The Poole Analysis in the New Open Economy Macroeconomic Framework

M. Hoffmann • B. Kempa
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Mathias Hoffmann* and Bernd Kempa†

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Abstract

This paper evaluates simple, non-optimising monetary policy rules in the tradition of the well-known Poole analysis within a general two-country open-economy model of the New Open Economy Macroeconomic framework. Pure money supply rules are compared with simple interest rate rules for the large and the small open economy. The results for the large economy resemble those of the original Poole analysis. This is also confirmed when considering welfare. In particular, an interest rate rule yields superior results with respect to liquidity shocks whereas a money supply rule is preferable when real shock predominate. In the small open economy scenario the results of the large economy case continue to hold for domestic shocks. For foreign shocks, welfare improves under an interest rate rule relative to a money supply rule when real shocks are considered and the impact on real money balances is neglected. The reverse holds for foreign liquidity shocks. In all scenarios an interest rate rule stabilises domestic consumption.

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*Mathias Hoffmann, Department of Economics, University of Cologne, Albertus-Magnus-Platz, 50931 Cologne, Germany. email: m.hoffmann@wiso.uni-koeln.de. Financial support of the German Research Foundation is gratefully acknowledged (Deutsche Forschungsgemeinschaft, HO 3282/1-1).
†Address of Correspondence: Bernd Kempa, Department of Economics, University of Duisburg-Essen, Universitätsstr. 12, 45117 Essen, Germany. email: bernd.kempa@uni-essen.de
1 Introduction

The New Neoclassical Synthesis (NNS) has become the workhorse model to analyse the impact of monetary policy on macroeconomic variables and welfare in both closed and open-economy contexts (see respectively, Blanchard and Kiyotaki, 1987, and Obstfeld and Rogoff, 1995). This paradigm has also opened up new avenues to assess the optimality of monetary policy as the traditional notions of stabilizing output or inflation are superseded by more rigorous welfare analyses. Monetary policy is usually introduced via an interest rate rule, formulated either as a function of the exogenous shocks of the model or in terms of current output and inflation. The optimal monetary policy is then described by the particular set of coefficients of the monetary policy rule which maximises economic welfare. Although this way of modelling monetary policy is theoretically appealing, it assumes that the monetary authorities are able to either uniquely identify macroeconomic shocks or at least perfectly monitor macroeconomic aggregates. In reality, usually neither central banks nor the general public possess timely information with respect to such variables. For example, Orphanides (2004) argues that monetary policy decisions are distorted by the use of output gap data, which are not available at the time of monetary policy decision making. Therefore the economic agents have to solve a signal extraction problem in the tradition of Lucas (1972). This can be accomplished by using currently observable variables or "information variables" in the sense of Kareken, Muench and Wallace (1973), such as central bank liabilities, money market interest rates or the exchange rate to infer the values of the unobservable variables such as output and inflation.

The pioneering study linking the choice of the desirable monetary policy instrument to currently observable information variables has been provided by Poole (1970). On the basis of a stochastic variant of the Keynesian IS-LM model, Poole compares a money supply rule with an interest rate rule and evaluates the outcomes in terms of the resulting levels of output variability. He shows that an interest rate rule is superior to a money supply rule whenever shocks originate primarily in the money markets, whereas a money supply rule delivers better results in an environment in which output shocks predominate. The Poole model not only inspired the theoretical literature on central bank targeting procedures (see Friedman, 1990, or Walsh, 2003), but has had a substantial impact on real-world monetary policy making as well. The increasing instability in money demand due to financial innovation in the 1970s and 1980s led many central banks to abandon money supply rules in favour of interest rate targets. By the same token, the Bundesbank maintained money supply targeting by referring explicitly to the stability of the German money demand. Even today, the two-pillar strategy of the European Central Bank with its particular focus on the money supply is often justified by referring to empirical studies.
pointing towards the apparent stability of European velocity.

The major extensions of the basic Poole model include Canzoneri et al. (1983), who rework the closed-economy Poole analysis within a rational-expectations model in which economic agents base their actions on the same imperfect information with respect to the incidence of shocks as does the central bank. They show that the results of the original Poole analysis with respect to liquidity as well as output demand shocks continue to hold. However, their model also allows for output supply shocks and these turn out to induce an ambiguity as to the relative performance of money supply and interest rate targets. An interest rate rule (money supply rule) is superior whenever the variability of output (inflation) is considered to be particularly costly. Roper and Turnovsky (1980) redo the Poole analysis within an expectations-augmented small-open-economy version of the stochastic IS-LM model with perfect capital mobility. In their model, the relevant monetary policy choice centres around a money supply rule and an exchange rate rule. In close analogy to the Poole model, an exchange rate target perfectly stabilises domestic output when the shocks originate in the domestic money market, whereas a money supply rule delivers better results in the alternative scenario in which shocks arise in the domestic output market. However, if shocks originate abroad, the case of targeting the money supply or the exchange rate become less clear-cut and the preferred policy turns out to be a dirty float.

Only very few studies have so far attempted to investigate Poole-type scenarios within modern general equilibrium models. Exceptions include Carlstrom and Fuerst (1995), who study a limited participation model of the closed economy with supply and demand shocks but without liquidity shocks, and Ireland (2000) and Galì (2002), who use the NNS model. However, Ireland incorporates no fiscal shocks while Galì analyses no liquidity shocks. Collard and Dellas (2005) analyse all three kinds of shocks in a model of the closed economy with staggered prices and capital accumulation in order to examine the properties of alternative targeting procedures. In their model, the original Poole results concerning the output stabilisation properties of money supply and interest rate targeting obtain when the degree of intertemporal substitution is low. Yet these output volatility rankings do not carry over to similar welfare rankings. For fiscal shocks, money supply targeting fares better for low and worse for high degrees of intertemporal substitution whereas the opposite pattern obtains for supply shocks. In line with the results of Roper and Turnovsky (1980), Parrado and Velasco (2002) show within the small-open-economy context that the optimal exchange rate policy turns out to be a dirty float and they rationalise their findings with the fear of floating phenomenon ascribed to a broad range of developing countries. However, the authors restrict their attention to an interest rate rule and allow the monetary authorities to respond contemporaneously to the shocks of the model, so their analysis does not fully conform to the Poole setup.
In NNS models, monetary policy can be geared towards neutralising the effects of nominal rigidities and can thus be utilised to replicate the flexible price equilibrium. In open-economy versions of the model with producer currency pricing, this usually requires a floating exchange rate in order to grant monetary policy the necessary flexibility required to achieve that goal (e.g. Obstfeld and Rogoff, 2000, 2002a as well as Gali and Monacelli, 2005). Under local-currency pricing, a fixed exchange rate may or may not deliver superior results (Devereux and Engel, 2003, Obstfeld, 2004). These results are derived under the assumption that the monetary authority can observe and react to shocks or macroeconomic aggregates contemporaneously. However, as outlined above, central banks do not have timely information about the underlying nature of the shocks, and observations on target variables such as real activity and inflation are obtained less frequently than data on interest rates, monetary aggregates or exchange rates. In such an environment, how should the central bank reach its policy goals such as stabilising inflation, output or the exchange rate? Should it attempt to target the short-run nominal interest rate or should it maintain a measure of the money supply on a target path? Which of the two policy instruments provides higher welfare for the economy and what is the implication for the country’s exchange rate regime? This paper aims to answer these questions by considering Poole-type policy rules, which are not optimal in the sense of the perfect-information literature. In a world of imperfect information, monetary policy will be unable to replicate the flexible price solution, and the derivation of optimal policy is not necessary for the evaluation of simpler and practically more important targeting procedures such as the money supply, the interest rate or the exchange rate.

This paper conducts the Poole analysis in a general two-country open-economy context of the New Open Economy Macroeconomic framework for a large economy and a small open economy. We consider liquidity shocks, fiscal shocks and supply shocks in each of the two countries, where all shocks are assumed to be contemporaneously unobservable to the central bank and the public alike. Within this environment, we study Poole-type policy rules in the form of pure money supply and interest rate rules. The remainder of the paper is structured as follows: Section 2 lays out the model and presents its closed form solution. Section 3 conducts the Poole analysis in terms of the level and volatility effects for consumption, output and the price level as well as for the resulting welfare implications. A final section concludes.

2 The model

We utilise a stochastic two-economy version of the New Open Economy Macroeconomic model pioneered by Obstfeld and Rogoff (1995, 2002b). In each country, consumers maximise utility,
producers maximise profits, and the government redistributes income by printing money and handing out lump-sum transfers to consumers.

2.1 Budget constraint and asset markets

The budget constraint for the domestic economy is given by

\[ M_t + B_t + \varepsilon_t B_t^* = M_{t-1} + (1 + \delta_{t-1})B_{t-1} + (1 + \delta^*_t)\varepsilon_t B_{t-1}^* + W_t L_t + \Pi_t - P_t C_t - P_{H,t} T_t, \]

where \( M, B, B^*, L, \Pi, C \) and \( T \) denote the money supply, the domestic and foreign bonds outstanding, labour supply, firms’ profits, consumption and taxes. The domestic and foreign nominal interest rates are reflected by \( i \) and \( i^* \), while \( \varepsilon \) equals the nominal exchange rate, denoted as the domestic currency price of foreign exchange, and \( W \) and \( P \) represent the nominal wage rate and the price index, respectively. Taxes are payable in terms of domestic goods, so the relevant price index \( P_H \) refers to home goods only. Finally, the time index is denoted by a lower-case \( t \). A similar condition holds for the foreign economy.

