Gjerdrum et al., 2001, Inderfurth and Minner, 2001, Sucky, 2004a, Sucky, 2004b, Sucky, 2005, Thun, 2005, Voß and Schneidereit, 2002. They all cover the scientific and practical problem of *fairness* of cooperation gain distributions only superficially. In most cases, fairness and acceptability of a presented solution concept from cooperative game theory is implied, but not followed through in detail. Only Fromen, 2004 discusses a wide selection of different solution concepts of cooperative game theory. He examines them mostly from a mathematical and analytical perspective, but not from the pragmatic viewpoint of their acceptability as fair solution concepts. For a detailed discussion of the current situation outlined above, see Zelewski (Zelewski, 2009, pp. 30–34).

In this article an *innovative approach* to fair distribution outcomes is presented. This approach rejects the idea of taking a solution concept from cooperative game theory as a “given” and applying it to a profit distribution problem on the naive assumption that the resulting distribution outcome will be accepted as fair. By making assumptions regarding the rationality of the actors it instead limits gradually the space of generally possible outcomes. If these assumptions are accepted as "reasonable", the result is a specific solution concept from cooperative game theory rarely found in the economic literature, the so-called χ-value. The *fairness* of the χ-value and the associated distribution outcomes is *justified* by the acceptability of the gradually established assumptions regarding the "reasonable" limitation of the valid solution space. It will be shown that these assumptions cannot be equated with the formalistic axioms of conventional game theory.

2. Requirements for a game theoretical solution concept

The following four requirements are considered to be important for game theory modeling of the problem of determining fair distributions of profits:

1. It must be possible to explicate the different scopes for alternative distribution outcomes that emerge from different assumptions regarding the rationality of the actors.
2. Distribution outcomes determined by the solution concepts must be justifiable in order for the proposed solutions to be accepted as fair distribution outcomes.
3. The solution of the distribution problem must be communicated easily in the supply chain.
4. There must be only one unique solution to the distribution problem.

The starting point for the cooperative distribution game is the *generic distribution problem* of distributing a profit or, synonymously, a cooperation gain G with $G \in \mathbb{R}_{>0}$ (where $\mathbb{R}_{>0}$ is the set of all positive real numbers) among the N autonomous actors A_n of a supply chain (with n = 1,…,N, $N \in \mathbb{N}$ and $N \geq 2$, where $\mathbb{N}$ is the set of all natural numbers). In cooperative game theory, a two-step standard approach to solving this generic distribution problem exists.

The first step is to develop a characteristic function c. This function refers to all possible coalitions which could be formed by the actors in the relevant supply chain. Moreover, "degenerate" coalitions formed by one actor are feasible. Therefore, a coalition C_m is a non-empty subset of the set A of all actors in the supply chain: $\varnothing \subset C_m \subseteq A$ with $A = \{A_1,...,A_N\}$. For each characteristic function c, it is assumed with $\wp$ as power set operator that: c: $\wp(A) \rightarrow \mathbb{R}_{\geq 0}$ with $C_m \rightarrow c(C_m)$ for each coalition C_m

and $\emptyset \rightarrow c(\emptyset) = 0$. Such a characteristic function assigns the amount $c(C_m)$ the respective coalition C_m can claim with good reason. In the case of the grand coalition $C_0 = A$, this is the overall cooperation gain G: $c(C_0) = G$. For all other coalitions C_m with $\emptyset \subset C_m \subset A$, these are the amounts $c(C_m)$ these coalitions C_m could realize on their own outside the grand coalition C_0 and therefore in competition with the rest of the grand coalition, i.e. the residual coalition RC_m where $RC_m = C_0 \setminus C_m$.

In the second step, the shape of a distribution function v where $v: A \rightarrow \mathbb{R}_{\geq 0}$ and $A_n \rightarrow v(A_n) = v_n$ for each actor A_n is determined by calculating the distribution function values v_n. Only two information sources are considered to calculate these values. These are the amounts each feasible coalition C_m can claim due to the characteristic function c from the first step. At the same time the applied game theory solution concept specifies how the distribution function values v_n are calculated based on the values $c(C_m)$ of the characteristic function c for all feasible coalitions C_m where $m = 0,1,...,2^N$-2. When all distribution function values v_n are determined, the result is a N-tuple $v = (v_1,...,v_N)$ as a solution v for the respective regarded instance of the generic distribution problem. Every solution v assigns a share v_n of the cooperation gain G to each actor A_n of the supply chain. This N-tuple v is formally equivalent to a solution point L in the N-dimensional non-negative real number space $\mathbb{R}_{\geq 0}{}^N$. The solution point L is represented as a column vector $\vec{v}$, whose transposed representation denoted by a superscript letter (^{T}) is: $\vec{v} = (v_1,...,v_N)^T$.

