Balancing Financial and Non-Financial Performance Measures

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Abstract

Non-financial performance measures are often used for performance evaluation. They are especially relevant if the available financial performance measures not completely reflect the manager’s contribution to the firm’s total value. Then, non-financial performance measures serve as an indicator for the firm’s long-term performance and may therefore be included in incentive contracts. In the paper we analyze the incentive weights placed on non-financial performance measures and the firm’s short-term financial return. We determine the consequences of a non-contractable long-term financial return and of private pre-decision information for the incentive weights of non-financial performance measures. We explore the extent to which the strength of the statistical relation between the non-financial performance and the firm-value, and the limitations of financial data as a measure of total firm performance influence the incentive weights. Specifically, we determine conditions where the incentive weight of the short-term financial return increases, although the firm’s interest in the long-term financial return increases.

Key words: non-financial performance measures, financial performance, LEN-model, multi-task, performance measure congruity

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1 Financial and Non-financial Performance Measures

Non-financial performance measures are frequently used for performance evaluation; specifically, they are a central element of concepts such as the balanced scorecard\(^1\). From an incentive point of view non-financial measures can be helpful because any combination of costless performance measures that reduces the risk imposed on the agent through an incentive contract is beneficial to the principal. Furthermore, combining different performance measures may help the principal in inducing specific activities, and thereby to reduce managerial myopia.

Non-financial performance measures such as customer satisfaction, product quality, or employee turnover are especially relevant in cases where market-based performance measures showing the total firm value are not available. This is true for the division of a firm, or when the firm is not listed on a stock market. Then, the director or owner of the firm can only use accounting-based and non-financial data for performance evaluation and management compensation.\(^2\)

Generally, non-financial measures have no intrinsic value for the director. Rather, they are leading indicators that provide information on future performance not contained in contemporaneous accounting measures. Empirical studies by Ittner et al. (1997) (quality – growth in profit margin), Ittner and Larcker (1998a) (customer satisfaction – future accounting performance), and Banker et al. (2000) (customer satisfaction – future accounting earnings) support the role of non-financial performance measures as a leading indicator of future financial results.\(^3\) Such leading indicators are especially necessary for performance measurement and management compensation when current managerial

\(^{1}\)See Kaplan and Norton (1992).


\(^{3}\)Furthermore, non-financial performance measures are also helpful for valuation purposes. See e.g. Amir and Lev (1996), Rajgopal et al. (2000).
actions influence the firm’s long-term financial return but are not reflected in the contemporaneous accounting measures. Examples refer to delaying costly maintenance activities at the expense of the future availability of the machinery and, therefore, a lower future financial return.

The use of non-financial performance measures in compensation schemes is limited by the fact that it is often problematic to relate non-financial data to accounting performance\textsuperscript{4}. An additional difference between accounting-based and non-financial performance measures refers to the circumstance that managers frequently have some pre-decision information regarding the non-financial performance measure that is not available to the director of the firm.

"Idiosyncratic knowledge of people, machines, organizations, customers, and suppliers, as well as knowledge of time and place, are examples of specific knowledge. ... Specific knowledge is also often obtained at low cost by individuals in an organization as a by-product of other activities, for example, the idiosyncratic knowledge about a machine that its operator gains over time. Prices and quantities are examples of general knowledge that are easily aggregated and are inexpensive to transmit among agents."\textsuperscript{5}

Compared with the accounting-based performance, the manager is therefore expected to have pre-decision information about his impact on the non-financial performance. To summarize the characteristics of the non-financial performance measures considered subsequently, we assume

1. the non-financial performance to be an indicator of the long-term financial return with no intrinsic value to the director, and

2. the manager’s influence on the non-financial performance to be unknown to the director.

\textsuperscript{4}See the empirical evidence provided in Ittner and Larcker (1998b).
Both, the relation between non-financial performance and long-term financial return as well as the uncertainty regarding the determinants of the non-financial performance are expected to influence the relevance of non-financial performance measures for performance evaluation and management compensation. This is supported by empirical studies. Lingle and Schiemann (1996) and Ittner et al. (1997) find that while non-financial performance measures such as customer satisfaction are frequently highly valued, they are only rarely linked to compensation. Instead accounting-based measures play a dominant role in bonus determination.

In the paper we analyze the tradeoffs underlying the weighting of the performance measures in the compensation scheme. Considering a moral hazard problem where the unobservable activities influence the accounting-based as well as the non-financial performance, we suppose the agent having private information w.r.t. his impact on the non-financial performance. In the model, the non-financial performance correlates with the firm’s long-term financial return. We assume that the total return to the firm consists of the short-term accounting-based performance and the long-term financial return. Moreover, only the short-term financial return and the non-financial performance are observable and contractible signals; the long-term financial return, however, is non-contractible because the manager may leave the firm before all outcomes of his activities are realized. Therefore, if the available performance measures do not perfectly reflect the firm’s total return, a problem of non-congruent performance measures exists.

Based on the principal/agent-model we explore the relevance of non-financial performance measures by analyzing the incentive weights placed on non-financial and accounting-based performance. Especially, we are interested in the influence of the strength of the statistical relation between the non-financial performance and the long-term financial return on the respective incentive weights. The analysis shows the extent to which the strength of the statistical relation may assist in assigning incentive weights to the performance measures. We find that the incentive weight placed on the non-financial signal increases with the strength of the statistical relation, whereas the incentive weight placed on the short-term financial return weakly decreases with the strength of the statistical
Often, the literature on performance measurement claims that due to limitations of financial data as a measure of total firm value, non-financial performance measures are necessary for an efficient performance evaluation.\textsuperscript{6} In order to explore this claim, we analyze the extent to which the shortcomings of the financial performance influence the incentive weights placed on the firm’s performance measures. Based on our model, we determine conditions where the incentive weight placed on the short-term financial return increases, although the firm’s relative interest in the short-term financial return decreases.

Section 2 gives a short summary of the related literature. In section 3 we describe the multi-task agency model with short-term financial and non-financial performance measures and derive the optimal incentive contract for a general case of the agent’s pre-decision information. In section 4 we determine the incentive weights when the productivities of a single non-financial performance measure are log-normally distributed and examine some special settings regarding the information content of the short-term financial return. We conclude in section 5 with a summary and some empirical implications.

