WACC, APV, and FTE revisited

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Abstract

Today, every textbook on corporate finance contains a chapter on valuation of levered firms. Typically, it is stated that the net present value of a project is exactly the same under the three methods APV, WACC, and FTE.

We will show in this paper that this is wrong: in general, WACC and APV will provide necessarily different values of the firm. Furthermore, the method that can be applied depends on the underlying financing assumption of the model. It turns out that the FTE method is not a suitable approach to firm valuation.

The literature states that there is a so-called circularity problem with the WACC approach. We show that this problem is a fictitious one.

1 Introduction

Today, every textbook on corporate finance contains a chapter on valuation of levered firms. As a rule, three methods are proposed: the APV (adjusted present value) approach, the WACC (weighted average cost of capital) approach and the FTE (flow to equity) method. The APV approach essentially going back to the fundamental result of \cite{ModiglianiMiller1958} was extended by Myers \cite{Myers1974} to normative capital budgeting analysis. The WACC approach was developed by Modigliani and Miller \cite{ModiglianiMiller1963}, extended by Miles and Ezzell \cite{MilesEzzell1980} and recently generalized by Löffler \cite{Loeffler1998}. Incomprehensibly, the topic is virtually not discussed in recent publications – although up to now there is no satisfying comparison of these methods.

For example, it is typically stated that "the net present value of our project is exactly the same under each of the three methods ... However, one method usually provides an easier computation than another ... " \cite[pp. 461]{Rossetal1996}. We will show in this paper that this statement is wrong: in general, the APV and the WACC methods will provide necessarily different values of the firm. Furthermore, the method that can be applied depends on the underlying financing assumption of the model and has nothing to do with computational considerations. In other words, WACC and APV are two completely different ways of evaluating tax savings on interest rates.

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For this reason, we carefully analyze the underlying assumptions of both the WACC and the APV method. It turns out that we have to consider two different cases depending on whether the future value of the firm is a random variable or not. Also, the literature proposes the FTE method as an approach to evaluate a levered firm. Yet, as we show, FTE needs information for valuation that is only given by WACC and APV together. Therefore we conclude that FTE is not an operationalizable approach for firm valuation.

The paper is organized as follows: in the next section we state the assumptions of the model. The third section considers the case of deterministic future value of the firm. In the fourth section we analyze the case of random future value. The last section concludes the paper.

2 Assumptions of the Model

If a firm is to be evaluated, one has to know what investments the managers are going to make. The investment decision will then determine the free cash flows of the firm. We will denote these cash flows at time \( t \) by \( \tilde{CF}_t \). Future cash flows are uncertain today, at time \( t = 0 \).

We assume that the firm at time \( t \) is financed by stocks \( S_t \) and bonds \( B_t \). Debt is certain. The cost of capital for debt will be denoted by \( r_f \), they remain constant through time. Furthermore, cost of capital for the common stock of an unlevered firm is given by \( r^*_S \) and remain constant through time, too. Both assumptions are made for simplicity. The calculation can be carried out analogously if \( r_f \) and \( r^*_S \) change with time.

There is an income tax, applied at rate \( \tau \), the tax rate is assumed constant. Debt reduces the taxable income base because interest payments are tax-deductible.

The value of the unlevered firm is given by

\[
V_{0u} = \sum_{t=1}^{T} \frac{(1-\tau)E[\tilde{CF}_t]}{(1+r^*_S)^t}.
\]

To determine the value of the tax shield we have to consider two cases. These cases differ concerning the value of the unlevered firm \( V_{t}^u \) at \( t > 0 \): up to now we do not know whether or not it is a random variable. In the second case (the value of the firm is a random variable) the financing decision of the firm has to be considered. Consequently, WACC and APV will inevitably give different values of the firm. In the first case, however, the value of the firm is independent of the method one uses.

3 First case: \( V_{t}^u \) not a random variable

We are aware of two special cases in which this assumption will hold:

- If the future cash flow is constant through time the value of the unlevered firm is always
  \[
  V_{t}^U = \frac{(1-\tau)E[CF]}{r^*_S},
  \]
  and this is not a random variable.
• If the cash flow follows a Markov process the values $\widetilde{CF}_{t+1}, \ldots$ are by assumption independent of the state of the world at time $t$. Hence, the value of the firm $V_t^u$ is also independent of the state at time $t$.