From a management point of view, this standard approach of cooperative game theory is *unsatisfactory*. Its main weakness lies in the characteristic function c, which is assumed to be known in conventional game theory analyses. This information premise is rather unrealistic since in actual practice it is often not known for each feasible coalition C_m which value $c(C_m)$ is reasonably appropriate for the respective coalition. A practicable game theory solution concept should therefore make it possible to calculate the values v_n without full knowledge of the characteristic function c. Such a solution concept should refer to as few coalitions as possible to calculate the values v_n for all actors A_n. Minimal knowledge is thus added as a fifth requirement to be satisfied by any solution concept for the fair distribution of profits achieved in a supply chain.

3. Fair distribution of profits in supply chains based on the χ-value

The χ-value hearkens back to contributions by Bergantiños and Massó (Bergantiños and Massó 1994, Bergantiños and Massó 1996, Bergantiños et al. 2000, Bergantiños and Massó 2002). Up to now, it has only been picked up on rarely (e.g. Sánchez-Soriano 2000) and, in the area of economic research at least, is still widely unknown. The χ-value is a remarkable game theory solution concept for the generic distribution problem, as the following paragraphs will show.

The basic idea of the reconstruction of the χ-value solution concept is to restrict the solution space ${\mathbb{R}_{\geq 0}}^N$ for the generic distribution problem by successively adding five assumptions which stem from the real problem of distributing profits achieved cooperatively in a supply chain among the cooperating actors. The following arguments yield to the χ-value as a “reasonable” solution to the generic distribution problem that is in principle acceptable as a fair distribution outcome.

The first assumption is the *condition of individual rationality*. This condition assumes that every actor in a supply chain acts rationally in the conventional sense of perfect rationality. This means that each actor maximizes his or her individual utility. The condition of individual rationality places a restriction on the solution space ${\mathbb{R}_{\geq 0}}^N$, since it would not be rational for an actor A_n to participate in the supply chain within the grand coalition C_0 if this coalition yields a smaller utility for this actor compared to if he or she left the coalition and realized the amount $c(\{A_n\})$ outside the supply chain. Thus the condition of individual rationality can be formulated with the characteristic function c and the feasible solution point L within the solution space as follows:

$$\forall L \in \mathbb{R}^N_{\geq 0}: \quad L = (v_1, \dots v_N)^T \geq \left(c\left(\{A_1\}\right), \dots, c\left(\{A_N\}\right)\right)^T \tag{1}$$

The second assumption is the *efficiency condition*. This condition requires the profit or cooperation gain G to be distributed exactly (“efficiently”) among all actors A_n of the grand coalition $C_0 = \{A_1, \dots, A_N\}$. While it would be irrational to distribute less than the profit G, because this would necessarily entail a loss of Pareto optimality, it is also impossible to distribute more than the profit G. Thus the following equation will hold true for every feasible solution L and the value $c(C_0)$ of the characteristic function c:

$$\forall L \in \mathbb{R}^N_{\geq 0}: \quad L = (v_1, ... v_N)^T \rightarrow \sum_{n=1}^{N} v_n = c(C_0) = G \tag{2}$$

A further restriction of the solution space $\mathbb{R}_{\geq 0}{}^N$ is implied by the efficiency condition.

Hence all the solutions of the distribution problem that fulfill the assumption of efficiency are solution points *L* on a hyperplane *H* in the *N*-dimensional solution space $\mathbb{R}_{\geq 0}{}^N$. This hyperplane *H* is the set of all solutions $v = (v_1,...,v_N)$ of the distribution problem that fulfill the equation on the right hand side of the sub-junction of formula (2).