2.2 Preferences and households’ choice

It is assumed that the agents in the home country, \( H \), and the foreign country, \( F \), produce traded goods only. Home agents are indexed by numbers in the interval \([0, 1]\) and foreign agents reside on \([0, P^*] \). The population size of the foreign country corresponds simply to \( P^* \). The agents in the two economies consume a basket consisting of home and foreign produced goods and the utility function of the representative household takes on the following form:

\[ U_t = E_t \left\{ \sum_{s=t}^{\infty} (1 + \delta)^{-(s-t)} \left[ \frac{C_t^{1-\rho}}{1-\rho} + \frac{\chi}{P_t} \left( \frac{M_t}{P_t} \right)^{1-\epsilon} - \frac{\kappa L_t^{\nu}}{\nu} \right] \right\}, \]

where \( \rho > 0 \) equates to the parameter of relative risk aversion and \( \delta \) to the rate of time preference. The parameter \( \kappa \) can be seen as a random shift in the marginal disutility of work effort or simply as an inversely related productivity shock. A positive productivity shock, a fall in \( \kappa \), allows the household to produce more in a given amount of time.\(^1\) The elasticity of marginal disutility from work effort equals \( \nu - 1 \), where \( \nu \geq 1 \). If \( \nu > 1 \), the labour supply schedule is downward sloping. A rise in \( \nu \) makes the labour supply more inelastic. The parameter \( \chi \) represents a random shift in the demand for real balances. An increase in \( \chi \) corresponds to a rise in the the demand for real balances. Real balances are denoted by \( \frac{M_t}{P_t} \), where the home price index equals \( P = P_H^\alpha P_F^{1-\alpha} \), with \( P_J = P_{J,1}^\alpha P_{J,2}^{1-\alpha} \) and \( J = H, F \). We assume a Calvo (1983) pricing rule in which the shares

\(^1\)Thus, the variable \( L(i) \) denotes efficient labour rather than the hours worked.
of flexible and fixed price firms are given by $\alpha$ and $1 - \alpha$, respectively, with $0 < \alpha \leq 1$. The price indices for the composite goods are defined as

$$P_{H,1} = \left( \frac{\int_0^\alpha P_H(z)^{\theta - 1} dz}{\alpha} \right)^{\frac{1}{\theta}}, \quad P_{H,2} = \left( \frac{\int_0^1 P_H(z)^{\theta - 1} dz}{1 - \alpha} \right)^{\frac{1}{\theta}},$$

$$P_{F,1} = \left( \frac{\int_0^{\alpha P^*} P_{F,1}(z)^{1 - \theta} dz}{\alpha P^*} \right)^{\frac{1}{1 - \theta}}, \quad P_{F,2} = \left( \frac{\int_0^{P^*} P_{F,2}(z)^{1 - \theta} dz}{P^* (1 - \alpha)} \right)^{\frac{1}{1 - \theta}},$$

where the elasticity of substitution between any two heterogeneous goods $z$ is reflected by $\theta > 1$. Total labour effort $L$ is given by labour effort provided to both, the sector producing the continuum of flexible price goods $C_{H,1}(i, z)$ and $P^* C_{H,1}^*(i, z)$ as well as to the sector producing the continuum of fixed price goods $C_{H,2}(i, z)$ and $P^* C_{H,2}^*(i, z)$. In both sectors the government consumes an equal amount of flexible in fixed price goods, $G_{H,1}(i, z)$ and $G_{H,2}(i, z)$ respectively.

The resource constraint is then given by

$$Y_{H,1}(z) = \int_0^1 (C_{H,1}(i, z) + G_{H,1}(i, z)) \, di + \int_0^{P^*} C_{H,1}^*(i, z) \, di \quad \text{and} \quad (3)$$

$$Y_{H,2}(z) = \int_0^1 (C_{H,2}(i, z) + G_{H,2}(i, z)) \, di + \int_0^{P^*} C_{H,2}^*(i, z) \, di, \quad \text{whereby}$$

$$L(i) = \int_0^\alpha L_{H,1}(z) \, dz + \int_0^1 L_{H,2}(z) \, dz, \quad Y_{H,1}(z) = L_{H,1}(z), \quad Y_{H,2}(z) = L_{H,2}(z).$$

The resource constraint in the foreign country takes on a similar form. Ignoring the subscript $i$, the consumption index at home can be written as follows: $C_J^n C_{F}^{1 - n} / (n^n (1 - n)^{1 - n})$, where $C_J = C_J^\alpha C_J^{1 - \alpha} / (\alpha^n (1 - \alpha)^{1 - \alpha})$, with

$$C(i)_{H,1} = \left( \frac{\int_0^\alpha C_{H,1}(i, z)^{\frac{1}{\theta - 1}} dz}{\left( \frac{1}{\alpha} \right)^{\frac{1}{\theta - 1}}} \right)^{\frac{1}{\theta}}, \quad C(i)_{H,2} = \left( \frac{\int_0^1 C_{H,1}(i, z)^{\frac{1}{\theta - 1}} dz}{\left( \frac{1}{1 - \alpha} \right)^{\frac{1}{\theta - 1}}} \right)^{\frac{1}{\theta}},$$

$$C(i)_{F,1} = \left( \frac{\int_0^{\alpha P^*} C_{F,1}(i, z)^{\frac{1}{1 - \theta}} dz}{\left( \frac{1}{\alpha P^*} \right)^{\frac{1}{1 - \theta}}} \right)^{\frac{1}{1 - \theta}}, \quad C(i)_{F,2} = \left( \frac{\int_0^{P^*} C_{F,2}(i, z)^{\frac{1}{1 - \theta}} dz}{\left( \frac{1}{P^*} \right)^{\frac{1}{1 - \theta}}} \right)^{\frac{1}{1 - \theta}},$$

and $n = 1 - (1 - P) \tau$. In line with the work by Sutherland (2005), the parameter $0 \leq n < 1$ measures the overall share of home goods in the home consumption basket while the share of the home population in the world population equals $P = 1/(1 + P^*)$. The degree of trade openness is measured by the parameter $0 < \tau \leq 1$. Foreign agents (denoted by $\ast$) have identical preferences, except that $\tilde{\kappa}^\ast$ and $L^\ast$ may differ from $\tilde{\kappa}$ and $L$. It is assumed that foreign agents hold their own money, $M^\ast$, deflated by their general price level, $P^\ast = P_F^{\ast n^\ast} P_H^{1 - n^\ast}$, with $n^\ast = 1 - P \tau$. The foreign consumption and price indices are similar to the ones for the home country.
The conditional commodity demand functions in the home and foreign country are derived by minimising the expenditure for the composite goods $z$ and are given by

$$C_{H,1}(i, z) = n \left( \frac{P_{H,1}(z)}{P_{H,1}} \right)^{-\theta} CP^{P_{H,1}}$$
$$C_{H,2}(i, z) = n \left( \frac{P_{H,2}(z)}{P_{H,2}} \right)^{-\theta} CP^{P_{H,2}} \tag{6}$$

$$C_{F,1}(i, z) = (1 - n) \left( \frac{P_{F,1}(z)}{P_{F,1}} \right)^{\theta} CP^{P_{F,1}}$$
$$C_{F,2}(i, z) = (1 - n) \left( \frac{P_{F,2}(z)}{P_{F,2}} \right)^{\theta} CP^{P_{F,2}} \tag{7}$$

and similarly for the foreign economy. From the objective function, (2), and the budget constraint the following first order conditions can be derived. The money demand at home equates to

$$\left( \frac{M_t}{P_t} \right)^{\epsilon} = \chi C_t^{\rho} \frac{1 + i_t}{\bar{i}_t}, \tag{8}$$

with analogous expressions for the foreign economy. The money demand relates the desired real balances to the variable on which transactions are based, $C_t$. Real balances, $\frac{M_t}{P_t}$, increase in the level of consumption, $C_t$, but also depend negatively on the nominal interest rate, $i_t$. A rise in $\chi$ leads to an increase in real balances. The size of this effect depends on the interest elasticity of money demand, $\rho$.

Given the portfolio choices, consumption at home and abroad evolves according to

$$\frac{1}{P_t C_t^{\rho}} = \left( \frac{1 + i_t}{1 + \delta} \right) E_t \left( \frac{1}{P_{t+1} C_{t+1}^{\rho}} \right), \quad \frac{\varepsilon_t}{P_t C_t^{\rho}} = \left( \frac{1 + \bar{i}_t}{1 + \delta} \right) E_t \left( \frac{\varepsilon_{t+1}}{P_{t+1} C_{t+1}^{\rho}} \right), \quad \text{and} \quad \frac{1}{P_t C_t^{\rho}} = \left( \frac{1 + i_t}{1 + \delta} \right) E_t \left( \frac{1}{P_{t+1} C_{t+1}^{\rho}} \right), \quad \frac{\varepsilon_t}{P_t C_t^{\rho}} = \left( \frac{1 + \bar{i}_t}{1 + \delta} \right) E_t \left( \frac{1}{P_{t+1} C_{t+1}^{\rho}} \right), \tag{9}$$

respectively. The Euler equations (9) and (10) illustrate that if the aggregate price level (nominal interest rate) is currently low relative to future values, present consumption will be preferred over future consumption.

### 2.3 Fiscal policy

Government spending $G_H$ is modelled as constant shares of the local flexible and fixed price products $z$ given by $G_{H,1} = \alpha G_H$ and $G_{H,2} = (1 - \alpha) G_H$, and the (per-capita) government real consumption index is analogous to equation (4). The home government demand for good $z$ takes on the following form:

$$G_{H,1}(i, z) = \frac{1}{\alpha} \left( \frac{P_{H,1}(z)}{P_{H,1}} \right)^{-\theta} G_{H,1}, \quad G_{H,2}(i, z) = \frac{1}{1 - \alpha} \left( \frac{P_{H,2}(z)}{P_{H,2}} \right)^{-\theta} G_{H,2}.$$

Similar relationships hold for the foreign country. The home government finances its spending by means of taxes and seigniorage, according to its budget constraint:
\[ G_H = T_H + \frac{M_t - M_t-1}{P_{H,t}}. \]

It is assumed that total government expenditure \( G_H \) is a stochastic proportion of overall domestic output \( Y_H \), so that \( Y_H - G_H = Y_H \exp(-\gamma) \), whereby \( \gamma \) denotes the stochastic (government) demand shock. A similar expression holds for the foreign country.

### 2.4 Consumption allocation and market clearing

Given the allocation of consumption across brands of equations (6) and (7), the overall demand for the home produced flexible price good \( z \) then equals

\[
Y^d_{H,1}(z) = n \left( \frac{P_{H,1}(z)}{P_{H,1}} \right)^{\theta} CP \frac{P^* (1 - n^*)}{P_{H,1}^*} \left( \frac{P_{H,1}(z)}{P^*_{H,1}} \right)^{-\theta} \frac{C^* P^*}{P^*_{H,1}} + G_{H,1}(i, z). \tag{11}
\]

For the fixed price producers it follows that

\[
Y^d_{H,2}(z) = n \left( \frac{P_{H,2}(z)}{P_{H,2}} \right)^{\theta} CP \frac{P^* (1 - n^*)}{P_{H,2}^*} \left( \frac{P_{H,2}(z)}{P^*_{H,2}} \right)^{-\theta} \frac{C^* P^*}{P^*_{H,2}} + G_{H,2}(i, z). \tag{12}
\]

with similar expressions for the foreign country. The law of one price (LOOP) holds, so that \( P_H(z) = \varepsilon P^*_H(z) \) and \( P_F(z) = \varepsilon P^*_F(z) \). Then equation (11) rearranges to

\[
Y^d_{H,1}(z) - G_{H,1}(i, z) = \left( \frac{P_{H,1}(z)}{P_{H,1}} \right)^{\theta} \frac{P}{P_{H,1}} \left( nC + P^* (1 - n^*) \frac{C^* \varepsilon P^*}{P} \right), \tag{13}
\]

and similarly for the fixed price producers. Analogous expressions obtain for the foreign country.