2 Related Literature

The incentive weights placed on performance measures are intensively analyzed within the principal/agent-framework. Following Banker and Datar (1989), the incentive weight increases with the sensitivity of the agent’s action and with the signal’s precision. The consequences of performance measure congruity and misalignment in a multi-task agency relationship\textsuperscript{7} are analyzed by Feltham and Xie (1994) and Feltham and Wu (1998). Given multiple activities of the agent, Datar et al. (1999) show that increasing the sensitivity does not necessarily increase the weight placed on that performance measure. The key to

\textsuperscript{6}See e.g. Kaplan and Norton (1992).
\textsuperscript{7}See Holmstrom and Milgrom (1991).
the result is that the principal must consider the congruity of an individual performance measure in relation to the congruity of alternative performance measures. Therefore, a non-congruent performance measure may enable the principal to provide precise incentives to the agent.

The incentive problem increases if the agent privately observes the congruity of the performance measures. Considering a single task agency relationship with agent pre-decision information, Baker (1992) shows that the incentive weight placed on a performance measure decreases for a decreasing congruity between signal and firm value.

3 The Model

The model is based on the assumptions of the LEN-model. A contract between a risk neutral principal and a risk averse agent specifies a compensation function linear in one or several performance measures. The agent’s preferences are described by a negative exponential utility function, with $r$ the coefficient of absolute risk aversion.

The agent controls a vector $\mathbf{a}$ of $n$ activities and chooses an effort $a_i$ devoted to each task. The activities influence both the short-term ($\pi_f$) and the discounted long-term ($\alpha\pi_l$) financial return to the principal. The short-term return may refer to a one period accounting profit of the whole firm or division, whereas the long-term financial return shows the discounted value of future profits; the latter represents “economic consequences of a manager’s actions that are not fully realized until after he leaves the firm.” Following the time line depicted in figure 1, the short-term financial return is realized at $t = 2$, whereas the long-term financial return is not realized until $t = 3$. We assume the total return $\tilde{\pi}$ to be given by

$$\tilde{\pi} = \tilde{\pi}_f + \alpha \tilde{\pi}_l,$$

with $\alpha \in R^+$ capturing the principal’s time value of money, i.e. the interest rate, together with the extent to which the agent’s effort choice spreads into the future. Hence, $\alpha$ may

\footnote{See Bamberg and Spremann (1981), Holmstrom and Milgrom (1987).}

\footnote{Feltham and Wu (1998, p. 5).}
Principal and Agent sign contract

Agent observes $\delta_{ij}$

Agent supplies effort $a$

Short-term financial ($\pi_f$) and non-financial ($y_j$) signals realized

Agent receives compensation $s$

Long-term financial return $\pi_l$ realized

$t = 0$  $t = 1$  $t = 2$  $t = 3$

Figure 1: Sequence of events

well obtain values greater than one$^{10}$. The parameter $\alpha$ indicates the principal’s interest in the long-term financial return. In addition, $\alpha$ shows the limitations of the short-term financial return as a measure of total financial return to the firm. The shortcoming of $\pi_f$ increases for increasing $\alpha$.

In order to simplify the analysis we suppose constant marginal productivities. Hence,  

$$\tilde{\pi}_f = \sum_{i=1}^{n} b_i a_i + \tilde{\epsilon}_f = b^t a + \tilde{\epsilon}_f \quad \text{and} \quad \tilde{\pi}_l = \sum_{i=1}^{n} \tilde{d}_i a_i + \tilde{\epsilon}_l = \tilde{d}^t a + \tilde{\epsilon}_l$$  

(2)

where $b = (b_1, ..., b_n)^t$ is the marginal productivity influencing the short-term payoff, and $\tilde{\epsilon}_f \sim N(0, \sigma_f^2)$ and $\tilde{\epsilon}_l \sim N(\varsigma_l, \sigma_l^2)$ are two normally distributed random components of the results$^{11}$. Due to the different time of realization we assume the two random components to be independent ($Cov(\tilde{\epsilon}_f, \tilde{\epsilon}_n) = 0$). Unlike the short-term marginal productivities, the long-term marginal productivities $\tilde{d} \equiv (\tilde{d}_1, ..., \tilde{d}_n)^t$ are unknown to the principal. At the time of contracting she only knows the expected productivities $\bar{d} \equiv (E[\tilde{d}_1], ..., E[\tilde{d}_n])^t$ and the standard deviation $\sigma_i^d$ for each task $i$. In order to exclude inconsistencies regarding $\alpha$ representing shortcomings of the short-term financial return as a measure of total firm value, we assume $b \neq \bar{d}$, i.e. there exists at least one task with $b_i \neq \bar{d}_i$.

Due to the late realization of $\tilde{\pi}_l$ the principal can only use $\tilde{\pi}_f$ as a performance measure. However, we assume that she can use $m$ (non-financial) signals $\tilde{y}_j$ as performance measures. The non-financial signals have no intrinsic value to the principal. The agent’s

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$^{10}$This is especially true if $\pi_l$ represents an infinite annuity, i.e. $\alpha = 1/i$, with $i$ the interest rate.

$^{11}$The reason for assuming a positive mean $\varsigma_l$ in the distribution of $\tilde{\epsilon}_l$ will be explained shortly. Because of the principal’s risk neutrality and the non-contractability of $\tilde{\pi}_l$ we may suppose $\sigma_l^2 = 0$ without loss of generality.
effort influences signal $j$ via

$$\tilde{y}_j = \sum_{i=1}^{n} \tilde{\delta}_{ji} a_i + \tilde{\epsilon}_{nj} = \tilde{\delta}_{ji}^t a + \tilde{\epsilon}_{nj} ,$$

with $\tilde{\delta}_j = (\tilde{\delta}_{j1}, ..., \tilde{\delta}_{jm})^t$ the marginal contributions to the signal, and $\tilde{\epsilon}_{nj} \sim N(0, \sigma_{nj}^2)$ an additive and uncontrollable random component of non-financial performance measure $\tilde{y}_j$. The marginal productivities $\tilde{\delta}_{ji}$ follow arbitrary distributions with only non-negative values.

We assume the non-financial performance measures to be indicators of the long-term financial return. Therefore, the long-term financial return is expected to correlate with the non-financial performance measures; specifically, this refers to the marginal contributions w.r.t. $\tilde{\pi}_l$ and $\tilde{y}_j$. The covariance between $\tilde{d}_i$ and $\tilde{\delta}_{ji}$ is $Cov(\tilde{d}_i, \tilde{\delta}_{ji}) = \rho_{d_{ji}} \sigma_{d_i} \sigma_{\delta_{ji}}$, with $\rho_{d_{ji}}$ the correlation coefficient between task $i$'s contribution to the long-term financial return ($\tilde{d}_i$) and task $i$'s contribution to the outcome of signal $y_j$ ($\tilde{\delta}_{ji}$). For performance measure $j$, the parameter $\psi_j = \sum_i \rho_{d_{ji}} \sigma_{d_i} \sigma_{\delta_{ji}}$ aggregates the covariances over all tasks. The aggregation is necessary because each performance measure aggregates the consequences of all tasks. Hence, the parameter $\psi_j$ reflects the strength of the statistical relation of performance measure $j$ and the long-term financial return $\pi_l$, i.e. ceteris paribus the strength of the statistical relation increases with increasing $\psi_j$.