The third assumption is the rationality *condition for maximum allocable shares of the profit*. This condition has the character of a condition of collective rationality, since it mirrors the rational consideration of all *N*-1 actors of the so-called marginal coalition MC_n where $MC_n = C_0 \backslash \{A_n\} = \{A_1,...,A_{n-1},A_{n+1},...,A_N\}$ to grant actor A_n at most the share $v_{n.max}$ of the profit *G*, so that the profit *G* would decrease if actor A_n left the grand coalition $C_0 = \{A_1,...,A_N\}$. This rationality condition requires the following where $c(C_0) = G$ from formula (2):

$$\forall n = 1,...,N \ \forall v_n \in \mathbb{R}_{\geq 0}: \\ v_n \leq v_{n.max} \wedge v_{n.max} = c(C_0) - c(MC_n) = G - c(MC_n) \tag{3}$$

This assumption can be generalized in such a way that the profit $c(C_m)$ of *each* coalition C_m *including* actor A_n would decrease if actor A_n left this coalition C_m. It follows that the maximum allocable share $v_{n.max}$ of the profit *G* for one actor A_n is measured by the maximum amount $c(C_m) - c(C_m \backslash \{A_n\})$ that the profit $c(C_m)$ of *each* coalition C_m *including* actor A_n would decrease if actor A_n left this coalition C_m. For the reasons mentioned above, the third assumption will be replaced for the χ-value by the following generalized rationality condition for maximum allocable shares $v^{\chi}_{n.max}$:

$$\forall n = 1,...,N \ \forall v_n \in \mathbb{R}_{\geq 0}: \quad v_n \leq v^{\chi}_{n.max} \wedge ... \\ v^{\chi}_{n.max} = max\left\{ c(C_m) - c(C_m \backslash \{A_n\}) \,\middle|\, \varnothing \subset C_m \subseteq A \wedge \{A_n\} \subset C_m \right\} \tag{4}$$

In the solution space, the point at which the maximum allocable share $v^{\chi}_{n.max}$ of the profit *G* is assigned to each actor A_n is called the upper bound *UB* or *ideal point* for the distribution of the profit *G*.

The fourth assumption is a *rationality condition for minimum allocable shares of the profit*. This condition also has the character of a collective rationality condition, since it reflects the rational consideration of all N-1 actors of the marginal coalition MC_n where $MC_n = C_0 \setminus \{A_n\}$ to grant actor A_n at least the share $v_{n.min}$ of the profit G with which he or she could credibly threaten to found at least one so-called outsider coalition $AC_{n.q.}$ An outsider coalition is a coalition $AC_{n.q}$ of former actors of the grand coalition, which leaves the grand coalition C_0 at least hypothetically and has at least the actor A_n as "leader". Since the same actor A_n can lead several outsider coalitions, the second index q is used to differentiate all outsider coalitions led by the same actor A_n.

For the χ-value, it is important which outsider coalitions $AC_{n.q}$ enable an actor A_n to threaten in a believable manner. In this paper, it is assumed that the characteristic function is partially known due to the amounts $c(AC_{n.q})$ for each outsider coalition led by an actor A_n. The actor A_n offers all other actors of the outsider coalition $AC_{n.q}$ an optimal incentive to defect. This incentive consists of so-called side payments and ensures that the utility of each other actor from the outsider coalition $AC_{n.q}$ is the same as his or her maximum utility as part of the grand coalition C_0. In this case, the actors in an outsider coalition have no incentive to remain in the grand coalition C_0. The operationalization of the side payments takes place in the following way, with the amount $c(\{A_n\}|AC_{n.q})$ realizable by actor A_n in the outsider coalition $AC_{n.q}$ and with the index set $IN_{n.q}$ of indices of all actors belonging to this outsider coalition:

$$\forall \varnothing \subset AC_{n.q} \subset A\text{: } \{A_n\} \subset AC_{n.q} \rightarrow \text{ ...} \\ c\left(\{A_n\}\middle|AC_{n.q}\right) = c\left(AC_{n.q}\right) - \sum\nolimits_{m \in \left(IN_{n.q} \setminus \{n\}\right)} v_{n.max}^{\chi} \tag{5}$$