If the value of the unlevered firm is not a random variable but deterministic today, everything turns out to be simple. The firm can be evaluated depending on the data the investor knows. Since under the assumption of the case debt and equity are certain (seen from $t=0$), the tax shield is certain too and has to be discounted with the cost of debt $r_f$. Hence, the value of the levered firm is given by the theory of Modigliani and Miller (see Brealey & Myers (1996))

$$V_0^l = V_0^u + \sum_{t=0}^{T-1} \frac{\tau r_f B_t}{(1+\tau r_f t+1)}.$$  \hspace{1cm} (2)

If the value of the unlevered firm is not a random variable (2) gives always the correct value of the firm. According to the literature we will denote this equation by the APV approach. If an investor knows the future amount of debt $B_t$ then equation (2) already determines the value of the levered firm.

But (2) cannot be applied if the investor knows only the future leverage ratio instead of the amount of debt. The leverage ratio will be denoted by

$$l_t = \frac{B_t}{S_t}.$$  \hspace{1cm} (3)

Since the underlying assumptions are not changed (2) still remains true. In Appendix 1 we have shown that this equation can be written as

$$V_0^l = \sum_{t=1}^{T-1} \frac{(r_f - r_s^*) V_t^u + (1 + r_f)(1 - \tau)E[\widetilde{CF}_t]}{(1 + r_s^*) \prod_{k=0}^{t-1} (1 + r_k)},$$  \hspace{1cm} (4)

where $r_k$ is given by:

$$1 + r_k = 1 + r_f \left(1 - \tau \frac{l_k}{1 + l_k}\right).$$  \hspace{1cm} (5)

We will denote this equation as modified APV approach. We are not aware of a formulation of (4) that looks similar to the WACC equation (see (9) below).

It is possible to modify equation (3) such that $S_0$ can be evaluated directly and this equation is denoted as FTE approach (see Brealey & Myers (1996), p.??). But to use this approach one has to know

\footnote{If expected cash flows and the leverage ratio are constant this formula reduces to the Modigliani–Miller equation

$$V_0^l = \frac{V_0^u}{r_s^* \left(1 - \tau \frac{l}{1 + l}\right)},$$

3}
1. the expected cash-flows after taxes and interest, hence the amount of future debt $B_t$
and
2. the cost of capital for stocks. The cost of capital can be derived from the textbook
formula (see for example Brealey & Myers (1996), p.??) if one knows the future
leverage ratio $l_t$.

As can be seen, the application of FTE presupposes the information provided by WACC and
modified APV together. Since the result is the same, there is no reason to use the FTE
approach.

To summarize: if the value of the unlevered firm is not a random variable, one can use the
APV approach (2) to determine the value of the levered firm. If the future amount of debt is
not known but the leverage ratio is known, one can modify the APV approach to an equation
that still determines the firm’s value. And both equations yield the same value. There is no
need to apply the FTE approach.

4 Second case: $V_t^u$ a random variable

We will now assume that the value of the unlevered firm at time $t$ is a random variable. We
believe that this assumption is (in contrast to section 3) the more realistic case. What are
the consequences for the APV and the WACC approach?

In the last section both debt and the leverage ratio were deterministic. If $V_t^U$ is a random
variable, the story is more complicated. We will illustrate that by means of an example.
Consider a model with two periods. It is assumed that the firm will be liquidated at the
end of period two. $\tilde{\text{CF}}_2$ is the cash flow of the firm at time $t = 2$. The owner receives the
after-tax-cash flow (dividend) $\tilde{D}_2$. $r_S$ are the cost of capital for equity in the levered firm.

Since the firm will be liquidated at time $t = 2$, and interest payments reduce the tax base,
we have

$$\tilde{D}_2 = E[(1 - \tau)\tilde{\text{CF}}_2 | \mathcal{F}_1] - (1 + r_f(1 - \tau))B_1.$$ 

Therefore, equity at time $t = 1$ has the value

$$\tilde{S}_1 = \frac{E[\tilde{D}_2 | \mathcal{F}_1]}{1 + r_S}.$$

Now assume, the dividend policy of the firm is determined at time $t = 0$. This is to say
that the amount of debt at time $t = 1$ is already known at $t = 0$ and is not a random variable.
Therefore, we can evaluate the leverage ratio at $t = 1$

$$\tilde{l}_1 = \frac{B_1}{\tilde{S}_1} = \frac{B_1(1 + r_S)}{E[(1 - \tau)\tilde{\text{CF}}_2 | \mathcal{F}_1] - (1 + r_f(1 - \tau))B_1}.$$ 

(6)

Now, particularly since the amount of debt is deterministic, the leverage ratio at time $t = 1$
becomes a random variable! Hence, with a given dividend policy the leverage policy will be
prescribed and not deterministic.
Vice versa, if we lay down the leverage ratio at time $t = 1$ to be determined at $t = 0$ the dividend policy is already specified. To prove this rearrange (6) and get

$$\tilde{B}_1 = \frac{l_1 E[(1-\tau)\tilde{CF}_2 | F_1]}{1 + r_S + l_1 (1 + r_f (1-\tau))}. \tag{7}$$

(7) shows that a deterministic leverage ratio will yield a random dividend policy. We summarize our interim results as follows.