Furthermore, the contribution of task $i$ to several non-financial performance measures may be correlated as well. W.r.t. the performance measures $\tilde{y}_j$ and $\tilde{y}_k$, the correlation of task $i$'s productivities $\tilde{\delta}_{ji}$ and $\tilde{\delta}_{ki}$ is described by the correlation coefficient $\rho_{d_{ji,ki}}$. For two signals $j$ and $k$, the vector $\lambda_{jk} = \sum_i \rho_{d_{ji,ki}} \sigma_{\delta_{ji}} \sigma_{\delta_{ki}}$ shows the respective strength of the statistical relation, aggregated over all tasks. Finally, we assume the short-term random components $\tilde{\epsilon}_f$ and $\tilde{\epsilon}_{nj}$ to be jointly normally distributed with covariance matrix $\Sigma$.

Using the $m+1$ performance measures, the principal may specify an incentive scheme

$$s = f + \nu_f \tilde{\pi}_f + \sum_{j=1}^{m} \nu_{nj} \tilde{y}_j = f + \nu^t y$$

with $f$ the fixed salary, $\nu_f$ and $\nu_{nj} \forall j$ the incentive weights for the financial and the non-financial performance measures, and $y \equiv (b^t a, \tilde{\delta}_{1}^t a, ..., \tilde{\delta}_{m}^t a)^t = \mu a$ a vector of the
outcome of the performance measures, including the short-term financial return. Hence, the matrix $\mu_{[m+1 \times n]}$ shows the marginal contributions of the $n$ tasks to the $m + 1$ performance measures. Depending on the incentive problem, however, the use of less than $m + 1$ performance measures may be optimal\textsuperscript{12}.

After signing the contract but before choosing and supplying his effort, the agent privately observes the marginal productivities $\delta_{ji}$. Due to the technical nature of the information we exclude any communication between principal and agent\textsuperscript{13}. Furthermore, we assume that the agent may quit after observing the productivities. Specifically, we assume that exogenous restrictions prevent the principal from including a damage payment for the jointly observable breach decision in the contract\textsuperscript{14,15}. Hence, the participation constraint must be satisfied for any productivity $\delta_{ji}$ when using signal $\tilde{\gamma}_j$ as a performance measure. This assumption is equivalent to assuming that it is impossible to require an agent to choose activities that result in an expected loss to him\textsuperscript{16}.

Due to the positive and effort-independent long-term contribution of the agent’s work ($\varsigma_l > 0$), the principal always prefers to contract with the agent. However, the principal may contract with an agent observing $\delta_{ji} = 0 \forall i$ who, hence, will not supply any effort regarding signal $\tilde{\gamma}_j$. Then, the principal has to compensate the agent for the risk $\tilde{\epsilon}_{nj}$ inherent in performance measure $\tilde{\gamma}_i$. In that case it might be optimal to not contract at all with the agent. In order to exclude cases like that we assume that these "inefficiencies"\textsuperscript{12,13,14,15,16}.

\textsuperscript{12}Since we focus the analysis on the variation of the incentive weights, we do not normalize the performance measures. However, the normalization seems to be crucial when considering the relative importance of the performance measures. See Lambert and Larcker (1987), Sloan (1993), Feltham and Wu (1998).

\textsuperscript{13}See also Lambert (1986), Demski and Sappington (1987).

\textsuperscript{14}Exogenous aspects like the notion of "fairness" may restrict the damages for breaching a contract. See Melumad (1989, p. 741).

\textsuperscript{15}The restriction placed on damage payments is consistent with excluding bonus payments as a part of the compensation function. The latter is necessary in order to prevent the Mirrlees-solution in case of an exponential utility function and a normally distributed performance measure. See Mirrlees (1974).

\textsuperscript{16}Kanodia (1993, p. 177), e.g., states that the "assumption that managers cannot be required to implement plans they know will impose losses on them is more consistent with the autonomy that divisional managers have in decentralized organizations."
are small relative to the result $\zeta$ of the long-term financial result. The effort independent result, however, only results if the agent does not breach the contract.\footnote{For ease of presentation we subsequently consider only the net long-term benefit, i.e. $\pi_l - \zeta$.}

Therefore, the principal’s problem is given by

$$\max_{f,\nu,a} E[\pi - s]$$

s.t.

$$PC \quad E[-\exp\{-r[s - C(a)]\}] \geq -\exp\{-rs_a\} \quad \forall \delta_{ji}$$

$$IC \quad a_i \in \arg\max_a E[-\exp\{-r[s - C(a)]\}] \quad \forall i$$

The risk neutral principal maximizes the expected net return after compensating the agent. With the participation constraints (6) she ensures that the agent receives always at least his reservation wage $s_a$. Supplying effort leads to private costs $C(a)$ to the agent. We assume the disutility of effort to be additively separable in each task, i.e. $C(a) = \frac{1}{2} \sum_{i=1}^n a_i^2 = \frac{1}{2}a^T a$. Therefore, the agent’s net result is $s - C(a)$. Following the incentive constraints (7), the agent chooses effort levels that maximize his expected utility.

Subsequently, we restrict the incentive weights to non-negative values. In general, negative values may provide incentives for sabotage ($a < 0$), and, hence, result in dysfunctional behavior of the agent. This seems to be obvious w.r.t. the short-term financial performance. There, without additional monitoring the agent may easily ‘throw away’ any short-term result. The incentive weights for the non-financial performance measures, however, are restricted to be non-negative as well. Although non-financial performance measures can be defined in a positive and a negative way\footnote{E.g., reliability rate and percentage of machine break downs.}, an implicit direction is given when considering the influence of the agent’s effort. Consider for example the case of a single non-financial performance measure $y$. For positive marginal productivities $\delta_i$ the agent increases $y$ by exerting effort. Hence, an incentive problem is supposed to exist w.r.t. the agent’s effort increasing the signal. For the opposite direction we may assume that the agent can costlessly throw away any result, i.e. without a significant personal
cost the agent can reduce the signal’s outcome. An example is the time consuming creation of customer satisfaction that can be destroyed quite easily. Then, the principal will only use non-negative incentive weights, since negative incentive weights would induce the agent to significantly reduce the signal’s outcome.\footnote{Analytically, the argumentation requires a more detailed description of the agent’s disutility of effort and the production functions under study. Considering a single task and a single performance measure \( \hat{y} \), these functions are \( C(a) = \begin{cases} 1/2a^2 & a \geq 0 \\ 0 & a < 0 \end{cases} \) and \( E[\hat{y}] = \begin{cases} \delta a & a \geq 0 \\ -\infty & a < 0 \end{cases} \). Hence, the principal always chooses a non-negative incentive weight \( \nu \geq 0 \). Otherwise, the agent would choose some \( a < 0 \) and receive an income of infinite size.\footnote{Bushman et al. (2000), however, do not preclude the existence of negative effort levels.}}