The amounts $c(\{A_n\}|AC_{n.q})$ utilized by actor A_n in threatening to found an outsider coalition may be negative. There are two reasons for this. Firstly, the sum $\sum_{m \in \left(IN_{n.q} \setminus \{n\}\right)} v_{n.max}^{\chi}$ of the side payments can be greater than the amount $c(AC_{n.q})$ realized by the outsider coalition $AC_{n.q}$. In this case, the leading actor A_n must withdraw the partial amount $\sum_{m \in \left(IN_{n.q} \setminus \{n\}\right)} v_{n.max}^{\chi} - c\left(AC_{n.q}\right)$ from savings or even incur debt. Secondly, if actor A_n is the sole actor in the outsider coalition $AC_{n.q}$ and thus the above mentioned side payments are not required, the amount $c(\{A_n\})$ may be negative as well. Actor A_n, for example, may not be competitive in the market without collaborating in the cooperation, for example, in a supply chain. In both cases where $c(\{A_n\}|AC_{n.q}) < 0$, a threat would not be believable. Thus both cases are excluded from the rationality condition for minimum allocable shares of the profit. The complete rationality condition for minimum shares $v^{\chi}_{n.min}$ of the profit *G* to be allocated is as follows:

$$\forall n = 1,...,N \; \forall v_n \in \mathbb{R}_{\geq 0} : v_n \geq v^{\chi}_{n.min} \;\; \wedge \;\; v^{\chi}_{n.min} = max\{c_{n.1}; c_{n.2}; 0\} \qquad (6)$$

where:

$$c_{n.1} = c(\{A_n\} \mid AC_{n.q}) = c(\{A_n\}) \quad for \quad AC_{n.q} = \{A_n\}$$

$$c_{n.2} = max\left\{ \begin{matrix} c(\{A_n\} \mid AC_{n.q}) = c(AC_{n.q}) - \sum_{m \in (IN_{n.q} \setminus \{n\})} v^{\chi}_{m.max} \;\middle|\; ... \\ \emptyset \subset AC_{n.q} \subseteq A \;\; \wedge \;\; \{A_n\} \subset AC_{n.q} \end{matrix} \right\}$$

As a side effect of this formulation of the rationality condition for minimum allocable shares of the profit, the condition of individual rationality according to formula (1) is implicitly covered as a borderline case of outsider coalitions $AC_{n.q}$ only including one actor A_n because of Term $c_{n.1}$ in formula (6). Hence the condition of individual rationality does not in principle need to be listed explicitly as an assumption according to formula (1). In this article, however, it will be used to show that the condition of individual rationality is always respected.

The lower bound *LB* for the distribution of the profit *G* is that point in the solution space $\mathbb{R}_{\geq 0}{}^N$ at which the minimum allocable share $v_{n.min}$ of the profit *G* is assigned to each actor A_n The lower bound $v^{\chi}_{n.max}$ is often called the *threat point*.

The fifth and last assumption is introduced as an *integrity condition* for the relation of the lower bound *LB* to the upper bound *UB* for the shares of the profit *G* to be distributed, as well as for the hyperplane *H* for compliance with the efficiency condition, in order to avoid certain complications outside the scope of this paper (for details of these complications due to the closely related τ-value see Zelewski (Zelewski, 2009, pp. 137-141 and 156-167):

$$\forall LB, UB \in \mathbb{R}^N_{\geq 0} \forall G \in \mathbb{R}_{>0} :$$

$$\left(LB = \begin{pmatrix} v^{\chi}_{1.min} \\ ... \\ v^{\chi}_{N.min} \end{pmatrix} \wedge UB = \begin{pmatrix} v^{\chi}_{1.max} \\ ... \\ v^{\chi}_{N.max} \end{pmatrix} \wedge c(C_0) = G \right) \qquad (7)$$

$$\rightarrow \quad \left(\sum_{n=1}^{N} v^{\chi}_{n.min} \;\leq\; G \;\leq\; \sum_{n=1}^{N} v^{\chi}_{n.max} \;\; \wedge \;\; LB \leq UB \right)$$