In equilibrium, \( P_{H,1}(z) = P_{H,1} \) and \( P_{H,2}(z) = P_{H,2} \). Therefore, it holds that \( C_{H,1}(i, z) = C_{H,2}(i, z) = C_{F,1}(i, z) = C_{F,2}(i, z) = \frac{C_{H,1}}{\alpha^{1-\alpha}}, \frac{C_{H,2}}{\alpha^{1-\alpha}}, \frac{C_{F,1}}{\alpha^{1-\alpha}}, \frac{C_{F,2}}{\alpha^{1-\alpha}} \), and as well as \( G_{H,1}(i, z) = \frac{G_{H,1}}{\alpha} \) and \( G_{H,2}(i, z) = \frac{G_{H,2}}{\alpha} \). Thus, the resource constraints at home, equation (3), and abroad allow to express the domestic output in the flexible and fix price sectors by the following equation:

\[
Y_{H,1} - \frac{G_{H,1}}{\alpha} = \left( n \frac{CP}{P_{H,1}} + P^* (1 - n^*) \frac{P^* C^*}{P_{H,1}} \right), \quad Y_{H,2} - \frac{G_{H,2}}{1-\alpha} = \left( n \frac{CP}{P_{H,2}} + P^* (1 - n^*) \frac{P^* C^*}{P_{H,2}} \right). \tag{14}
\]

It follows that that the revenue from producing flexible and fixed price goods equals

\[
Revenue = \Pi + WLH = nPC + P^* (1 - n^*) \varepsilon P^* C^* + P_H G_H. \tag{15}
\]

where overall profits \( \Pi \) is the sum of profits of the flex-price firms (\( \pi_1 \)) and fixed-price firms (\( \pi_2 \)).

Given isoclastic preferences over total consumption \( C \), countries always consume exactly their real incomes (cf. Corsetti and Pesenti, 2001) when the initial asset holdings \( B_0 = B^*_0 \) equate
to zero. It follows that for all $t \geq 0$ the equilibrium conditions are solved by the allocation $B_t = B_t^* = 0$. Then from the budget constraint and the condition that $G_{H,t} = T_{H,t} + \frac{M_t - M_{t-1}}{P_{H,t}}$ in equilibrium the real value of overall production equates to
\[
Y_H = \frac{CP}{P_H} \exp(\gamma) = L. \tag{16}
\]

A similar expression holds for the foreign country. Equation (16) induces a zero current account at all times. Thus, in equilibrium nominal spending is equalised in the two countries: $PC = nPC + P^*(1 - n^*) L^* C^*$. This implies that equation (14) can be written as $Y_{H,1} = \frac{CP}{P_{H,1}} + \frac{G_{H,1}}{\alpha} = \frac{CP\exp(\gamma)}{P_{H,1}}$ and $Y_{H,2} = \frac{CP}{P_{H,2}} + \frac{G_{H,2}}{1 - \alpha} = \frac{CP\exp(\gamma)}{P_{H,2}}$. A starred version holds for the foreign economy.

### 2.5 Profits and prices

It is assumed that firms do not price discriminate in foreign markets so that they only choose one price. Firms in the flexible price sector set their price every period after shocks are realised. Flexible prices at home and abroad equate to
\[
P_{H,1} = \frac{\theta}{\theta - 1} \frac{L^{u-1} P C^\rho}{\rho}, \quad P_{F,1}^* = \frac{\theta}{\theta - 1} \frac{L^{u-1} P^* C^*}{\rho}, \tag{17}
\]
whereby the consumption leisure trade-off, $\frac{W_L}{P_H} = \frac{L^{u-1}}{C^\rho}$, has been utilised. The fixed price firms supply their goods in a market where prices are set in advance of the realisation of shocks. It follows that they meet the demand at the pre-set price. Thus, when $P_{J,2}$ is chosen, exogenous changes realised at $t - 1$ or earlier of the economic environment are known, but not disturbances realised in period $t$. Thus, conditional on the information at $t - 1$ the firms maximise expected profits, $E_{-1}(\lambda \pi_2) = E_{-1} \left( (PC^\rho)^{-1} (P_{H,2}(z) Y_{H,2}(z) - WL_{H,2}) \right)$, where $\lambda$ equates to the appropriate discount factor $\frac{CP}{C^\rho}$, the households’ marginal utility $U'(C)$. Also, the expectations operator $E_{t-1} = E_{-1}$ has been used since prices $P_{H,2}$ are set every period. It follows that
\[
P_{H,2} = \frac{\theta}{\theta - 1} \frac{E_{-1}(\exp(\gamma) WC^{1-\rho})}{E_{-1}(\exp(\gamma) C^{1-\rho})}. \tag{18}
\]

Utilising the consumption leisure trade-off, equation (18) and the foreign counterpart become
\[
P_{H,2} = \frac{\theta}{\theta - 1} \frac{E_{-1}(\exp(\gamma) \frac{L^{u-1}}{C^\rho})}{E_{-1}(\exp(\gamma) \frac{L^{u-1}}{C^\rho})}, \quad P_{F,2}^* = \frac{\theta}{\theta - 1} \frac{E_{-1}(\exp(\gamma) \frac{L^{u-1}}{C^\rho})}{E_{-1}(\exp(\gamma) \frac{L^{u-1}}{C^\rho})}. \tag{19}
\]

Defining $\exp(\gamma) CP = AD$ as aggregate demand, the price equation can be interpreted as follows: The marginal gain from a small reduction in price, $E_{-1}(ADU'(C)) (\theta - 1)$, has to equal the expected utility cost from higher work effort, $\frac{E_{-1}(ADU'(C))}{P_{H,2}} \theta$. Thus, fixed price firms will set
lower prices when the marginal utility of consumption is high, as consumption is low or when the disutility of work effort \( \kappa L^{\nu-1} \) is low, as \( \kappa \) is low.

2.6 Monetary policy

We consider Poole-type monetary policy rules, in which the domestic and foreign central banks target a combination of their respective money supplies and interest rates. In particular, we assume that the national money supplies are the actual policy instruments, adjusted in response to interest-rate movements (cf. Canzoneri, Henderson and Rogoff, 1983, McCallum, 1986). In log, defining \( \log(X) = x \), the policy rule is given by

\[
m_t = m + \Phi (i_t - E_{-1} (i_t)),
\]

where \( E_{-1} (i_t) \) is the expected value of the nominal interest rate \( i_t \) and \( m \) corresponds to the initial (or steady state) value of the domestic money supply. The money supply path of future values of money \( m_s, s \geq t \) is specified by \( E_t (m_s) = m \). Thus, the average value of \( m_t \) is tied down by \( m \). An interest rate operating target involves letting \( \Phi \) go to infinity. In this case the monetary authority adjusts the money supply growth rate, \( m_t - m \), in response to deviations of the nominal interest rate from its expected value. In contrast, if \( \Phi \) is zero, we have the case of a monetary (base) target in which money supply is not adjusted in response to interest rate movements. Thus, the policy reaction functions can be utilised to model pure money supply rules by setting the adjustment parameters \( \Phi \) to zero, or pure interest rate rules by letting the adjustment parameters go to infinity. The constant term \( m \) assures price-level determinacy for any realisations of the adjustment parameters (cf. Walsh, 2003, and Obstfeld and Rogoff, 1996, as well as Appendix B). For the foreign country a similar monetary policy rule exists except that \( \Phi^* \) can differ from \( \Phi \).

2.7 Log-linear versions

In order to solve the model in closed form, we go on to derive its log-linear version.\(^2\) Given the assumption of a log-linear distribution the domestic Euler consumption equation follows from (9),

\[
\rho (E_t c_{t+1} - c_t) = i_t - \delta - (E_t p_{t+1} - p_t) + \frac{1}{2} (\rho^2 \sigma_c^2 + \sigma_p^2 + 2 \rho \sigma_c p).
\]

The home consumption Euler equation for foreign bonds is derived as

\[2\]The random vector \( \bar{X} = (X_1, ..., X_N) \) is normally distributed with a mean vector \( \bar{u} \) and a variance-covariance matrix \( \Sigma \). Then, for a moment generating function, \( G_X(l) \), a multinormal distribution of the form \( G_X(l) = E(\exp(lX)) = \exp(l\cdot \bar{u} + \frac{1}{2} l' \cdot \Sigma \cdot l) \) is assumed which reflects the first and second moments of the model’s variables.
\[
\rho (E_t c_{t+1} - c_t) = i^*_t - \delta - (E_t e_{t+1} - e_t) - (E_t p_{t+1} - p_t) + \frac{1}{2} \left( \rho^2 \sigma^2_e + \sigma^2_e + \sigma^2_p - 2\rho \sigma_e \sigma_p + 2\rho \sigma_e \rho \sigma_p + 2\sigma_p \right).
\] (22)

The foreign consumption Euler equation follows from (10):

\[
\rho (E_t c^*_t - c^*_t) = i_t - \delta - (E_t e_{t+1} - e_t) - (E_t p^*_t - p^*_t) + \frac{1}{2} \left( \rho^2 \sigma^2_e + \sigma^2_e + \sigma^2_p - 2\rho \sigma_e \sigma_p + 2\rho \sigma_e \rho \sigma_p + 2\sigma_p \right).
\] (23)

and the foreign consumption Euler equation for the domestic bond equates to

\[
\rho (E_t c^*_t - c^*_t) = i_t - \delta - (E_t e_{t+1} - e_t) - (E_t p^*_t - p^*_t) + \frac{1}{2} \left( \rho^2 \sigma^2_e + \sigma^2_e + \sigma^2_p - 2\rho \sigma_e \sigma_p + 2\rho \sigma_e \rho \sigma_p + 2\sigma_p \right).
\] (24)

Equating (22) and (21) as well as (23) and (24) yields a version of the uncovered interest rate parity (UIP) condition:

\[
i_t - i^*_t = (E_t e_{t+1} - e_t) - \frac{1}{2} (\rho \sigma_e + \sigma_p + \rho \sigma_e \rho + \sigma_p).
\] (25)

An expected depreciation of the domestic currency implies that the domestic return must increase. Hence, UIP predicts that higher domestic interest rates are associated with an expected depreciation. However, equation (25) also includes a risk premium, \((\rho \sigma_e + \sigma_p + \rho \sigma_e \rho + \sigma_p)\), which does not simply reflect the reward for taking foreign currency positions. Rather, it is a reward for non-diversifiable risk. The risk premium would be negative if the covariance between consumption (prices) and the exchange rate is positive.

The log-linear versions of the money demand, equation (8) and its foreign counterpart, require an approximation around a non-stochastic initial level of the nominal interest rate, with solutions given by

\[
\epsilon (m_t - p_t) = \Delta + \chi - \rho \left[ E_t c_{t+1} - c_t - \frac{1}{2} \rho^2 \right] - \frac{1}{\delta} \left[ E_t p_{t+1} - p_t + \frac{\sigma^2_p}{2} + \rho \sigma_e \right] + \rho c_t,
\]

\[
\epsilon (m^*_t - p^*_t) = \Delta + \chi^* - \rho \left[ E_t c^*_t - c^*_t - \frac{1}{2} \rho^2 \right] - \frac{1}{\delta} \left[ E_t p^*_t - p^*_t + \frac{\sigma^2_p}{2} + \rho \sigma_e \rho \sigma_p \right] + \rho c^*_t,
\]

where \(\Delta = \log \left( \frac{1+\delta}{2} \right)\) and \(\delta = \frac{1-\beta}{\rho}\), with \(\beta\) equal to the subjective discount factor.