Since the agent observes the marginal productivities \( \delta_{ji} \) before choosing \( a_i \), only uncertainties regarding the random components \( \tilde{\epsilon}_f \) and \( \tilde{\epsilon}_{nj} \) remain when choosing the effort levels. Due to their joint normal distribution we can express the agent’s expected utility by its certainty equivalent, i.e.

\[
CE = f + \nu' \mu - \frac{1}{2} \omega \nu - \frac{1}{2} r \text{Var}[s].
\]  

(8)

Maximizing \( CE \) is equivalent to maximizing the expected utility. Using (8) in (7), first order conditions yield \( a^* = \mu' \nu \). Then, the certainty equivalent for the participation constraints is

\[
f + \frac{1}{2} \nu' \mu \nu - \frac{1}{2} r \nu' \Sigma \nu \geq s_a \quad \forall \delta_{ji}.
\]  

(9)

The participation constraints must be fulfilled for all values \( \delta_{ji} \) of the marginal productivities. In the optimum, the principal will choose \( f \) so that (9) is just fulfilled with equality. With \( \nu \) non-negative, the minimum LHS of (9) results for the minimum contribution margins \( \delta_{ji} \). Then, the principal chooses

\[
f = s_a - \frac{1}{2} \nu' \mu_0 \nu + \frac{1}{2} r \nu' \Sigma \nu
\]  

(10)

with \( \mu_0 \equiv [b, \tilde{\delta}_1, \ldots, \tilde{\delta}_m]' \), \( \tilde{\delta}_j \equiv (\tilde{\delta}_{j1}, \ldots, \tilde{\delta}_{jn})' \forall j \), and \( \tilde{\delta}_{ji} \equiv \min\{\tilde{\delta}_{ji}\} \forall i, j \). It is straightforward to show that an agent observing \( \delta_{ji} > \tilde{\delta}_{ji} \) receives a rent. Without loss of generality we assume \( s_a = 0 \).
Using the condition for the optimal effort levels together with (10), we can reduce the principal’s decision problem. Hence, we obtain the following unconstrained decision problem

\[
\max_{\nu} \quad E \left[ b^t \mu^t \nu + \alpha \bar{d}^t \mu^t \nu - \frac{1}{2} r \nu^t \Sigma \nu + \frac{1}{2} \nu^t \mu_0 \mu_0^t \nu - \nu^t \mu \nu \right]
\]

\[
= b^t \bar{\mu}^t \nu + \alpha (\bar{d}^t \bar{\mu}^t \nu + \Psi^t \nu) - \frac{1}{2} r \nu^t \Sigma \nu + \frac{1}{2} \nu^t \mu_0 \mu_0^t \nu - \nu^t (\bar{\mu} \bar{\mu}^t + \Lambda) \nu \quad (11)
\]

where

\[
\bar{d} \equiv E[\bar{d}]
\]

\[
\bar{\mu} \equiv E[\mu]
\]

\[
\Psi \equiv [\psi_1, \ldots, \psi_{m+1}]^t \quad \text{with} \quad \psi_j = \sum_{i=1}^n \rho_{ji} \sigma_i^d \sigma_{ji}
\]

\[
\Lambda \equiv [\lambda_{jk}]_{m+1 \times m+1} \quad \text{with} \quad \lambda_{jj} = \sum_{i=1}^n \sigma_{ji}^2, \lambda_{jk} = \sum_{i=1}^n \rho_{ji,ki} \sigma_{ji} \sigma_{ki}
\]

Here, vector \( \Psi \) describes for all performance measures the covariance of task \( i \)'s contribution to the long-term financial return and the contribution to the outcome of the performance measure, aggregated over all tasks. The matrix \( \Lambda \) shows the covariance of task \( i \)'s contribution to performance measures \( j \) and \( k \), aggregated over all tasks. The scalars \( \lambda_{jj} \) aggregate the variances of the individual productivities. Hence, they show the information asymmetry between principal and agent, i.e. the agent’s informational advantage increases with increasing \( \lambda_{jj} \). Moreover, the non-diagonal scalars \( \lambda_{jk} \) show the strength of the statistical relation between performance measures \( j \) and \( k \). Since the agent’s contributions to performance measure \( \tilde{\pi}_f \) are well known to the principal, we obtain \( \psi_1 = 0 \), \( \lambda_{1k} = 0 \forall k \), and \( \lambda_{j1} = 0 \forall j \).

Given (11), it is straightforward to determine the optimal incentive weights \( \nu^* \). Due to the concave nature of the problem the first order conditions are also sufficient\(^{21}\). Then, the vector of optimal incentive weights is

\[
\nu^* = \left[ 2(\bar{\mu} \bar{\mu}^t + \Lambda) - \mu_0 \mu_0^t + r \Sigma \right]^{-1} \left\{ \bar{\mu} \Delta + \alpha \Psi \right\} \quad (12)
\]

\(^{21}\)Here, we assume interior solutions with \( \nu^* \geq 0 \).
with $\Delta \equiv b + \alpha \bar{d}$ showing the agent’s expected impact on the total firm value (here, $\Delta_i = b_i + \alpha \bar{d}_i$). Condition (12) is a general result for the incentive weights given that the agent has private pre-decision information regarding the signal’s productivities, and the principal having preferences for a non-contractable return $\tilde{\pi}_l$. Relaxing these assumptions we obtain the incentive weights for some special cases. Considering all marginal productivities to be public knowledge at the time of contracting, we have $\bar{\mu} = \mu_0 = \mu$, and, hence, $\mu_0 \mu_0^t = \bar{\mu} \bar{\mu}^t = \mu \mu^t$, $\bar{d} = d$, $\Lambda = 0$, and $\Psi = 0$. Then, the incentive weights become

$$\nu^* = \left[ \mu \mu^t + r \Sigma \right]^{-1} \{ \mu \Delta \}$$

Furthermore, for $\alpha = 0$ the principal is not interested in the non-contractable return $\tilde{\pi}_l$, and the short-term financial return perfectly represents the principal’s financial interests. Then, the incentive weights become

$$\nu^* = \left[ 2(\bar{\mu} \bar{\mu}^t + \Lambda) - \mu_0 \mu_0^t + r \Sigma \right]^{-1} \{ \bar{\mu} b \}$$