It can be shown that exactly one solution point *L* in the *N*-dimensional non-negative real number space $\mathbb{R}_{\geq 0}{}^N$ fulfills all five aforementioned assumptions for the generic distribution problem concerning individual and collective rationality, as well as efficiency and integrity, i.e. the formulas (1), (2), (4), (5), and (7). This unique solution point is the χ-value. The χ-value is a special solution point L_{χ}, which is determined by a convex or, in less precise but more intuitive terms, linear combination of the upper bound (ideal point) *UB* and the lower bound (threat point) *LB* with the weighting factor γ and $0 \leq \gamma \leq 1$. Therefore it must hold true that:

$$\forall L, LB, UB \in \mathbb{R}^N_{\geq 0} \forall G \in \mathbb{R}_{>0}: \left(L = \begin{pmatrix} v_1 \\ \ldots \\ v_N \end{pmatrix} \wedge \sum\nolimits_{n=1}^N v_n = G \wedge LB = \begin{pmatrix} v^{\chi}_{1.min} \\ \ldots \\ v^{\chi}_{N.min} \end{pmatrix} \wedge UB = \begin{pmatrix} v^{\chi}_{1.max} \\ \ldots \\ v^{\chi}_{N.max} \end{pmatrix} \wedge G \geq \sum\nolimits_{n=1}^N c(\{A_n\}) \right)$$

$$\rightarrow \left(\dot{\exists} L_{\chi} \in \mathbb{R}^N_{\geq 0} \; \dot{\exists} \lambda \in \mathbb{R}_{\geq 0}: \; L_{\chi} = \lambda \bullet LB + (1-\lambda) \bullet UB \wedge 0 \leq \lambda \leq 1 \right) \quad (8)$$

After some simple transformations using the efficiency condition and with special regard to the frequently neglected degenerated case $\sum\nolimits_{n=1}^N v^{\chi}_{n.max} = \sum\nolimits_{n=1}^N v^{\chi}_{n.min}$, the common formula for calculating the χ-value produces:

$$\forall n = 1,\ldots,N: \quad v^{\chi}_{n.\tau} = \gamma \bullet v^{\chi}_{n.max} + (1-\gamma) \bullet v^{\chi}_{n.max} \quad (9)$$

where:

$$\gamma = \frac{G - \sum\nolimits_{n=1}^N v^{\chi}_{n.min}}{\sum\nolimits_{n=1}^N v^{\chi}_{n.max} - \sum\nolimits_{n=1}^N v^{\chi}_{n.min}}; \quad \text{if } \sum\nolimits_{n=1}^N v^{\chi}_{n.max} \neq \sum\nolimits_{n=1}^N v^{\chi}_{n.min}$$

$$\gamma \in [0;1]; \quad \text{if } \sum\nolimits_{n=1}^N v^{\chi}_{n.max} = \sum\nolimits_{n=1}^N v^{\chi}_{n.min} \quad (10)$$

4. A practical example for calculating the χ-value

The following example shows how the χ-value can be applied in management practice to solve the problem of fair distribution of profits in supply chains. For illustrative purposes, a simply structured fictitious example is considered. It is restricted to the number of *N* = 5 actors. The numerical values are chosen so that the necessary calculations remain relatively easy.

The following example should illustrate what information is required in management practice in order to apply the χ-value in calculating profit distributions. It should be noted, however, that the example is not concerned with the gathering of information. In management practice, obtaining all values of the characteristic function *c* for all possible coalitions could prove particularly difficult.

The numerical example considers a supply chain with 5 actors $A_1,...,A_5$. In the last business year, the actors jointly realized a profit *G* of $ 100,000. This profit is to be distributed among the actors in a manner that these actors accept as fair. Firstly, to ensure the comparability with other game theory solution concepts, it is assumed that the values of the characteristic function *c* for the generic distribution game are known. Thus the values $c(C_m)$ are known for every possible coalition C_m which can be formed from the set of actors $A = \{A_1,...,A_5\}$. The values $c(C_m)$ are given in Table 1 for all 2^5-1 = 31 coalitions C_m where $m = 0,1,2,\dots,30$.