Utilising equation (19), the price of the domestic fixed price producer equates to

\[
p_{H2} = \Omega + E^{-1} \left( (v - 1) (l + p + \rho c) + (v - 1) \sigma_{d,1} + \sigma_{d,p} \right) + (v - 1) (\sigma_{1,p} + \sigma_{1,c}) + \sigma_{p,c} + \frac{1}{2} \left( \sigma^2_{d} + (v - 1)^2 \sigma^2_{p} + \rho (2 - \rho) \sigma^2_{d} + \sigma^2_{p} \right) + (1 - \alpha) \left( (v - 1) \sigma_{d,1} + \sigma_{d,p} + \rho \sigma_{d,c} \right).
\]
where \( \log(\tilde{\kappa}) = \kappa, \Omega = \log(\theta/(\theta - 1)) \) and the shock parameter have zero mean. The fixed price \( p_{H,2} \) increases with the variability of the productivity disturbance as well as with the volatility in overall prices, \( \sigma_p^2 \), consumption, \( \sigma_c^2 \), and labour supply, \( \sigma_L^2 \). Furthermore, the covariances between these variables determine the level of \( p_{H,2} \). A similar expression holds for the foreign fixed price producers.

Lastly, consider the disutility of work effort, \((\tilde{k}L^\nu)/v\). Equation (16) implies that the expected disutility of work effort equates to

\[
E_{-1}(\tilde{k}L^\nu) = \exp(\sigma_k^2 + v \left( (\sigma_k^2 + \sigma_p^2 + \sigma_{\mu}^2 + \sigma_c^2) / 2 + (\sigma_{k,c} + \sigma_{k,p} - \sigma_{k,\mu}) \right) + v^2 (\sigma_{c,p} + \sigma_{c,\gamma} - \sigma_{c,\mu} + \sigma_{p,\gamma} + \sigma_{p,\mu}) + vE_{-1}(c + p - p_H)).
\]

Thus, the expected disutility of work effort increases for example with the variability in consumption, \( \sigma_c^2 \), but also with the variance in prices, \( \sigma_p^2 + \sigma_{\mu}^2 \). The more variable are prices, the higher will be the utility cost of work effort.

Equations (21) to (26) depend on their expected values and second moments. In order to obtain these expressions it is necessary to solve the variables for their steady state realisation. This is done in the next section.

### 2.8 Equilibrium

We assume that shocks are temporary and that therefore the expectations of a variable are invariably the steady state. It follows that \( E_t(x_{t+1}) = x \) and similarly \( E_{t-1}(x_t) = x \), where variables without a time subscript denote the steady state.

**Definition 1.** A rational expectations equilibrium of the model outlined above is a set of sequences \( \{c_t, c_t^*, m_t, m_t^*, p_t, p_t^*, i_t, i_t^*, e_t, y_{H,t}, y_{F,t} \} \) satisfying the following equations at all dates \( t \geq 0 \):

<table>
<thead>
<tr>
<th>Equation</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \rho(c - c_t) = i_t - \delta - (p - p_t) + \frac{1}{2} \left( \rho^2 \sigma_c^2 + \sigma_p^2 + 2\rho\sigma_{cp} \right) )</td>
<td></td>
</tr>
<tr>
<td>( \rho(c^* - c_t^<em>) = i_t^</em> - \delta - (p^* - p_t^*) + \frac{1}{2} \left( \rho^2 \sigma_c^2 + \sigma_p^2 + 2\rho\sigma_{cp} \right) )</td>
<td></td>
</tr>
<tr>
<td>( \epsilon(m_t - p_t) = \Delta + x - \frac{2}{\delta} \left[ c - c_t - \frac{\rho_1}{\rho_2} \sigma_c^2 \right] - \frac{1}{\delta} \left( p - p_t + \frac{\sigma_p^2}{\delta} + \rho\sigma_{cp} \right) + \rho c_t )</td>
<td></td>
</tr>
<tr>
<td>( \epsilon(m_t^* - p_t^<em>) = \Delta + x^</em> - \frac{2}{\delta} \left[ c^* - c_t^* - \frac{\rho_1}{\rho_2} \sigma_c^2 \right] - \frac{1}{\delta} \left( p^* - p_t^* + \frac{\sigma_p^2}{\delta} + \rho\sigma_{cp} \right) + \rho c_t^* )</td>
<td></td>
</tr>
<tr>
<td>( i_t - i_t^* = (e - e_t) - \frac{1}{\rho} \left( \rho\sigma_{ce} + \sigma_{ep} + \rho\sigma_{c,e} + \sigma_{ep} \right) )</td>
<td></td>
</tr>
<tr>
<td>( c_t + p_t + p_{H,t} = y_{H,t} = \gamma )</td>
<td></td>
</tr>
<tr>
<td>( c_t^* + p_t^* + p_{F,t} = y_{F,t} = \gamma^* )</td>
<td></td>
</tr>
<tr>
<td>( p_t = n \cdot (p_{H,t} + (1 - n) \cdot (e_t + p_{F,t}) )</td>
<td></td>
</tr>
<tr>
<td>( p_t^* = n^* \cdot (p_{F,t}^* + (1 - n^*) \cdot (p_{H,t} - e_t) )</td>
<td></td>
</tr>
</tbody>
</table>
and the money supply rules, \( m_t - m = \Phi (i_t - i) \) and \( m^*_t - m^* = \Phi^* (i^*_t - i^*) \). The stochastic steady state is given by

\[
\begin{align*}
\epsilon (m - p) &= \Delta + \frac{\epsilon}{\delta} \left[ \frac{\epsilon}{\delta} \sigma^2_t - \frac{1}{\delta} \left( \frac{\epsilon}{\delta} \sigma^2_p + \rho \sigma c_p \right) + \rho c \\
\epsilon (m^* - p^*) &= \Delta + \frac{\epsilon}{\delta} \left[ \frac{\epsilon}{\delta} \sigma^2_t - \frac{1}{\delta} \left( \frac{\epsilon}{\delta} \sigma^2_p + \rho \sigma c_p \right) + \rho c^* \\
-% \frac{\epsilon}{\delta} (\rho \sigma c_e + \sigma c_p + \rho c^* \rho c + \sigma c^* p) &= \frac{1}{\delta} (\rho^2 \sigma^2_c + \sigma^2_p + 2 \rho \sigma c_p) - \frac{1}{\delta} (\rho^2 \sigma^2_c + \sigma^2_p + 2 \rho \sigma c_p)
\end{align*}
\]

The eleven equilibrium equations together with the solution for the steady state can be utilised to solve the model’s 11 endogenous variables \( \{c_t, c^*_t, m_t, m^*_t, p_t, p^*_t, i_t, i^*_t, c_t, y_{H,t}, y_{F,t} \} \) and the expected, i.e. steady state values, as functions of the exogenous disturbances. To do so we express the system of equations in form of deviations from steady state:

\[
\begin{align*}
\rho (c_t - c) &= -\alpha n (p_{H,t} - p_{H,1}) - \alpha (1 - n) (p_{F,t} - p_{F,1}) - (i_t - i) - (1 - n) \epsilon_t (c_t - c) \\
\rho (c^*_t - c^*) &= -\alpha n^* (p_{F,t}^* - p_{F,1}^*) - \alpha (1 - n^*) (p_{H,t} - p_{H,1}) - (i^*_t - i^*) + (1 - n^*) \epsilon_t (c^*_t - c^*) \\
\epsilon (m_t - m) &= \alpha + \frac{1+\delta}{\delta} \alpha n (p_{H,t} - p_{H,1}) - \frac{\alpha (1 - \delta)}{\delta} \epsilon_t (c_t - c) \\
&+ \frac{1+\delta}{\delta} \alpha (1 - n) (p_{F,t}^* - p_{F,1}^*) + \frac{1+\delta}{\delta} \epsilon_t (c_t - c) \\
\epsilon (m^*_t - m^*) &= \alpha^* + \frac{1+\delta}{\delta} \alpha^* n^* (p_{F,t}^* - p_{F,1}^*) - \frac{\alpha^* (1 - \delta)}{\delta} \epsilon_t (c^*_t - c^*) \\
&+ \frac{1+\delta}{\delta} \alpha^* (1 - n^*) (p_{H,t} - p_{H,1}) + \frac{1+\delta}{\delta} \epsilon_t (c^*_t - c^*) \\
\epsilon_t (c_t - c) &= (i_t - i) - (1 - i) \\
y_{H,t} - y_{H,1} &= \gamma - \alpha (1 - n) (p_{H,t} - p_{H,1}) + \alpha (1 - n) (p_{F,t} - p_{F,1}) \\
&+ (1 - n) (c_t - c) + (c_t - c) \\
y_{F,t}^* - y_{F,1} &= \gamma^* - \alpha (1 - n^*) (p_{F,t}^* - p_{F,1}^*) + \alpha (1 - n^*) (p_{H,t} - p_{H,1}) \\
&- (1 - n^*) (c^*_t - c^*) + (c^*_t - c^*) \\
\end{align*}
\]

The steady state solution and its deviations form can be used to compute the closed form solutions of the model for the two alternative simple monetary policy rules as suggested by Poole (see also Appendix B). The simple money supply rule is implemented by setting \( \Phi \) and \( \Phi^* \) to zero. It follows that \( m_t - m = 0 \) in the domestic country and \( m^*_t - m^* = 0 \) in the foreign country, whereas the simple interest rate rules follow by letting \( \Phi \) and \( \Phi^* \) go to infinity, implying that \( i_t - i = 0 \) and \( i^*_t - i^* = 0 \), respectively. The money supply rules can be evaluated by first setting the left-hand sides of the equilibrium money demand equations equal to zero and substituting one into the other. The resulting equations are then plugged back into the other equations of the system to obtain the reduced form. The interest rate rules are implemented by simply setting the interest rate differentials in the system to zero, which directly delivers the reduced form.
3 Results

The model can be used to study the small open-economy case from the perspective of the domestic economy and the large-economy case from the viewpoint of the foreign economy. The computations for the various scenarios are rather tedious, and the resulting expressions in themselves are not very illuminating (for details see Appendix A). Therefore we do not present explicit solutions, but instead discuss the various model scenarios on the basis of some calibration results. Unless indicated otherwise, we chose the following set of parameter values for the calibration: $\rho = 2$, $v = 1.5$, $\alpha = 0.5$, $\theta = 5$, $\delta = 0.05$ and $\epsilon = 5$. The particular choice of the parameter values is motivated in part by recourse to previous literature. Chari, Kehoe, and McGrattan (1998) assume that $v = 1.5$ when $\left( \frac{v - \kappa L}{v} \right)$ is the disutility of labour. Chari, Kehoe, and McGrattan (2002) parameterise the absolute value of the interest elasticity of money demand as $\frac{1}{\epsilon} = 0.39$, whereas Ireland (2001) provides empirical estimates for the interest elasticity in US data before and after 1979 and comes up with values of $\frac{1}{\epsilon} = 0.19$ and $\frac{1}{\epsilon} = 0.12$, respectively. In our calibration we set $\epsilon = 5$ at the conservative end of the empirical estimates. We follow other work in this literature, e.g. Devereux and Engel (2003), and set $\rho$ greater one. Finally, we maintain the assumption of the uncorrelatedness of the stochastic disturbances in the model and analyse the impact on the endogenous variables by setting the variances of the liquidity as well as the demand and supply shocks to unity one at a time.