Finally, neglecting both pre-decision information and a non-contractable result yields

$$\nu^* = \left[ \mu \mu^t + r \Sigma \right]^{-1} \{ \mu b \}$$

Comparing the closed form solutions, we observe a tendency that introducing agent pre-decision information reduces the incentive weights, given that the matrix of the expected marginal contributions equals the marginal contributions for the case of publicly observable productivities ($\bar{\mu} = \mu$) and a positive correlation between the aggregated uncertain productivities ($\Lambda \geq 0$). Furthermore, the comparison shows that introducing a non-contractable return strictly increases the incentive weights, given a positive discount factor ($\alpha > 0$), a positive contribution ($d > 0$), and a non-negative statistical relation of the non-financial performance measures and the long-term financial return ($\Psi \geq 0$).

In order to gain additional insights we restrict our analysis to a setting with only one non-financial performance measure and suppose specific probability distribution functions for the marginal productivities. For this setting, we analyze the balancing of the

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22See also the solution to the comparable problem in Feltham and Xie (1994) with $b = 0$ and $\alpha = 1$. 

incentive weights for the short-term financial return and the non-financial performance measure.

4 BALANCING OF INCENTIVE WEIGHTS

4.1 Single Non-financial Performance Measure

Subsequently, we consider a single non-financial performance measure $\tilde{y}$ whose productivities are correlated with the marginal contributions to the long-term return. When signing the contract, the principal expects the marginal contributions to be log-normally distributed, i.e. $\tilde{d}_i \sim LN(\mu_{d_i}, \sigma_{d_i}^2) \forall i$. The marginal contributions are independent random variables. Equivalently, the marginal productivities $\tilde{\delta}_i$ of signal $\tilde{y}$ follow a log-normal distribution, with $\tilde{\delta}_i \sim LN(\mu_{\delta_i}, \sigma_{\delta_i}^2) \forall i$ that are independently distributed as well. Regarding the correlation between $\tilde{y}$ and $\tilde{\pi}_l$, we assume for each task $i$ the contribution $\tilde{d}_i$ and the productivity $\tilde{\delta}_i$ to be correlated, with a covariance of $Cov(\tilde{d}_i, \tilde{\delta}_i) = \rho_{d_i} \sigma_{d_i} \sigma_{\delta_i}$. Therefore, $\psi = \sum_i \rho_{d_i} \sigma_{d_i} \sigma_{\delta_i}$ aggregates the covariances over all tasks. Ceteris paribus, a larger value of $\psi$ indicates a higher statistical relation between $\tilde{y}$ and $\tilde{\pi}_l$. Furthermore, $\lambda = \sum_i \sigma_{\delta_i}^2$ aggregates the variances of the non-financial productivities. Finally, the random error terms are supposed to be non-correlated: $Cov(\tilde{\epsilon}_f, \tilde{\epsilon}_n) = 0$.

Applying the result specified in (12) we obtain the following closed form solutions for the two incentive weights:

$$\nu_f^* = \frac{2\Phi_{b\delta} \Delta - 2\alpha \psi b^t \delta + b^t \Delta (2\lambda + r \sigma_n^2)}{2(\Phi_{b\delta} - b^t \delta \delta^t b + \lambda b^t b) + 2(\delta^t \delta + \lambda) r \sigma_f^2 + (b^t b + r \sigma_f^2) r \sigma_n^2} \tag{16}$$

$$\nu_n^* = \frac{\Phi_{b\delta} \Delta - \delta^t b b^t \Delta + \alpha \psi b^t b + (\delta^t \Delta + \alpha \psi) r \sigma_f^2}{2(\Phi_{b\delta} - b^t \delta \delta^t b + \lambda b^t b) + 2(\delta^t \delta + \lambda) r \sigma_f^2 + (b^t b + r \sigma_f^2) r \sigma_n^2} \tag{17}$$

with

$$\Phi_{b\delta} \equiv \delta^t b^t \Delta - b^t \delta \delta^t \Delta = \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} (b_i \delta_j - b_j \delta_i) (\Delta_i \delta_j - \Delta_j \delta_i)$$

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See the appendix for the derivation of eqs. (16) and (17).
\[ \Phi_{\delta b \Delta} \equiv b' b \bar{\delta} \Delta - \delta' bb' \Delta = \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} (\bar{\delta}_i b_j - \bar{\delta}_j b_i)(\Delta_i b_j - \Delta_j b_i) \]
\[ \Phi_{b \delta} \equiv b' b \bar{\delta} \bar{\delta} - b' \bar{\delta} \delta' b = \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} (b_i \bar{\delta}_j - b_j \bar{\delta}_i)^2 \]

First, the incentive weights depend on parameters reflecting the congruency of \( \pi_f \) and \( y \), and their alignment with \( \pi \). Here, \( \Phi_{b \delta \Delta} \) compares the agent’s expected impact on the non-financial signal \( (\bar{\delta}) \) with the impact on the second performance measure \( (b) \) and the expected impact on the firm’s total return \( (\Delta) \). Because of \( \Delta = b + \alpha \bar{d} \), \( \Phi_{b \delta \Delta} \) aggregates the congruency \( \Phi_{b \delta} \) of the two performance measures and the comparison \( \Phi_{b \delta \Delta} \) of the agent’s expected impact on the non-financial signal with the impact on \( \pi_f \) and the expected impact on \( \pi_t \) (\( \Phi_{b \delta \Delta} = \Phi_{b \delta} + \alpha \Phi_{b \delta \Delta} \), with \( \Phi_{b \delta \Delta} = \bar{\delta} \delta b' \bar{d} - b' \delta \bar{\delta} b \)). For a similar relative impact on the two performance measures (\( \bar{\delta} = \omega b, \omega \in \mathbb{R}^+ \)), and for a similar relative impact on \( y \) and \( \pi \) (\( \bar{\delta} = \omega \Delta, \omega \in \mathbb{R}^+ \)), \( \Phi_{b \delta \Delta} = 0 \) and a lower incentive weight \( \nu_f^* \) results. First, the expected outcomes of the two performance measures differ only by a constant factor (\( E[\bar{g}] = \omega E[\bar{\pi}_f] \)), and second the expected outcome of the non-financial signal perfectly represents the expected total firm return (\( E[\bar{g}] = \omega E[\bar{\pi}] \)). In both cases, the relevance of \( \bar{\pi}_f \) as a performance measure is reduced.\(^{24}\) Regarding the incentive weight \( \nu_n^* \) placed on the non-financial signal, however, the agent’s impact on \( \pi_f \) as compared to \( y \) and \( \pi \) must be considered (\( \Phi_{b \delta \Delta} \)).