C_m	$c(C_m)$	C_m	$c(C_m)$	C_m	$c(C_m)$
$C_0=\{A_1,A_2,A_3,A_4,A_5\}$	100,000				
$C_1=\{A_1\}$	0	$C_{11}=\{A_2,A_4\}$	25,000	$C_{21}=\{A_1,A_4,A_5\}$	55,000
$C_2=\{A_2\}$	0	$C_{12}=\{A_2,A_5\}$	30,000	$C_{22}=\{A_2,A_3,A_4\}$	50,000
$C_3=\{A_3\}$	0	$C_{13}=\{A_3,A_4\}$	30,000	$C_{23}=\{A_2,A_3,A_5\}$	55,000
$C_4=\{A_4\}$	5,000	$C_{14}=\{A_3,A_5\}$	35,000	$C_{24}=\{A_2,A_4,A_5\}$	65,000
$C_5=\{A_5\}$	10,000	$C_{15}=\{A_4,A_5\}$	45,000	$C_{25}=\{A_3,A_4,A_5\}$	70,000
$C_6=\{A_1,A_2\}$	0	$C_{16}=\{A_1,A_2,A_3\}$	25,000	$C_{26}=\{A_1,A_2,A_3,A_4\}$	60,000
$C_7=\{A_1,A_3\}$	5,000	$C_{17}=\{A_1,A_2,A_4\}$	35,000	$C_{27}=\{A_1,A_2,A_3,A_5\}$	65,000
$C_8=\{A_1,A_4\}$	15,000	$C_{18}=\{A_1,A_2,A_5\}$	40,000	$C_{28}=\{A_1,A_2,A_4,A_5\}$	75,000
$C_9=\{A_1,A_5\}$	20,000	$C_{19}=\{A_1,A_3,A_4\}$	40,000	$C_{29}=\{A_1,A_3,A_4,A_5\}$	80,000
$C_{10}=\{A_2,A_3\}$	5,000	$C_{20}=\{A_1,A_3,A_5\}$	45,000	$C_{30}=\{A_2,A_3,A_4,A_5\}$	90,000

Tab. 1: Values of the characteristic function *c* for all coalitions C_m

A prerequisite for calculation of the χ-value as a solution $\underline{v}_\chi$ where $\underline{v}_\chi = (v_{1.\chi},\dots, v_{N.\chi})$ for the generic distribution game is that the values of the characteristic function *c* for all three types of coalition are available. That is, $c(C_0)$ must be available for the grand coalition $C_0 = \{A_1,...,A_5\}$, while $c(MC_n)$ is required for each marginal coalition MC_n where $n = 1,...,5$ and $c(AC_{n.q})$ must be known for each outsider coalition $AC_{n.q}$. The value $c(C_0) = 100,000$ for the grand coalition C_0 is immediately available from Table 1, since, according to the efficiency condition, the entire profit $G = 100,000$ must be distributed exactly among all 5 actors $A_1,...,A_5$ in the supply chain. The values $c(MC_n)$

for the marginal coalitions MC_n where n = 1,...,5 can be determined with the aid of the definition $MC_n = C_0 \backslash \{A_n\}$ (results in Table 2).

MC_n	$c(MC_n)$
MC_1	$c(\{A_1,...,A_5\} \setminus \{A_1\}) = c(\{A_2,A_3,A_4,A_5\}) = 90{,}000$
MC_2	$c(\{A_1,...,A_5\} \setminus \{A_2\}) = c(\{A_1,A_3,A_4,A_5\}) = 80{,}000$
MC_3	$c(\{A_1,...,A_5\} \setminus \{A_3\}) = c(\{A_1,A_2,A_4,A_5\}) = 75{,}000$
MC_4	$c(\{A_1,...,A_5\} \setminus \{A_4\}) = c(\{A_1,A_2,A_3,A_5\}) = 65{,}000$
MC_5	$c(\{A_1,...,A_5\} \setminus \{A_5\}) = c(\{A_1,A_2,A_3,A_4\}) = 60{,}000$

Tab. 2: Values of the characteristic function c for all marginal coalitions MCn

The values $c(AC_{n.q})$ for the outsider coalitions $AC_{n.q}$ where n = 1,...,5 can be obtained immediately from Table 1. However, calculation of these values $c(AC_{n.q})$ requires a tremendous amount of work, since 75 (5•15 = n•q) feasible outsider coalitions must be considered. This calculation is therefore omitted for space reasons. It is significant that for the calculation of the values $c(AC_{n.q})$ for all possible combinations outsider coalitions $AC_{n.q}$, the values of the characteristic function c for all possible coalitions C_m with $\emptyset \subset C_m \subset C_0$ need to be determined. Hence the above mentioned fifth requirement of minimal knowledge is not fulfilled by the χ-value. This is surprising, since it appears from the formulas (1) to (6) that, to determine the χ-value, only those values of the characteristic function c must be known that refer to the grand coalition C_0, the marginal coalitions MC_n and the outsider coalitions $AC_{n.q}$. Only the concrete numeric calculation of the χ-value for the example considered here shows that calculation of the values $c(AC_{n.q})$ for the outsider coalitions $AC_{n.q}$ indirectly leads to the fact that the values $c(C_m)$ of the characteristic function c for *all* possible coalitions C_m with $\emptyset \subset C_m \subset C_0$ must be known.