We start with the case of the large foreign economy as the benchmark scenario as it lends itself most easily to a comparison with the results of the basic Poole model. This scenario is implemented by letting $P^*$ go to infinity, so that the restrictions $n = 1 - \tau$ and $n^* = 1$ hold. The results are summarised in Tables 1 and 2 for the scenarios of perfect price flexibility ($\alpha = 1$) and partial price rigidity ($\alpha = 0.5$), respectively. We then move on to consider the case of the small open economy, the results for which are presented in Tables 3 to 6. In what follows, the expression $\Delta x = x_t - x$ represents deviations from the steady state. Ex ante welfare can be expressed by equation (2) as

$$E_{-1} \left\{ \sum_{s=t}^{\infty} (1 + \delta)^{-(s-t)} U_s \right\} = \frac{E_{-1} (U)}{1 - \beta},$$

where

$$E_{-1} (U) = \frac{E_{-1} (C^{1-\rho})}{1 - \rho} + \frac{E_{-1} \left( \chi \left( \frac{M}{P} \right)^{1-\epsilon} \right)}{1 - \epsilon} - \frac{E_{-1} (\bar{\kappa} L^n)}{\nu}. \quad (27)$$

The welfare results provided below contain the case with real balances denoted by $U_{RB}$ and $U^*_{RB}$, and without real balances, $U$ and $U^*$. Thus, in the latter case the term $\frac{M}{P}$ is neglected.\(^3\)

\(^3\)For more details see Appendix C.
3.1 Large economy

In the flex-price scenario of Table 1, the model turns out to be dichotomous for both the money supply and interest rate rules.

Table 1: The large economy with perfect price flexibility

<table>
<thead>
<tr>
<th></th>
<th>money supply rule</th>
<th>interest rate rule</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \kappa^* )-shock</td>
<td>0</td>
<td>-0.2000</td>
</tr>
<tr>
<td>( \gamma^* )-shock</td>
<td>-0.2000</td>
<td>-0.4000</td>
</tr>
<tr>
<td>( \kappa^* )-shock</td>
<td>( \kappa^* )-shock</td>
<td>( \kappa^* )-shock</td>
</tr>
<tr>
<td>( \gamma^* )-shock</td>
<td>( \gamma^* )-shock</td>
<td>( \gamma^* )-shock</td>
</tr>
<tr>
<td>( \kappa^* )-shock</td>
<td>( \kappa^* )-shock</td>
<td>( \kappa^* )-shock</td>
</tr>
</tbody>
</table>

Notes: The share of flexible price firms equals \( \alpha = 1 \), \( U_{RB}^{*} \) and \( U^{*} \) denote foreign welfare with and without consideration of the real balance effect.

For clarity, we summarise our findings in a series of results. >From Table 1 the following results can be established:

**Result 1**  
*Under completely flexible prices*

a) Liquidity shocks do not impact any of the real variables,

b) Real shocks affect consumption and output under the different monetary policy rules equally,

c) In terms of welfare, an interest rate rule is preferable for liquidity shocks but performs worse for real shocks,

d) If the real balance effect on welfare is ignored, welfare is exactly identical under either the money supply or the interest rate rules.

A positive liquidity disturbance (\( \kappa^* \)-shock) is absorbed either by adjustments in the price level and the nominal interest rate under the money supply rule, or in the level of the money supply under an interest rate rule such that the real interest rate remains the same under either
monetary policy rules.\footnote{The real interest rate equates to \((1 + r_t) = (1 + i_t) / E_t \left( \frac{P_{t+1}}{P_t} \right)\), so that \((r_t - r) = (i_t - i) + (p_t - p)\).} Real shocks in form of a rise in government spending \((\gamma^*\text{-shock})\) or a negative supply disturbance \((\kappa^*\text{-shock})\) affect consumption and output levels, but they do so in a symmetric fashion as the real interest rate responds equally under either a money supply or an interest rate rule. Hence the choice of the monetary instrument is of no consequence for the resulting rates of change and levels of variability of the endogenous real variables as the only difference occurs in the adjustments of nominal variables. However, the welfare effects are identical across the two regimes only if the impact of real balances on welfare is ignored. Incorporating the latter, an interest rate rule raises overall welfare through its stabilising impact on real balances in the instance of liquidity shocks, but has the opposite effect in the case of real shocks.

Table 2 reports the results for the scenario of partial price rigidity.

<table>
<thead>
<tr>
<th></th>
<th>money supply rule</th>
<th>interest rate rule</th>
</tr>
</thead>
<tbody>
<tr>
<td>(</td>
<td>\Delta c^*</td>
<td>)</td>
</tr>
<tr>
<td>(</td>
<td>\Delta y_P^*</td>
<td>)</td>
</tr>
<tr>
<td>(</td>
<td>\Delta p^*</td>
<td>)</td>
</tr>
<tr>
<td>(</td>
<td>\Delta i^*</td>
<td>)</td>
</tr>
<tr>
<td>(</td>
<td>\Delta m^*</td>
<td>)</td>
</tr>
<tr>
<td>(\sigma^2_c)</td>
<td>.0001</td>
<td>.0766</td>
</tr>
<tr>
<td>(\sigma^2_{y_P})</td>
<td>.0001</td>
<td>.0572</td>
</tr>
<tr>
<td>(\sigma^2_p)</td>
<td>.0006</td>
<td>.1615</td>
</tr>
<tr>
<td>(U^*)</td>
<td>-1.6255</td>
<td>-1.9252</td>
</tr>
<tr>
<td>(U_{RB}^*)</td>
<td>-1.8652</td>
<td>-1.8458</td>
</tr>
</tbody>
</table>

Notes: The share of flexible price firms equals \(\alpha = .5\).

In this case, the dichotomy breaks down and the different economic disturbances have an impact on the variability of the real variables under the money supply and interest rate rules. The following results can be established:

**Result 2** Under sticky prices the traditional Poole results are replicated in the sense that

\(a)\) an interest rate rule delivers a lower level of output variability when liquidity shocks dominate.
b) a money supply rule fares better when demand shocks are the dominant source of disturbance,

c) supply shocks lead to relatively lower levels of output variability under an interest rate rule, whereas a money supply rule performs better in terms of price level variability.

Result 2a) follows from the fact that under a money supply rule, a rise in the demand for money induces a fall in the price level and an increase in the nominal interest rate. As the nominal interest rate effect is quantitatively stronger than the price level effect, the real interest rate rises. The higher real interest rate lowers consumption, and due to the need of market clearing, output declines as well. With an interest rate rule, however, the liquidity shock is perfectly neutralised by an adjustment in the money supply, and neither the price level nor the nominal or real interest rates are affected.

In response to a positive government spending shock, both monetary policy rules induce a rise in the real interest rate which crowds out household demand and drives a wedge between output and consumption such that the former rises and the latter falls. The crowding out becomes relatively stronger under the money supply rule as the rise in the nominal interest rate reinforces the real interest rate response, thus strengthening the consumption effect. With a negative supply shock, the fall in consumption aggravates the resulting output effect. Result 2b) and the first part of result 2c) follow directly from the above observations. The second part of result 2c) follows from the fact that an interest rate rule channels a bigger fraction of volatility onto the price level through its impact on the money supply. The stronger movements of the real interest rate under a money supply rule also implies the following result:

**Result 3** Under sticky prices the consumption variability is always smaller under an interest rate rule.

The stabilisation of domestic consumption arises because the interest rate rule mitigates the real interest rate response and therefore allows consumption to adjust more smoothly to shocks hitting the economy. Finally, the model lends itself to an explicit welfare analysis, which provide the following results:

**Result 4** The Poole results are also exactly replicated in that

a) an interest rate rule is preferable in terms of welfare in an environment of dominant liquidity shocks,

b) a money supply rule yields better welfare results when real shocks are the predominant source of economic disturbances.
Result 4a) follows immediately from Table 2 since the liquidity shock is completely absorbed under an interest rate rule. Consequently, relative welfare is higher under this particular policy rule. To understand result 4b), one needs to consider that the country’s welfare increases with a reduction of the variability in (expected) consumption and labour effort (equation (27)). The higher welfare losses under the interest rate rule with respect to real shocks can be explained as follows: The pegged nominal interest rate requires the price level to respond to real disturbances relatively more strongly to achieve the necessary changes in the real interest rate. As a consequence, the relatively stronger price movements cause a higher variability in expected domestic labour supply (equation (26)). The variability of expected labour supply increases the utility cost of work effort and outweighs the mitigated variability in consumption when prices are sticky, i.e. for any value of $0 < \alpha < 1$. Hence, relative welfare declines under an interest rate rule when the large economy is hit by real disturbances. These results obtain independently of whether or not real balances are included in the welfare function.

### 3.2 Small open economy

For the small open-economy the impact of domestic shocks is reported in Table 3.

<table>
<thead>
<tr>
<th>Table 3: The small open economy: domestic shocks</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>$\Delta c$</td>
</tr>
<tr>
<td>$\Delta y_H$</td>
</tr>
<tr>
<td>$\Delta p$</td>
</tr>
<tr>
<td>$\Delta i$</td>
</tr>
<tr>
<td>$\Delta m$</td>
</tr>
<tr>
<td>$\Delta e$</td>
</tr>
<tr>
<td>$\sigma^2_{c}$</td>
</tr>
<tr>
<td>$\sigma^2_{y_H}$</td>
</tr>
<tr>
<td>$\sigma^2_{p}$</td>
</tr>
<tr>
<td>$\sigma^2_{e}$</td>
</tr>
<tr>
<td>$U_{RB}$</td>
</tr>
</tbody>
</table>

Notes: The share of flexible price firms equals $\alpha = .5$ while the degree of trade openness is $\tau = .2$. 

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It turns out that for domestic shocks, all qualitative results of the large economy case go through here as well. This evidence is laid down in the following result:

**Result 5** The Poole results are also replicated for domestic shocks in a small open economy.

The transmission of foreign shocks on domestic variables is influenced by the flexibility of the nominal exchange rate. The exchange rate effect in turn depends on the adopted monetary policy rules both at home and abroad. Given the different responses of the nominal exchange rate, we report results for the domestic monetary policy rules separately for the scenarios of the foreign central bank pursuing a money supply rule (Table 4) and an interest rate rule (Table 5).