<table>
<thead>
<tr>
<th>finance policy</th>
<th>leverage policy</th>
</tr>
</thead>
<tbody>
<tr>
<td>dividend policy</td>
<td>leverage policy</td>
</tr>
<tr>
<td>$B_t$ deterministic</td>
<td>$l_t$ deterministic</td>
</tr>
<tr>
<td>$l_t$ random variable</td>
<td>$B_t$ random variable</td>
</tr>
<tr>
<td>$\Rightarrow$ certain tax shield</td>
<td>$\Rightarrow$ uncertain tax shield</td>
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</tbody>
</table>

This reveals that the investor has to make a decision about financing policy: There is a considerable difference between determining the dividend versus the leverage policy. Hence, we will expect two different values of the firm according to the financing policy that will be followed. We consider the two policies separately.

### 4.1 Deterministic debt or the APV method

Throughout this section we assume that future debt of the firm is deterministic. As we have seen already, the future leverage ratio of the firm will then be a random variable. The theory of Modigliani and Miller now implies that the tax shield will be deterministic and therefore certain. Consequently, the cost of capital for the tax shield is $r_f$.

Suppose that the investor knows the future amount of debt $B_t$. As we have mentioned in the preceding section, the value of the firm is given by the APV method:

$$V_0^L = V_0^U + \sum_{t=0}^{T-1} \frac{\tau r_f B_t}{(1 + r_f)^{t+1}}. \tag{8}$$

What happens if the investor does not know the future amount of debt but instead the expected leverage ratios (remember that the leverage ratios are uncertain)? Is it still possible to value the firm using this information? Since the underlying assumptions of the model do not change, equation (8) still remains true. The question is whether it is possible to evaluate the future amount of debt by knowing only the expected leverage ratios. Equation (8) should then be transformed such that it contains the expected leverage ratios $E[\tilde{l}_t]$.

We will now show that such a transformation is impossible. We illustrate that with a simple example which shows that the knowledge of the expected leverage ratios does not suffice to value the firm if the future amount of debt is certain.

Consider again equation (6)

$$E[\tilde{l}_t] = E \left[ \frac{B_1 (1 + r_S)}{E[(1-\tau)\tilde{CF}_2 | F_1] - (1 + r_f (1-\tau))B_1} \right].$$
At first, it is easy to see that (given the expected leverage ratio) a unique solution of the amount of debt exists: if $B_1 = 0$ then the right hand side of the equation is zero. If $B_1$ increases, the right hand sides strictly increases. Hence, there must be a unique $B_1$ such that the right hand side equals the expected leverage ratio.

But there does not necessarily exist a formula to determine $B_1$. Consider the case where the conditional expectation of future cash flow $E[\tilde{\text{CF}}_2 | \mathcal{F}_1]$ has two realisations $\text{CF}_{2,1}$ and $\text{CF}_{2,2}$ with probabilities $p_1$ and $p_2$. Then (6) can be written as

$$E[\tilde{l}_1] = p_1 \frac{B_1(1 + r_S)}{(1 - \tau)\text{CF}_{2,1} - (1 + r_f(1 - \tau))B_1} + p_2 \frac{B_1(1 + r_S)}{(1 - \tau)\text{CF}_{2,2} - (1 + r_f(1 - \tau))B_1},$$

and we can calculate $B_1$ by solving a quadratic equation. But if the conditional expectation of future cash flow has $n$ realisations, this leads us to a polynomial of degree $n$ and there does not exist a closed form solution for such a polynomial in general.

To summarize: If the future amount of debt is deterministic and known, then the APV method gives the correct value of the firm. If only the expected leverage ratios are known, APV is still the right method but there does not exist a formula involving $E[\tilde{l}_1]$ to determine the value of the firm.

### 4.2 Deterministic leverage ratio or the WACC method

We will now assume that the leverage ratio is determined at time $t = 0$. This implies that the tax shield is not certain and therefore cannot be discounted at $r_f$. Equation (8) does not yield the correct value of the firm:

“Even though the firm might issue riskless debt, if financing policy is targeted to realized market values, the amount of debt outstanding in future periods is not known with certainty (unless the investment is riskless) . . . ” [Miles & Ezzell (1980)], p. 721.