The components $v_{n.max}$ of the upper bound *UB* (ideal point) are calculated with formula (3) on the basis of the values $c(C_0)$ and $c(MC_n)$ instead of the more complicated formula (4). This possiblity of simplification relies on the fact that the example used here is a *convex game* (Curiel 1997, p. 3; Fromen 2004, p. 87; Zelewski 2009, p. 216). It was proven for the class of *convex games* that the χ-value coincides with the τ-value. Because the τ-value is calculated with the aid of formula (3), it is sufficient to use this formula here.

The value $c(C_0)$ is immediately given by the profit G to be distributed: $c(C_0) = G$ =100,000. Thus the components $v_{n.max}^{\chi}$ of the upper bound *UB* of the χ-value are those shown in Table 3.

A_n	$v_{n.max}^{\chi}$
A_1	$c(C_0)-c(MC_1)$ = 100,000 - 90,000 = 10,000
A_2	$c(C_0)-c(MC_2)$ = 100,000 - 80,000 = 20,000
A_3	$c(C_0)-c(MC_3)$ = 100,000 - 75,000 = 25,000
A_4	$c(C_0)-c(MC_4)$ = 100,000 - 65,000 = 35,000
A_5	$c(C_0)-c(MC_5)$ = 100,000 - 60,000 = 40,000

Table 3: Components $v_{n.max}^{\chi}$ of the upper bound *UB* of the χ-value

The components $v_{n.min}^{\chi}$ of the lower bound *LB* (threat point) of the χ-value are calculated with formula (6) for each of the 5 actors A_1 to A_5. This calculation is shown as an example for actor A_4:

$$v_{4.min}^{\chi} = max\{c_{4.1};c_{4.2};0\} = max\{5{,}000;5{,}000;0\} = 5{,}000$$

because:

$$c_{4.1} = c\left(\{A_4\} \mid AC_{4.1}\right) = c\left(\{A_4\}\right) = 5{,}000$$

$$c_{4.2} = max\left\{c\left(\{A_4\} \mid AC_{4.q}\right) = c\left(AC_{4.q}\right) - \sum\nolimits_{m\in\left(IN_{4.q}\setminus\{4\}\right)} v_{m.max}^{\chi} \;\middle|\; q = 2,...,15\right\} = 5{,}000$$

From the components $v_{n.max}^{\chi}$ of the upper bound *UB* and the components $v_{n.min}^{\chi}$ of the lower bound *LB* calculated above, it follows that the standard case for calculation of the χ-value with $\sum\nolimits_{n=1}^{N} v_{n.max}^{\chi} \neq \sum\nolimits_{n=1}^{N} v_{n.min}^{\chi}$ applies. According to formula (10), the weighting factor γ is as follows:

$$\gamma = \frac{G - \sum\nolimits_{n=1}^{N} v_{n.min}^{\chi}}{\sum\nolimits_{n=1}^{N} v_{n.max}^{\chi} - \sum\nolimits_{n=1}^{N} v_{n.min}^{\chi}}$$

$$= \frac{100{,}000 - (0+0+0+5{,}000+10{,}000)}{(10{,}000+20{,}000+25{,}000+35{,}000+40{,}000) - (0+0+0+5{,}000+10{,}000)}$$

$$= \frac{17}{23} \approx 0{,}74$$

The components $v_{n.\chi}$ of the χ-value $\underline{v}_{\chi}$ are then calculated in Table 4 for each actor A_n using formula (10) and the weighting factor $\gamma = {}^{17}/_{23}$ as the convex combination of the components $v^{\chi}_{n.max}$ of the upper bound *UB* (threat point) and the components $v^{\chi}_{n.min}$ of the lower bound *LB* (threat point) for the χ-value.