Table 4 reports the different domestic policy scenarios for the case in which the foreign economy adopts a money supply rule. In the first case the domestic country also implements a money supply rule. This scenario implies a dirty float, as the domestic interest rate always moves in the same direction as the foreign rate (compare Tables 2 and 4). In contrast, a domestic interest rate rule implies a clean float (in what follows referred to as clean float I).

<table>
<thead>
<tr>
<th></th>
<th>money supply rule (dirty float)</th>
<th>interest rate rule (clean float I)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta c$</td>
<td>-0.0022</td>
<td>-0.0275</td>
</tr>
<tr>
<td>$\Delta y_{H}$</td>
<td>0.015</td>
<td>0.0185</td>
</tr>
<tr>
<td>$\Delta p$</td>
<td>0.0037</td>
<td>0.0463</td>
</tr>
<tr>
<td>$\Delta i$</td>
<td>0.0007</td>
<td>0.0088</td>
</tr>
<tr>
<td>$\Delta m$</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$\sigma^2_c$</td>
<td>0.0000485</td>
<td>0.0008</td>
</tr>
<tr>
<td>$\sigma^2_{y_{H}}$</td>
<td>0.0000219</td>
<td>0.0003</td>
</tr>
<tr>
<td>$\sigma^2_p$</td>
<td>0.000054</td>
<td>0.0019</td>
</tr>
<tr>
<td>$\sigma^2_i$</td>
<td>0.0018</td>
<td>0.0009</td>
</tr>
<tr>
<td>$U$</td>
<td>-1.629411</td>
<td>-1.6120</td>
</tr>
<tr>
<td>$U_{RB}$</td>
<td>-1.7292</td>
<td>-1.7042</td>
</tr>
</tbody>
</table>

Notes: The share of flexible price firms equals $\alpha = .5$ while the degree of trade openness is $\tau = .2$

In Table 4 both foreign liquidity and real shocks shock lead to a rise in the foreign interest rate under this particular foreign monetary policy rule (compare Table 2). The rise in the foreign
interest rate depreciates the nominal and the real exchange rate, \((\Delta p^* + \Delta e - \Delta p)\), switching relative demand towards domestic goods, thus raising domestic output. The relative price effect requires a decline in domestic consumption which is brought about by a rise in the domestic (real) interest rate. The rise in the domestic nominal interest rate and the decline in consumption both contribute to lowering the domestic money demand so that an increase in the domestic price level is required to restore equilibrium in the money market.

Table 5 presents the domestic monetary policy options when the foreign economy adopts an interest rate rule. A domestic money supply rule induces the nominal exchange rate to fluctuate freely, resulting in an alternative scenario of a clean float (clean float II), whereas a domestic interest rate rule delivers a fixed exchange rate.

A foreign liquidity shock is completely absorbed under a foreign interest rate rule with no effect on either real or nominal domestic variables (Table 5).

<table>
<thead>
<tr>
<th>Table 5: Foreign shocks with an interest rate rule in the large economy</th>
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</thead>
<tbody>
<tr>
<td><strong>money supply rule (clean float II)</strong></td>
</tr>
<tr>
<td>(\Delta c)</td>
</tr>
<tr>
<td>(\Delta y_H)</td>
</tr>
<tr>
<td>(\Delta p)</td>
</tr>
<tr>
<td>(\Delta i)</td>
</tr>
<tr>
<td>(\Delta m)</td>
</tr>
<tr>
<td>(\sigma^2_{c})</td>
</tr>
<tr>
<td>(\sigma^2_{y_H})</td>
</tr>
<tr>
<td>(\sigma^2_{p})</td>
</tr>
<tr>
<td>(\sigma^2_{e})</td>
</tr>
<tr>
<td>(U)</td>
</tr>
<tr>
<td>(U_{RB})</td>
</tr>
</tbody>
</table>

Notes: The share of flexible price firms equals \(\alpha = 0.5\) while the degree of trade openness is \(\tau = 0.2\).

Foreign real shocks exert similar effects compared to the foreign money supply rule of Table 4. The only difference occurs with respect to the reaction of the nominal exchange rate, which now appreciates under a domestic money supply rule, but remains constant under a domestic interest rate rule. This latter effect is entirely due to the increase in the foreign money supply, which also induces a stronger price response in the domestic economy under an interest rate rule. Both
the exchange rate and the domestic price effects weaken the real exchange rate depreciation of the domestic economy and thus the quantitative impact of the foreign real shocks on domestic production and consumption.

Comparing Tables 4 and 5, a domestic interest rate rule again stabilises domestic consumption for any of the two foreign monetary policy rules. Consequently, Result 3 is also valid for the small open economy. With respect to the variability of domestic output and prices, a money supply rule is preferable in the case of foreign demand shocks, which is in line with the findings for the scenario of domestic shocks of Result 2b). With respect to foreign supply shocks, the following results obtain:

Result 6  
Foreign shocks have implications for monetary policy that differ from the traditional Poole results in that

a) a domestic money supply rule is preferable in terms of the resulting output variability for both foreign liquidity and real shocks independently of the monetary rule chosen by the foreign monetary authority,

b) there is no trade-off between the output and price level stabilisation goals for foreign supply shocks, and a money supply rule always delivers lower levels of variability.

Result 6a is due to the combined response of the nominal and real interest rates as well as the nominal exchange rate. The flexibility of the nominal exchange rate associated with a domestic money supply rule contributes to bringing about the requisite relative price responses in the wake of foreign shocks, thus mitigating the absolute price level variability under the domestic money supply rule. At the same time, the amplified response of the nominal and real interest rates induces consumption to decline more strongly in the wake of an adverse supply shock than under an interest rate rule. Together with the mitigated price response, this causes domestic output to react more smoothly to supply disturbances under a domestic money supply rule, so that result 6b immediately follows.

With respect to welfare the following results are established:

Result 7  
In the small open economy relative welfare

a) improves under an interest rate rule when real shocks are considered and the real balance effect is neglected,

b) improves only when the domestic monetary authority adopts the same policy rule as the foreign economy when the real balance effect is factored in,

c) is at least as high as under a money supply rule compared to an interest rate rule when foreign liquidity shocks are considered.
Neglecting real balances in the utility function induces a dominance of an interest rate rule over a money supply rule for the small open economy when foreign real shocks are assessed. This result obtains independently of whether the foreign economy adopts a money supply rule or an interest rate rule. The relative higher welfare with respect to foreign real disturbances under an interest rate rule occurs because domestic consumption is stabilised under this particular policy rule.

If the real balance effect is included in the welfare analysis, the domestic monetary authority always improves welfare when it adopts the same monetary policy strategy as the foreign economy as such a strategy stabilises the domestic real money supply (compare $\Delta m - \Delta p$ and the resulting levels of variability in Tables 4 and 5).

Regardless of the real balance effect, foreign liquidity shocks tilt the balance towards adopting a money supply rule over an interest rate rule for the small open economy when the foreign monetary authority does not neutralise the foreign money market disturbance (see Table 4). This result is due to fact that the relatively stronger price movements under the interest rate rule induce the variability of expected domestic labour supply to increase. This raises the utility cost of work effort, which outweighs the mitigated variability in consumption. In summary, the choice of the monetary policy instruments is exactly reversed when foreign shocks are considered and real balances are neglected from the welfare evaluation. When real balances are factored in, the small open economy should always adopt the same monetary policy rule as the large economy.

How important are the welfare differences between the resulting exchange rate regimes? To answer this question, Table 6 gives a measure of the relative benefits of the adopted exchange rate regimes.

<table>
<thead>
<tr>
<th>Table 6: Consumption Costs</th>
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<tr>
<td></td>
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<tr>
<td>Cons.Cost(dirty float)</td>
</tr>
<tr>
<td>Cons.Cost$_{RB}$(dirty float)</td>
</tr>
<tr>
<td>Cons.Cost(clean float I)</td>
</tr>
<tr>
<td>Cons.Cost$_{RB}$(clean float I)</td>
</tr>
<tr>
<td>Cons.Cost(clean float II)</td>
</tr>
<tr>
<td>Cons.Cost$_{RB}$(clean float II)</td>
</tr>
<tr>
<td>Cons.Cost(fixed exchange rate)</td>
</tr>
<tr>
<td>Cons.Cost$_{RB}$(fixed exchange rate)</td>
</tr>
</tbody>
</table>

Notes: The share of flexible price firms equals $\alpha = .5$, the degree of openness is $\tau = .2$. Values of .0000 or -.0000 indicate very small numbers.
More precisely, Table 6 reports the resulting consumption costs, \( Cons.Cost \), in terms of the different combinations of the adopted monetary policy rules of Tables 4 and 5, where the dirty-float scenario of simultaneous money supply rules in both countries are taken as the reference case. We follow Schmitt-Grohé and Uribe (2004) and characterise \( Cons.Cost \) as the fraction of annuity consumption that the households would be willing to give up in order to make them indifferent between the different exchange rate regimes (see Appendix D).

In terms of a foreign demand shock in the absence of a real balance effect, households would be willing to give up 0.07 percentage points of their annuity consumption to be in the clean float regime I relative to the dirty float. In contrast, when real balances are factored in, households must be offered a positive fraction of annuity consumption of 1.83 percentage points to be as well off under the clean float as under the dirty float regime.

The insights of these consumption cost comparisons are summarised as follows: A foreign interest rate rule completely neutralises foreign liquidity shocks. Consequently, the clean float II and the fixed exchange rate regime dominate the alternative scenarios of Table 4. Furthermore, with respect to the other foreign disturbances the following results emerge:

**Result 8** When real shocks are the major source of disturbance, then in terms of consumption costs

\( a) \) a clean float I is preferable if real balances are neglected,

\( b) \) a dirty float becomes the preferred exchange rate regime for demand shocks if real balances are included, whereas

\( c) \) a fixed exchange rate dominates the other exchange regimes under the inclusion of real balances when supply shocks dominate.

Result 8\( a) \) is due to the interplay of the insulating role of the exchange rate in smoothing the impact of foreign real shocks on the domestic economy and the property that a domestic interest rate rule stabilises domestic consumption. When real balances are included, the interaction of the variability of consumption and real domestic money supply is of importance. Results 8\( b) \) and 8\( c) \) follow from the combined stabilisation of the domestic real money supply and consumption in these regimes (compare Tables 4 and 5).

### 3.3 Sensitivity analysis

The above results are derived on the basis of a particular parameterisation of the model. Although the choice of the parameters appears well specified in light of the related literature, we
have nevertheless checked the robustness of our results by modifying all parameters in the calibration exercise. For wide ranges of parameter values, the results change quantitatively but not qualitatively.