In this case [Modigliani & Miller (1963)] developed a theory for the case of a perpetual cash flow. This approach was later generalized by [Miles & Ezzell (1980)] to the case of a constant leverage ratio. [Löffler (1998)] recently proved a general WACC formula that is already applicable if the leverage ratio is deterministic. The main part of the proof was to show that the correct cost of capital for the tax shield is $r^*_S$, the cost of capital of the unlevered firm.

Suppose, the investor knows the future leverage ratios of the firm. As was shown in [Löffler (1998)], the value of the firm is given by the WACC method

$$V_0^L = \sum_{t=1}^{T} \frac{(1 - \tau)E[\text{CF}_t]}{\prod_{k=0}^{t-1} \left[1 - \frac{r_f}{1+\tau}\right](1 + r^*_S)}.$$  

(9)

What happens if the investor knows the expected amount of debt in future periods? Since the underlying assumptions do not change, equation (8) still determines the correct value of
the firm. But this equation has to be modified to include $E[\tilde{B}_t]$. In appendix 2 we show that the value of the firm is now given by the equation

$$V_0^L = V_0^U + \sum_{t=0}^{T-1} \frac{\tau r_f E[\tilde{B}_t]}{(1 + r_f)^t} \cdot (1 + r_f).$$  \hspace{1cm} \text{(10)}$$

As one can see, the difference of this modified APV–equation compared to equation (8) is the cost of capital for the tax shield.

Usually, the literature states that there is a theoretical problem with the WACC approach (the so–called circularity problem):

“To calculate the changing WACC, one must know the market value of a firm’s debt and equity. But if the debt and equity values are already known, the total market value of the company is also known. That is, one must know the value of the company to calculate the WACC” \textit{Ross et al. (1996)}, p.480.

As we have shown the WACC method can only be applied

- if the future leverage ratios are deterministic and
- if these ratios are known.

In this case there does not exist any circularity problem. In all other cases however, WACC is not applicable. The circularity problem is a fictitious one.

What can be said about the FTE approach? First, since FTE uses the textbook formula, the underlying assumption has to be the same as in the WACC method: a deterministic leverage ratio. Similarly to the case of certain $V_t^U$ one then has to know

1. the expected cash–flows after taxes and interest and hence the expected amount of future debt and
2. the cost of capital for stocks. The cost of capital can be derived via the textbook formula if one knows the future leverage ratio.

Again, the application of FTE presupposes the information provided by WACC. Since the result is the same, there is no reason to use the FTE approach, one can instead use WACC.

To summarize: if the value of the unlevered firm is not a random variable, one can use the APV approach (2) to determine the value of the levered firm. If the future amount of debt is not known but the leverage ratio is known, one can modify the APV approach to an equation that still determines the firm’s value. And both equations yield the same value. The FTE approach is not an operationalizable method for firm valuation.

5 Summary

We have analyzed the underlying assumptions of the WACC and the APV approach. We have shown that WACC can only be applied if the leverage ratio of the company is deterministic
and known at time \( t = 0 \). APV can be used to evaluate the firm if the amount of future debt is deterministic and known today. Since both methods use different assumptions on the financing decision of the firm, one cannot expect the same value by WACC and APV. The FTE approach is not a suitable method for firm valuation.

We summarize our results as follow:

<table>
<thead>
<tr>
<th>Finance assumptions</th>
<th>Information</th>
</tr>
</thead>
<tbody>
<tr>
<td>( B_t ) deterministic</td>
<td>( l_t ) (or ( E[l_t] )) known</td>
</tr>
<tr>
<td>( l_t ) deterministic</td>
<td>APV method (( \Box ))</td>
</tr>
<tr>
<td>mod. APV method (( \Box ))</td>
<td>WACC method (( \Box ))</td>
</tr>
</tbody>
</table>

### Appendix 1

To prove (11) we assume the following equations. The value of the levered firm at time \( t \) is given by the value of the unlevered firm \( V_t^u \) and the value of the tax shield \( T_t \):

\[
V_t^l = V_t^u + T_t. \tag{11}
\]

Since the cost of capital for the unlevered firm remain constant we have

\[
V_t^u = \frac{V_{t+1}^u + (1 - \tau)E[ CF_{t+1}^r ]}{1 + r_S^*}, \tag{12}
\]

The APV method implies the following iteration for the tax shield

\[
T_t = \frac{T_{t+1}}{1 + r_f} + \frac{\tau r_f}{1 + r_f} B_t. \tag{13}
\]