Second, the strength of the statistical relation \( \psi \) influences the incentive weights. The closed form solutions in (16) and (17) show differing consequences of \( \psi \): while the incentive weight placed on the non-financial performance measure increases with the strength of the statistical relation, the incentive weight placed on the short-term financial return decreases for increasing \( \psi \). However, given that the agent’s activities either influence \( \pi_f \) or \( y \), i.e. \( b' \bar{\delta} = 0, \psi \) does not influence the incentive weight \( \nu_f^* \).

The results in (16) and (17) show an interior solution for the incentive weights. Boundary solutions result if one of the incentive weights is negative. They follow from (12) with

\(^{24}\)Furthermore, \( \Phi_{b \delta \Delta} = 0 \) when the agent’s activities influence either \( \pi_f \), or \( y \) and \( \pi \), i.e. \( b' \bar{\delta} = 0 \) and \( b' \Delta = 0 \). Due to \( \Delta = b + \alpha \bar{d} \), however, \( b' \Delta = 0 \) requires a specific distribution of the expected productivities \( \bar{d} \), with at least one negative productivity.
some necessary assumptions regarding the problem's parameters. Considering the case where only the short-term financial return $\tilde{\pi}_f$ is used as a performance measure, we have $\tilde{\mu} = \mu_0 = b^t, \nu = \nu_0^f, \Sigma = \sigma_f^2$, and $\Lambda = \Psi = 0$. Then, the incentive weight is

$$\nu_0^f = \frac{b^t \Delta}{b^t b + r\sigma_f^2}. \quad (18)$$

Alternatively, if the principal only uses the non-financial performance measure we have $\tilde{\mu} = \tilde{\delta}_t, \mu_0 = 0, \nu = \nu_0^0, \Sigma = \sigma_n^2, \Lambda = [[0, 0], [0, \lambda]],$ and $\Psi = [0, \psi]$. Then, we obtain the incentive weight as

$$\nu_0^0 = \frac{\tilde{\delta}_t \Delta + \alpha \psi}{2(\tilde{\delta}_t \tilde{\delta}_t + \lambda) + r\sigma_n^2}. \quad (19)$$

Based on eq. (18) we observe that for an exogenous restriction of the performance measures to $\pi_f$, the corresponding incentive weight $\nu_0^f$ is non-negative. Moreover, when exogenously restricting the performance measure to $y$, a negative incentive weight $\nu_0^y$ may result for negative correlations between the marginal benefits and the marginal contribution to the signal ($\psi < -\tilde{\delta}_t \Delta / \alpha$). Then, it is optimal for the principal to not use an incentive contract at all.

We illustrate the result for the incentive weights with the following example. There, the agent has to choose his effort for three different tasks. We assume his coefficient of absolut risk aversion to be $r = 1$, and standard deviations for the two performance measures are $\sigma_n = 15$ and $\sigma_f = 22$. Table 1 shows the remaining parameter values.

![Table 1: Data for the example](image)

Figure 2 shows an increasing incentive weight for the non-financial performance signal, and a decreasing incentive weight for the short-term financial return for an increasing parameter $\alpha$. For values $\alpha \geq 1.3097$ the principal only uses the non-financial signal $y$ as
a performance measure. Furthermore, we observe $\nu_n^* \leq \nu_n^0$ for $\alpha \leq \hat{\alpha}_f$, i.e. the additional use of the short-term financial return reduces the incentive weight for the non-financial performance measure. Figure 2 illustrates that for increasing limitations of the short-term financial return and an increasing interest in the long-term financial return, the principal decreases the incentive weight placed on $\pi_f$ and increases the incentive weight placed on $y$.

The closed form solutions of (16) and (17) indicate that the incentive weights crucially depend on the congruency of the performance measures. In order to additionally explore this aspect we consider two settings differing in the degree of myopia of the short-term financial return. There, the agent has to perform two tasks where task 1 influences the short-term financial return, and task 2 determines the non-financial signal and the long-term financial return. The two settings differ in that task 2 is not contained in $\pi_f$ (myopic short-term financial return), and that task 2 also influences the outcome of $\pi_f$ (forward-looking short-term financial return).
4.2 Myopic Short-term Financial Return

The short-term financial return is myopic when the agent’s activity determining the firm’s long-term financial return does not influence $\pi_f$.\footnote{Here, \(\alpha > 0\) is necessary for the short-term financial return to differ from the firm’s total financial return.} Such a separation of the consequences of the agent’s activities on the two performance measures results for $b = (b_1, 0)^t$, $\bar{\delta} = (0, \delta_2)^t$, and $\bar{d} = (0, d_2)^t$. In addition, the aggregated variances of the non-financial productivities ($\lambda$), and the aggregated covariances of the task’s productivities w.r.t. the long-term financial return and the non-financial performance ($\psi$) must be adjusted accordingly. Based on (16) and (17) it is straightforward to determine the incentive weights for the short-term financial return and the non-financial signal as

$$\nu_f^* = \frac{b_1^2}{b_1^2 + r\sigma_f^2} \quad \text{and} \quad \nu_n^* = \frac{\alpha(\delta_2 \bar{d}_2 + \psi)}{2(\delta_2^2 + \lambda) + r\sigma_n^2}.$$ \hspace{1cm} (20)

Here, the incentive weight for the short-term financial return does not depend on the strength $\psi$ of the statistical relation between $\tilde{y}$ and $\tilde{\pi}_l$, nor does it depend on the shortcomings of $\pi_f$ as a measure of total firm performance.

The incentive weight for the non-financial performance, however, increases with the strength of the statistical relation ($\frac{\partial \nu_n^*}{\partial \psi} \geq 0$). Furthermore, the principal may well choose a positive incentive weight $\nu_n^*$ even when no statistical relation exists between the non-financial signal and the long-term financial return ($\psi = 0$). Despite the missing statistical relation, the use of $y$ as a performance measure enables the principal to induce effort in task 2. In this case, the risk neutral principal uses the expected productivities w.r.t. $y$ and $\pi_l$ when determining $\nu_n^*$.