In particular, all qualitative results obtain for any positive realisation of the rate of time preference $\delta$. The same applies for the elasticity of substitution $\theta$ between the different goods $z$ and the interest elasticity of money demand $\epsilon$ for any values above unity. Neither does the parameter of relative risk aversion $\rho$ play any role in driving the qualitative results. For realisations of $\rho$ below unity, welfare assume positive values throughout, but the welfare rankings remain unaffected.

The elasticity of marginal disutility from work effort $v - 1$ has been calibrated for different values above zero without in any way impacting the qualitative results. However, if $v$ takes on a unit value, the labour supply decision of the household becomes perfectly elastic in the real wage rate. As a consequence, any government spending shock is perfectly accommodated by labour supply. As this accommodation of demand shocks occurs independently of the particular monetary policy strategy adopted, the latter does not affect any of the endogenous variables and thus has no differential impact on welfare.

Within the small open-economy context, the degree of openness has been set at $\tau = 0.2$. Increasing $\tau$ towards unity leaves all results qualitatively unchanged with two exceptions both related to the resulting levels of welfare in the absence of the real balance effect. The one exception occurs at $\tau = 1$, where the choice of the domestic monetary policy rule does not exert any differential impact on welfare. The other exception is related to foreign productivity disturbances. In the model, exchange rate movements are an integral part in the adjustment of relative prices in bringing about the requisite expenditure switching effect. As the degree of openness increases, the exchange rate loses its shock absorber function for the domestic economy. As a consequence, the stabilising impact on domestic consumption imparted by an interest rate rule loses its importance. At the same time, the reduced volatility in prices and output induced by a domestic money supply rule turns the latter into the preferred monetary policy instrument in an environment of foreign productivity shocks and high degrees of openness. This switch occurs when $\tau$ reaches 0.5.

4 Conclusion

This paper has evaluated simple, non-optimising monetary policy rules in the tradition of the well-known Poole analysis within a two-country open-economy version of the New Open Economy Macroeconomic Framework. We analyse the impact of liquidity, demand and supply shocks on
consumption, output, prices and the nominal exchange rate by means of a calibration exercise.

Pure money supply rules are compared with simple interest rate rules for the large economy and the small open economy. The results for the large economy resemble those of the original Poole analysis. If evaluated in terms of the resulting levels of output variability, an interest rate rule is preferable whenever liquidity shocks are the major source of economic disturbances, whereas a money supply rule yields better results when demand shocks dominate. Similar results also obtain when both output and price level variability are used as the relevant metric, with the well-documented exception that supply shocks lead to relatively lower levels of output variability under an interest rate rule, whereas a money supply rule performs better in terms of price level variability. When measured in terms of consumption variability, however, an interest rate rule fares better for both liquidity and real shocks.

Our framework also allows for an explicit welfare analysis. For the large-economy scenario, the results of the Poole model are exactly replicated as an interest rate rule is preferable in an environment of dominant liquidity shocks, whereas a money supply rule yields better results when demand or supply shocks are the predominant source of economic disturbances.

In the small open-economy scenario all results of the large-economy case continue to hold for domestic shocks. For foreign shocks, the choice of the monetary policy instrument is exactly reversed. A money supply rule is preferable in an environment of foreign liquidity shocks as it helps to stabilise the domestic price level and hence the domestic labour supply. With foreign real shocks, an interest rate rule turns out to be the preferred monetary policy instrument as it contributes to stabilising consumption. Interestingly, when the real balance effect is factored in, the domestic monetary authority improves welfare only when it adopts the same monetary policy stance as the foreign economy, as such a strategy turns out to stabilise domestic real money balances.

These insights also determine the choice of the preferred exchange rate system. The domestic economy is perfectly insulated from foreign liquidity shocks when the foreign economy implements an interest rate rule, and this scenario turns out to be compatible with either a clean float or with a fixed exchange rate. When real shocks are the major source of disturbance, results depend on whether or not real balances are included in the welfare analysis. Neglecting the real balance effect, a clean float associated with a domestic interest rate rule is always preferable due to the insulating role of the exchange rate and the consumption stabilisation property. However, if real balances are factored in, the preferred regime turns out to be a dirty float for foreign demand shocks and a fixed exchange rate for foreign supply shocks. The latter results are due to the combined importance of the resulting variability in consumption and real money balances.
Appendix A: The large and small open economy

The deviations from the steady state, section 2.10, allow to rewrite the system of equations for the large and small open economy in the following way: Utilising the fact that \( n = 1 - \tau \) and \( n^* = 1 \) (for \( \mathcal{D}^* \) going to infinity) and defining the constant terms

\[
\begin{align*}
\Lambda_1 &= \frac{1-\alpha(1-\tau)(1-\alpha(1-\nu)) - \tau(1-\alpha(\nu-1)+\alpha(1-\nu)(1-\alpha))}{\rho(1-\alpha(1-\nu)) + \alpha(1-\nu)(1-\alpha)} \\
\Lambda_2 &= \tau \left[ \frac{1-\alpha(1-\nu) + \alpha(1-\nu)(1-\alpha)}{\rho(1-\alpha(1-\nu)) + \alpha(1-\nu)(1-\alpha)} \right] \\
\Lambda_3 &= \alpha(1-\tau)(1-\alpha) \\
\Lambda_4 &= \alpha \tau \left[ \frac{\alpha(1-\tau)(1-\alpha) + \alpha(1-\tau)(1-\nu)}{\rho(1-\alpha(1-\nu)) + \alpha(1-\nu)(1-\alpha)} \right] \\
\Lambda_5 &= \left[ \rho \frac{1+\delta}{\delta} + \left( \frac{1+\epsilon \delta}{\delta} \right) \left( \frac{\alpha(1-\tau)}{1-\alpha} \right) \left( \rho + (v-1) \frac{1-\alpha(1-\tau)}{1-\alpha(1-\nu)} \right) \right] \\
\Lambda_6 &= (1+\epsilon \delta) \frac{\alpha(1-\tau) - \alpha(1-\nu)}{1-\alpha(1-\nu)} \\
\Lambda_7 &= (1+\epsilon \delta) \left( \frac{\alpha}{\rho} \right) \left( v-1 \right) \left( \frac{1-\alpha(1-\nu)}{1-\alpha(1-\nu)} + 1 \right) \\
\Lambda_8 &= \left( \frac{1+\epsilon \delta}{\delta} \right) \left( \frac{\alpha}{\rho} \right) \left( \frac{\alpha(1-\tau) - \alpha(1-\nu) - (1-\alpha)(1-\nu)}{1-\alpha(1-\nu)} \right) \\
\Lambda_9 &= \frac{1-\nu}{\rho} \alpha(1-\nu) \\
\Lambda_{10} &= \alpha \frac{1-\nu}{\rho} \\
\Lambda_{11} &= \rho \left[ \frac{1+\delta}{\delta} + \frac{\nu+v-1}{\rho} \frac{1+\epsilon \delta}{\delta} \frac{\alpha}{1-\alpha} \right] \\
\Lambda_{12} &= \frac{1+\epsilon \delta}{\delta} \frac{\alpha}{1-\alpha}
\end{align*}
\]

the endogenous variables can then be written as

\[
\begin{align*}
e_t - c &= - (\Lambda_2 - \Lambda_4 \Lambda_9 (\rho + v - 1)) (i_t^* - i^*) - \Lambda_3 [\kappa + (v-1) \gamma] \\
&\quad - (\Lambda_4 - \Lambda_4 \Lambda_{10} (\rho + v - 1)) [\kappa^* + (v-1) \gamma^*] - \Lambda_1 (i_t - \tilde{i}) \\
e_t^* - c^* &= - \Lambda_9 (i_t^* - i^*) - \Lambda_{10} [\kappa^* + (v-1) \gamma^*] \\
\epsilon (m_t - m) &= \kappa + (\Lambda_8 - \Lambda_1 \Lambda_5) (i_t - \tilde{i}) + (\Lambda_6 - \Lambda_3 \Lambda_5) [\kappa + (v-1) \gamma] \\
&\quad + (\Lambda_7 - \Lambda_5 (\Lambda_4 - \Lambda_4 \Lambda_{10} (\rho + v - 1)) - \Lambda_7 \Lambda_{10} (\rho + v - 1)) [\kappa^* + (v-1) \gamma^*] \\
&\quad - (\Lambda_8 + \Lambda_5 (\Lambda_2 - \Lambda_4 \Lambda_3 (\rho + v - 1)) + \Lambda_7 \Lambda_9 (\rho + v - 1)) (i_t^* - i^*) \\
\epsilon (m_t^* - m^*) &= \kappa^* - \Lambda_9 \Lambda_{11} (i_t^* - i^*) + (\Lambda_{12} - \Lambda_{10} \Lambda_{11}) [\kappa^* + (v-1) \gamma^*]
\end{align*}
\]

26
Utilising the money supply rules, equation (20), the system can be solved for the endogenous variables as functions of the exogenous disturbances. These equations can then be utilised to obtain expressions for second moments of the endogenous variables in the system.

**Appendix B: The home price level of the small open economy**

Equation (22),

\[
\rho (E_t c_{t+1} - c_t) = i_t - \delta + (E_t e_{t+1} - e_t) - (E_t p_{t+1} - p_t) + \frac{1}{2} (\rho^2 \sigma_e^2 + \sigma_p^2 + \sigma_e^2 - 2 \rho \sigma_{cc} + 2 \rho \sigma_{cp} - 2 \sigma_{cp}) ,
\]

can be rewritten in terms of deviations from the steady state as

\[
\rho (c_t - c) = -(i_t^* - i^*) + (e_t - e) - (p_t - p) - (r_t^* - r^*) + (p_t^* - p^*) + (e_t - e) - (p_t - p) - (q_t - q) ,
\]

where the steady state relationship

\[
i^* = \delta - \frac{1}{2} (\rho^2 \sigma_e^2 + \sigma_p^2 + \sigma_e^2 - 2 \rho \sigma_{cc} + 2 \rho \sigma_{cp} - 2 \sigma_{cp}) ,
\]

and the definition of the real exchange rate \( Q = \frac{e^*}{p^*} \) has been applied. Utilising the domestic Euler equation (21),

\[
\rho (E_t c_{t+1} - c_t) = i_t - \delta - (E_t p_{t+1} - p_t) + \frac{1}{2} (\rho^2 \sigma_e^2 + \sigma_p^2 + 2 \rho \sigma_{cp}) ,
\]
and combining it with equation (22) yields for the steady state relation,

\[ i = \delta - \frac{1}{2} \left( \rho^2 \sigma^2 + \sigma^2 p + 2 \rho \sigma e \right), \]

the following condition:

\[
\begin{align*}
(e_t - e) &= (i_t^* - i^*) - (i_t - i) \\
(e_t - e) + (p_t^* - p) - (p_t - p) &= (i_t^* - i^*) + (p_t^* - p^*) - (i_t - i) - (p_t - p) \\
(q_t - q) &= (r_t^* - r^*) - (i_t - i) - (p_t - p).
\end{align*}
\]