From (11), (12) and (13) we get using the definition of the leverage ratio \( l_t \)

\[
V_t^l = \frac{V_{t+1}^u + E[ CF_{t+1}^r ]}{1 + r_S^*} + \frac{T_{t+1} + \tau r_f l_t}{1 + r_f} V_t^l.
\]

Rearranging this equation gives

\[
\left( 1 - \frac{\tau r_f}{1 + r_f} \frac{l_t}{1 + l_t} \right) V_t^l = \frac{V_{t+1}^u + E[ CF_{t+1}^r ]}{1 + r_S^*} + \frac{T_{t+1}}{1 + r_f} \frac{V_t^l - V_{t+1}^u}{1 + r_f}
\]

\[
= \frac{V_{t+1}^u + E[ CF_{t+1}^r ]}{1 + r_S^*} + \frac{V_t^l}{1 + r_f}
\]

\[
= \frac{V_{t+1}^u + E[ CF_{t+1}^r ]}{1 + r_S^*} + \frac{1 + r_f E[ CF_{t+1}^r ]}{1 + r_f} + \frac{V_t^l}{1 + r_f}
\]

\[
\left( 1 + r_f - \tau r_f \frac{l_t}{1 + l_t} \right) V_t^l = \frac{V_{t+1}^u + E[ CF_{t+1}^r ]}{1 + r_S^*} + \frac{1 + r_f E[ CF_{t+1}^r ]}{1 + r_f} + \frac{V_t^l}{1 + r_f}
\]

\[
V_t^l = \frac{V_{t+1}^u(r_f - r_S^*)}{(1 + r_S^*)(1 + r_f)} + \frac{E[ CF_{t+1}^r](1 + r_f)}{(1 + r_S^*)(1 + r_f(1 - \tau l_t))} + \frac{V_t^l}{1 + r_f(1 - \tau l_t)}
\]
By applying this iteration we show that (4) is indeed a consequence. Using (5) we get

\[ V_{T-2}^l = \frac{E[\tilde{CF}_T](1 + r_f)}{(1 + r_T^s)(1 + r_{T-2})}. \]

At time \( T - 2 \) we have for the value of the firm

\[ V_{T-2}^l = V_{T-1}^u (r_f - r_T^s) + \frac{E[\tilde{CF}_{T-1}](1 + r_f)}{(1 + r_T^s)(1 + r_{T-2})} + \frac{E[\tilde{CF}_T](1 + r_f)}{(1 + r_T^s)(1 + r_{T-2})(1 + r_{T-3})}. \]

At \( T - 3 \) we have

\[ V_{T-3}^l = \frac{V_{T-2}^u (r_f - r_T^s)}{(1 + r_T^s)(1 + r_{T-3})} + \frac{V_{T-1}^u (r_f - r_T^s)}{(1 + r_T^s)(1 + r_{T-2})(1 + r_{T-3})} + \frac{E[\tilde{CF}_{T-2}](1 + r_f)}{(1 + r_T^s)(1 + r_{T-3})} + \frac{E[\tilde{CF}_{T-1}](1 + r_f)}{(1 + r_T^s)(1 + r_{T-2})(1 + r_{T-3})} + \frac{E[\tilde{CF}_T](1 + r_f)}{(1 + r_T^s)(1 + r_{T-1})(1 + r_{T-2})(1 + r_{T-3})}. \]

As one can see, (14) indeed determines the value of the levered firm at time \( t = 0 \).

**Appendix 2**

We assume that the leverage ratio is deterministic but unkown to the investor. Instead the investor knows the expected amount of debt \( E[\tilde{B}_t] \).

We follow the theory in Löffler (1998) and know that the value of the levered firm is given by the value of the unlevered firm and the value of the tax shield

\[ V_0^l = V_0^u + T_0. \]

This (uncertain) tax shield satisfies the following iteration

\[ T_t = \frac{E[T_{t+1} | \mathcal{F}_t]}{1 + r_T^s} + \frac{\tau r_f F_t}{1 + r_f}. \]

Using the law of iterated expectation

\[ t < s \implies E[X | \mathcal{F}_s] | \mathcal{F}_t] = E[X | \mathcal{F}_t] \]

we get for the tax shield at time \( t = 0 \)

\[ T_0 = \sum_{t=0}^{T-1} \frac{\tau r_f E[\tilde{B}_t]}{(1 + r_T^s)^t \cdot (1 + r_f)} \]

and therefore the assertion.
References