A_n	$v_{n.\chi}$
A_1	${}^{17}/_{23} \bullet 10{,}000 + {}^{6}/_{23} \bullet 0 = {}^{1}/_{23} \bullet 170{,}000$
A_2	${}^{17}/_{23} \bullet 20{,}000 + {}^{6}/_{23} \bullet 0 = {}^{1}/_{23} \bullet 340{,}000$
A_3	${}^{17}/_{23} \bullet 25{,}000 + {}^{6}/_{23} \bullet 0 = {}^{1}/_{23} \bullet 425{,}000$
A_4	${}^{17}/_{23} \bullet 35{,}000 + {}^{6}/_{23} \bullet 5{,}000 = {}^{1}/_{23} \bullet 625{,}000$
A_5	${}^{17}/_{23} \bullet 40{,}000 + {}^{6}/_{23} \bullet 10{,}000 = {}^{1}/_{23} \bullet 740{,}000$

Table 4: Components $v_{n.\chi}$ of the χ-value v_{χ}

In the end, exactly one χ-value with $\underline{v}_{\chi} = {}^{1}/_{23} \bullet (170{,}000; 340{,}000; 425{,}000; 625{,}000; 740{,}000)$ exists as a unique solution.

5. Conclusion

This article has shown how the vague *understanding of fairness* can be *defined* by applying game theory solution concepts to the generic distribution problem.

In the authors view, the assumptions it is necessary to accept for use of the χ-value are so straightforward that the solution concept has *great potential* for *general acceptance.* Other game theory solution concepts, for example the Shapley value and the nucleolus, demand the acceptance of far more abstract, often only formally precisely definable assumptions. Hence they have considerably lower general acceptance potential. Additionally, other game theory solution concepts, for example the core of a game, can be traced back to a few plausible assumptions. However, they have the disadvantage that they do not exist for many instances of the generic distribution problem or have multiple, often even infinite, solutions.

As *managerial insights* three aspects can be gained from above explanations. Firstly, game theory solution concepts such as the χ-value offer a “reasonable”, that is, provable with *good reason*, and justifiable *basis* for the *distribution of profits* in supply chains. Thanks to the explicability of the good reasons, there is a high chance that the companies will accept the distribution as *fair*. However, distribution of profits calculated using the χ-value can always only represent the basis of a discussion about the fair distribution of a collectively realized profit, not the final outcome of the distribution. Like any other concept for distributing profits, the χ-value is based on specific assumptions, which can, but must not, be accepted as „reasonable“.

Secondly, it was implied in this contribution that the *profit G can be defined precisely* and quantified monetarily, but that this assumption will only rarely be fulfilled in practice. This can lead to two basic practical problems. On the one hand, agreement needs to be reached as to the concrete economic scale on which the profit to be distributed is to be determined and from which sources the information required to determine it can be drawn. On the other hand, how the management of a supply chain is defined needs to be clarified. If a supply chain is dominated by one focal company, it is relatively simple to equate the management of a supply chain with the management of the focal company. However, as a side condition it must be considered that the management of the focal company can only make decisions that do not jeopardize the stability of the supply chain. There is also the question of what the management of a supply chain is, if the special case of a focal company does not apply. In this non-focal case, one option is to revert to the game theory concept of coalition formation games. With the aid of this concept, it is possible to examine how coalitions of legally autonomous companies in (the form of) a supply chain come about. However, even such coalition formation games so far offer no starting points at which to determine how, in supply chains without a focal company, the profit to be distributed should be determined in concrete terms. Extensive academic research is still required on this point.

Thirdly, the management of companies cooperating in a supply chain must always be aware of the fact that game theory solution concepts assume the rationality of all involved actors (companies). Negotiations in real existing supply chains about the “fair” distribution of profits are by no means always guided by the rationality of the negotiating partners. Rather, management must be aware that the process of negotiation on the fair distribution of profits also influences the fact that conceptions of rationality do not correspond to classic game theory. Influences “beyond” the conceptions of rationality of classic game theory are not covered by the game theory solution concept introduced here.

The χ-value thus represents an interesting approach and allows the aspect of bargaining power to be included in determining distribution outcomes which can be accepted as fair.

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