(29)

Noting that for the small open economy (as \( P^* \) goes to infinity) the real exchange rate equals

\[ Q = \frac{e^{1-\tau} P_{F}^{1-\tau}}{P_{H}}, \]

it follows that

\[ P = P_{H}^{1-\tau} (eP_{F})^{\tau} = P_{H} \left( \frac{eP_{F}^{\tau}}{P_{H}} \right)^{\tau} = P_{H} Q^{\tau}. \]

Consequently, equation (29) can be rewritten as

\[
\begin{align*}
(q_t - E_tq_{t+1}) &= (r_t^* - r^*) - (i_t - i) - (p_t - E_t p_{t+1}) \\
(q_t - E_tq_{t+1}) &= (r_t^* - r^*) - (i_t - i) - \left( p_{H,t} + \frac{\tau}{1-\tau} q_t - E_t \left( p_{H,t+1} + \frac{\tau}{1-\tau} q_{t+1} \right) \right) \\
E_tq_{t+1} - q_t &= - (r_t^* - r^*) + (i_t - i) - (E_t p_{H,t+1} - p_{H,t}) - \frac{\tau}{1-\tau} (E_tq_{t+1} - q_t) \\
- (r_t^* - r^*) &= (E_t p_{H,t+1} - p_{H,t}) - (i_t - i) + \frac{1}{1-\tau} (E_t q_{t+1} - q_t).
\end{align*}
\]

(30)

Substituting equation (30) into (28) results in

\[
\begin{align*}
\rho (c_t - c) &= (E_t p_{H,t+1} - p_{H,t}) - (i_t - i) - \frac{1}{1-\tau} (q_t - q) + (q_t - q) \\
&= (E_t p_{H,t+1} - p_{H,t}) - (i_t - i) - \frac{\tau}{1-\tau} (q_t - q) \\
(E_t p_{H,t+1} - p_{H,t}) &= \rho (c_t - c) + (i_t - i) + \frac{\tau}{1-\tau} (q_t - q).
\end{align*}
\]

Utilising nominal income relationship (16),

\[ y_{H,t} - y = (c_t - c) + \frac{\tau}{1-\tau} (q_t - q) + \gamma_t, \]

it follows that

\[ (E_t p_{H,t+1} - p_{H,t}) = (i_t - i) + (1 - \rho) \frac{\tau}{1-\tau} (q_t - q) + \rho (y_{H,t} - y) - \rho \gamma_t. \]

(31)

Given the money supply rule, (20), the difference equation in domestic prices, (31), equates to

\[ (E_t p_{H,t+1} - p_{H,t}) = \left( \frac{m_t - m}{\Phi} \right) + (1 - \rho) \frac{\tau}{1-\tau} (q_t - q) + \rho (y_{H,t} - y) - \rho \gamma_t. \]

28
Forwarding this difference equation and ruling out bubbles, it follows that

\[ p_{H,t} = E_t \sum_{s=0}^{\infty} \left( \frac{m_{t+s} - m}{\Phi} + (1 - \rho) \frac{\tau}{1 - \tau} (q_{t+s} - q) + \rho (y_{H,t+s} - yH) - \rho \gamma_{t+s} \right). \] (32)

Taking expectations of \( t - 1 \), noting that future values of money \( m_{t+s} \) are given by \( m \) and that shocks are temporary (with zero mean), output and the real exchange rate are at their steady state value in all future periods. Then equation (32) becomes

\[ E_{t-1} (p_{H,t}) = 0. \]

Leading (32) one period results in

\[ p_{H,t+1} = E_{t+1} \sum_{s=0}^{\infty} \left( \frac{m_{t+1+s} - m}{\Phi} + (1 - \rho) \frac{\tau}{1 - \tau} (q_{t+1+s} - q) + \rho (y_{H,t+1+s} - yH) - \rho \gamma_{t+1+s} \right) \] (33).

Taking expectation of \( t \) it follows that

\[ E_t (p_{H,t+1}) = 0. \]

Thus, we have that \( E_t (p_{H,t+1} - E_{t-1} p_{H,t}) = 0 \) for all \( t \). For the foreign price level similar results can be derived.

**Appendix C: Expected welfare**

Given the assumption of a log-normal distribution, equation (27),

\[ E_{t-1} (U_t) = \frac{E_{t-1} (C^{1-\rho})}{1 - \rho} + \frac{E_{t-1} (\chi (\frac{M}{P})^{1-\epsilon})}{1 - \epsilon} - \frac{E_{t-1} (\kappa L^\nu)}{v}, \]

can be written as

\[ E_{t-1} (U_t) = \frac{\exp \left( (1 - \rho) c + (1 - \rho)^2 \frac{\sigma^2}{2} \right)}{1 - \rho} + \frac{\exp \left( \frac{\sigma^2}{2} + (1 - \epsilon) (m - p) + (1 - \epsilon)^2 \left( \frac{\sigma^2}{2} + \sigma_p^2 - \sigma_{mp} \right) + (1 - \epsilon) (\sigma_{cm} - \sigma_{cp}) \right)}{1 - \epsilon} - \frac{\exp \left( \frac{\sigma^2}{2} + vl + v^2 \frac{\sigma^2}{2} + v \sigma_{l,\nu} \right)}{v}. \]

The endogenous expected values, variances and covariances of \( c, p \) and \( l \) can be derived on the basis of section 2.8 and Appendix A.

**Appendix D: Consumption equivalent welfare measure**

For the Policy regime \( R \) the expected utility can be written as

\[ U^R = E_t \left\{ \sum_{\delta=t}^{\infty} (1 + \delta)^{-(s-t)} \left[ \frac{C^R_s^{1-\rho}}{1 - \rho} + \frac{\chi (M_s^R)^{1-\epsilon}}{1 - \epsilon} - \frac{\kappa L_s^R}{v} \right] \right\}. \]
whereby \( \{C^r_s\}, \{M^r_s\}, \{P^r_s\} \) and \( \{L^r_s\} \) are the stream of consumption, money supply, price and labour supply under policy regime \( R \). To compare across different regimes we define \( C^r, M^r, P^r \) and \( L^r \) as the annuity consumption, money supply, price and labour supply due to regime \( R \). It follows that

\[
E_t \left\{ \sum_{s=t}^{\infty} (1 + \delta)^{-(s-t)} \left[ \frac{C^R_s(1-\rho)}{1 - \rho} + \frac{\chi}{1 - e} \left( \frac{M^R_s}{P^R_s} \right)^{1-\epsilon} \right] - \frac{\tilde{\kappa} L^R_s v}{v} \right\}
\]

It follows that the expected utility under regime \( R \) equates to

\[
U^R = \frac{E (C^{r(1-\rho)})}{(1-\beta)(1-\rho)} + \frac{E \left( \chi \left( \frac{M^r}{P^r} \right)^{1-\epsilon} \right)}{(1-\beta)(1-\epsilon)} - \frac{E (\tilde{\kappa} L^{r v})}{(1-\beta) v}.
\]

Similarly, for the monetary policy regime \( R' \) the expected utility can be written as

\[
U^{R'} = \frac{E (C^{r'(1-\rho)})}{(1-\beta)(1-\rho)} + \frac{E \left( \chi \left( \frac{M^{r'}}{P^{r'}} \right)^{1-\epsilon} \right)}{(1-\beta)(1-\epsilon)} - \frac{E (\tilde{\kappa} L^{r' v})}{(1-\beta) v}.
\]

In a next step define \( \mathcal{E} \) as the fraction of the annuity consumption that a household governed by the monetary policy regime \( R \) would be willing to give up in order to make the household indifferent between monetary policy \( R \) and the alternative monetary policy regime \( R' \). \( \mathcal{E} \) can be derived as follows:

\[
\frac{(1 - \mathcal{E}) E (C^{r(1-\rho)})}{(1-\beta)(1-\rho)} + \frac{E \left( \chi \left( \frac{M^r}{P^r} \right)^{1-\epsilon} \right)}{(1-\beta)(1-\epsilon)} - \frac{E (\tilde{\kappa} L^{r v})}{(1-\beta) v} = \frac{E (C^{r'(1-\rho)})}{(1-\beta)(1-\rho)} + \frac{E \left( \chi \left( \frac{M^{r'}}{P^{r'}} \right)^{1-\epsilon} \right)}{(1-\beta)(1-\epsilon)} - \frac{E (\tilde{\kappa} L^{r' v})}{(1-\beta) v},
\]

which can be written as

\[
U^{R'} = \frac{(1 - \mathcal{E}) E (C^{r(1-\rho)})}{(1-\beta)(1-\rho)} + \frac{E \left( \chi \left( \frac{M^r}{P^r} \right)^{1-\epsilon} \right)}{(1-\beta)(1-\epsilon)} - \frac{E (\tilde{\kappa} L^{r v})}{(1-\beta) v} = \frac{U^{R'} + E (\tilde{\kappa} L^{r v})}{(1-\beta) v} - \frac{E \left( \chi \left( \frac{M^{r'}}{P^{r'}} \right)^{1-\epsilon} \right)}{(1-\beta)(1-\epsilon)} = 1 - \frac{U^{R'} + E (\tilde{\kappa} L^{r v})}{(1-\beta) v} - \frac{E \left( \chi \left( \frac{M^{r'}}{P^{r'}} \right)^{1-\epsilon} \right)}{(1-\beta)(1-\epsilon)} \left[ \frac{(1 - \mathcal{E}) E (C^{r(1-\rho)})}{(1-\beta)(1-\rho)} \right] (1-\beta)(1-\rho)
\]

Equation (34) can be written so that

\[
E (C^{r(1-\rho)}) = (1-\beta)(1-\rho) \left[ U^{R'} - \frac{E \left( \chi \left( \frac{M^r}{P^r} \right)^{1-\epsilon} \right)}{(1-\beta)(1-\epsilon)} + \frac{E (\tilde{\kappa} L^{r v})}{(1-\beta) v} \right].
\]
Given (35), equation (36) can be rewritten so that the fraction of the annuity consumption that a household governed by the monetary policy regime $\mathcal{R}$ would be willing to give up in order to make the household indifferent between monetary policy $\mathcal{R}$ and the alternative monetary policy regime $\mathcal{R}'$ equates to

$$
\mathcal{E} = 1 - \frac{U^{\mathcal{R}'} + \frac{E(\lambda L^{\mathcal{R}'})}{(1-\beta)v} - \frac{E(\lambda L^{\mathcal{R}'})^{1-\epsilon}}{(1-\beta)(1-\epsilon)}}{U^{\mathcal{R}} + \frac{E(\lambda L^{\mathcal{R}})}{(1-\beta)v} - \frac{E(\lambda L^{\mathcal{R}})^{1-\epsilon}}{(1-\beta)(1-\epsilon)}} = \text{Cons.Cost.}
$$
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