Since negative incentive weights are supposed to motivate gaming of the performance measure by the agent, the principal includes the non-financial signal in the compensation scheme only for $\psi = \rho_d^d \sigma_2^d \sigma_2 > -\delta_2 \bar{d}_2$. Therefore, the non-financial signal may not be used for a negative statistical relation $\rho_d^d$ combined with large variations $\sigma_2$ and $\sigma_2^d$ of the productivities for $\delta_2$ and $d_2$. In addition, (20) shows that an increasing interest of the...
principal in the long-term financial return increases the incentive weight placed on the non-financial signal. Equivalently, increasing shortcomings of \(\pi_f\) motivate the principal to place a larger emphasis on the non-financial signal for performance evaluation and management compensation. Therefore, she increases the agent’s effort directed towards the long-term financial return via a higher incentive weight placed on the non-financial signal. Finally, the incentive weight \(\nu_n^*\) decreases for an increasing variation of \(\delta_2\), i.e. \(\partial \nu_n^*/\partial \lambda < 0\).

The principal’s expected return for a myopic short-term financial return is

\[
E[U_P] = \frac{b_1^4}{b_1^2 + r\sigma_f^2} + \frac{\alpha^2(\delta_2 d_2 + \psi)^2}{2(\delta_2^2 + \lambda) + r\sigma_n^2}.
\]

(21)

Here, the expected return can be separated into the value of the two performance measures. The second part in (21) shows the expected value of the non-financial performance measure. When including the non-financial signal into the compensation scheme \((\psi > -\delta_2 d_2)\), an increasing strength of the statistical relation between the non-financial signal and the firm’s long-term financial return improves the expected return to the principal. Moreover, the value of the non-financial signal decreases with the principal’s uncertainty \(\lambda\) regarding the agent’s influence on the non-financial signal.

To summarize, for a myopic short-term financial return no balancing is necessary in order to determine the incentive weights placed on the short-term financial return and the non-financial performance. Of course, one crucial assumption for this result is that the error terms \(\tilde{\epsilon}_f\) and \(\tilde{\epsilon}_n\) are uncorrelated. Then, incentive weights and expected values of the performance measures depend on factors idiosyncratic to the production of the non-financial outcome and the short-term financial return.

4.3 Forward-looking Short-term Financial Return

A forward-looking short-term financial return follows when the agent’s activity determining the long-term financial return also influences the short-term financial return.

\(^{26}\)Since \(\lambda = \sigma_2^2\) and \(\psi = \rho^d_2 \sigma^d_2 \sigma_2\), we have to substitute \(\psi\) with \(\psi' \lambda^{1/2}\) in order to show \(\partial \nu_n^*/\partial \lambda < 0\).
Therefore, \( \delta = (0, \delta_2)^t \) and \( \bar{d} = (0, \bar{d}_2)^t \), and the short-term financial return has multiple determinants with \( b = (b_1, b_2)^t \). Based on eqs. (16) and 17, the incentive weights are

\[
\nu_f^* = \frac{2b_1^2\delta_2^2 - 2\alpha\psi b_2\delta_2 + (b_1^2 + b_2^2 + \alpha b_2\bar{d}_2)(2\lambda + r\sigma_n^2)}{-4b_1^2\delta_2^2 + (b_1^2 + b_2^2 + r\sigma_n^2)(2(\lambda + \delta_2^2) + r\sigma_n^2)}
\]

(22)

\[
\nu_n^* = \frac{-b_2\delta_2(b_1^2 + b_2^2) + \alpha((b_1^2 - b_2^2)\bar{d}_2\delta_2 + \psi(b_1^2 + b_2^2)) + (\alpha\psi + (b_2 + \alpha\bar{d}_2)\bar{d}_2)r\sigma_n^2 + (b_1^2 + b_2^2 + r\sigma_n^2)(2(\lambda + \delta_2^2) + r\sigma_n^2)}{-4b_2^2\delta_2^2 + (b_1^2 + b_2^2 + r\sigma_n^2)(2(\lambda + \delta_2^2) + r\sigma_n^2)}
\]

(23)

Since in this setting one activity of the agent influences the outcome of the non-financial performance as well as the short-term financial return, factors idiosyncratic to the production of \( y \) influence the incentive weight placed on the short-term financial return. Equal to the general result for eq. (16), we find that \( \nu_f^* \) decreases for an increasing strength between the non-financial signal and the long-term financial return \((\partial \nu_f^*/\partial \psi \sim -2\alpha b_2\bar{d}_2)\), and \( \nu_n^* \) increases with the statistical relation: \( \partial \nu_n^*/\partial \psi \geq 0 \).

Moreover, the incentive weights vary with the limitations of \( \pi_f \) as a measure of total firm profit and the principal’s interest in the long-term financial return. Regarding the short-term financial return we determine \( \partial \nu_f^*/\partial \alpha \sim b_2(\bar{d}_2(2\lambda + r\sigma_n^2) - 2\psi\bar{d}_2) \). Therefore, the incentive weight \( \nu_f^* \) may increase as well as decrease for increasing shortcomings of \( \pi_f \). Especially, \( \nu_f^* \) will increase for

\[
\psi < \bar{\psi} \equiv \frac{\bar{d}_2(2\lambda + r\sigma_n^2)}{2\delta_2}.
\]

(24)

For a rather low correlation of \( \tilde{y} \) and \( \tilde{\pi}_l \), the incentive weight \( \nu_f^* \) increases with \( \alpha \). The threshold \( \bar{\psi} \) of the statistical relation increases with the variance \( \lambda \) of the non-financial productivity \( \delta_2 \), the variance \( \sigma_n^2 \) of the random noise \( \tilde{\epsilon}_n \), the expected contribution \( \bar{d}_2 \) to the long-term financial return, and it decreases with the agent’s expected influence on the signal’s outcome \( \bar{\delta}_2 \).

The key to the result is that for a high \( \alpha \) the principal has a large interest in the long-term financial return. Then, given a low statistical relation of \( \tilde{y} \) and \( \tilde{\pi}_l \), the principal extensively uses the short-term financial return in the compensation contract. By increasing \( \nu_f^* \), the principal motivates the agent to provide more effort in task 2, which
also improves $\tilde{\pi}_l$. This is especially of benefit to the principal when she is rather uncertain regarding the non-financial signal’s productivity. Moreover, an increasing variance $\sigma^2_n$ increases the risk imposed on the agent through $\nu^*_n$ as compared to $\nu^*_f$. Then, the risk premium necessary to contract with the agent is lower for the short-term financial return. Equation (24) indicates the combined influence of the strength of the statistical relation $\psi$, the variance $\lambda$ of the non-financial productivities, and the non-financial’s precision ($\sigma^{-2}_n$) on the relevance of the short-term financial return for increasing $\alpha$. Given a high statistical relation of $\tilde{y}$ and $\tilde{\pi}_l$, however, the principal decreases the incentive weight placed on $\pi_f$ for increasing shortcomings of the short-term financial return.

The incentive weight placed on the non-financial signal may also increase and decrease with the limitations of the financial data. Based on $\partial \nu^*_n / \partial \alpha$ we find a decreasing incentive weight $\nu^*_n$ for

$$\psi < \bar{d}_2 \bar{\delta}_2 \quad \text{and} \quad b^*_2 > \frac{\bar{d}_2 \bar{\delta}_2 + \psi (b^*_1 + \sigma^2_f)}{\bar{d}_2 \bar{\delta}_2 - \psi \bar{d}_1^2}.$$  

The first condition implies that the agent’s expected impact on the non-financial signal is only poorly correlated with his impact on the long-term financial return, indicating a rather low statistical relation between $\tilde{y}$ and $\tilde{\pi}_l$. In addition, the second condition requires the consequences of task 2 to be adequately represented in the short-term financial return relative to task 1. As a consequence, the short-term financial return is “sufficiently forward-looking” and can be used for motivating the agent to supply effort in task 2. Hence, for an increasing interest in the long-term financial return the principal focuses on the short-term financial return for management compensation as compared to the non-financial signal.

To summarize, a forward-looking short-term financial return requires a balancing of the incentive weights. Then, the incentive weight placed on the short-term financial return decreases for an increasing strength of the statistical relation between the non-financial signal and the long-term financial return. Furthermore, given a low correlation between the non-financial signal and the long-term financial return, the incentive weight placed on the short-term financial return increases with the principal’s interest in the long-term financial return. Hence, despite increasing shortcomings of the short-term
financial return as a measure of total firm value the director increases its incentive weight in the compensation scheme.

Moreover, the forward-looking character of the short-term financial return also influences the incentive weight placed on the non-financial signal. Especially, we find that for a sufficiently forward-looking short-term financial return and for a low statistical relation between the non-financial signal and the long-term financial return the incentive weight placed on the non-financial signal decreases for an increasing interest of the principal in the long-term financial return.

5 Empirical Implications and Concluding Remarks

In the paper we analyze the impact of the agent’s pre-decision information and non-congruent performance measures on the incentive weights placed on the short-term financial return and the non-financial results in a multi-task principal/agent-relationship. The analysis identifies conditions where the incentive weights may actually increase, although the relative importance of the signal to the principal decreases.

The analysis shows that the magnitude of the agent’s pre-decision information reduces the incentive weights placed on non-financial signals. Therefore, we predict the incentive weights to increase with the knowledge of the manager’s superiors or his compensation committee regarding the manager’s impact on the non-financial signal. The manager’s information advantage, on the other side, is expected to increase with the size of his division. Based on the analytical results, we expect to observe a negative relationship between division size and incentive weight\textsuperscript{27}. Furthermore, this result seems to correspond with the empirical results provided by Lingle and Schiemann (1996) and Ittner et al. (1997), that non-financial performance measures are only rarely linked to compensation, and that accounting-based performance measures, despite their limitations as a measure of total firm value, play a dominant role for managerial compensation.

In addition, we expect the weight placed on non-financial performance measures in

\textsuperscript{27}See the empirical results provided by Balkin and Gomez-Mejia (1987).
compensation contracts to increase with the strength of the statistical relation between the non-financial signal and the firm’s long-term financial return. Regarding customer satisfaction, e.g., we expect a higher statistical relation to the long-term financial return in a customer-oriented market as compared to a supplier-oriented market. Hence, we expect the influence of customer satisfaction on managerial compensation to be higher in a customer-oriented market as compared to a supplier-oriented market.

Finally, we expect a balancing of the incentive weights placed on the performance measures to be necessary only when the long-term managerial decisions show short-term consequences. Given that the long-term managerial decisions do not influence the short-term performance, however, the balancing of the performance measures reduces to choosing the optimal non-financial signal.
Appendix

Derivation of the incentive weights for a single non-financial performance measure

For the problem considered, we have $\bar{\mu} = [b, \bar{\delta}]^t$, $\mu_0 = [b, 0]^t$, $\bar{\delta} = [\bar{\delta}_1, \ldots, \bar{\delta}_n]^t$, and $\bar{d} = [\bar{d}_1, \ldots, \bar{d}_n]^t$, where (DeGroot (1970))

$$\bar{d}_i \equiv E[\tilde{d}_i] = e^{\mu_d + \sigma_d^2/2} \forall i$$
$$\bar{\delta}_i \equiv E[\tilde{\delta}_i] = e^{\mu_\delta + \sigma_\delta^2/2} \forall i$$

Furthermore, we have

$$\Lambda = \begin{bmatrix} 0 & 0 \\ 0 & \sum_{i=1}^n \sigma_i^2 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & \lambda \end{bmatrix} \quad \text{and} \quad \Psi = \begin{bmatrix} 0 \\ \sum_{i=1}^n \rho_i \sigma_i \sigma_i \end{bmatrix} = \begin{bmatrix} 0 \\ \psi \end{bmatrix} \quad (A.1)$$

where

$$(\sigma_i^d)^2 \equiv \text{Var}[\tilde{d}_i] = e^{2\mu_d + \sigma_d^2} (e^{\sigma_d^2} - 1) \forall i$$
$$\sigma_i^2 \equiv \text{Var}[\tilde{\delta}_i] = e^{2\mu_\delta + \sigma_\delta^2} (e^{\sigma_\delta^2} - 1) \forall i$$

$$E[\tilde{\delta}_i^2] = E[\tilde{\delta}_i]^2 + \text{Var}[\tilde{\delta}_i] = \bar{\delta}_i^2 + \sigma_i^2 \forall i$$

Finally, for uncorrelated random components $\tilde{\epsilon}_f$ and $\tilde{\epsilon}_n$ the covariance matrix $\Sigma$ is

$$\Sigma = \begin{bmatrix} \sigma_f^2 & 0 \\ 0 & \sigma_n^2 \end{bmatrix} \quad (A.2)$$

Hence, the optimal incentive weights depend on the following inverse matrix

$$\left[2(\bar{\mu}\bar{\mu}^t + \Lambda) - \mu_0\mu_0^t + r\Sigma\right]^{-1} = \begin{bmatrix} b'b + r\sigma_f^2 & 2b'\bar{\delta} \\ 2b\bar{\delta} & 2\bar{\delta}'\bar{\delta} + 2\lambda + r\sigma_n^2 \end{bmatrix}^{-1} \quad (A.3)$$

which can be determined using a standard approach for a $2 \times 2$-matrix.
References


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