On the Notion of Responsibility in Organizations

Dirk Sliwka*
University of Bonn

September 2002

Abstract

A theory of responsibility is provided in a simple model where agents care for their career prospects. First, two agents with uncertain abilities work on a task. Second, a principal decides to promote one of them. Three types of equilibria occur. One in which no agent is responsible for the task and no effort is contributed and two *sole responsibility* equilibria where exactly one agent contributes effort. The less able agent works harder when he is responsible but produces less output. If incentives matter strongly, the agent whose ability is more precisely known should be responsible, but the opposite is true if selection is important.

Key Words: Responsibility, Career Concerns, Delegation, Incentives

JEL Classification: D 23, M 12

*Betriebswirtschaftliche Abteilung II, University of Bonn, Adenauerallee 24-42, D-53113 Bonn, Germany, tel: +49 228 739214, fax: +49 228 739210, e-mail: dirk.sliwka@uni-bonn.de.

The sole and undivided responsibility of one man will naturally beget a livelier sense of duty and a more exact regard to reputation.
Alexander Hamilton

1 Introduction

When a certain task has to be carried out in an organization several people may contribute to the success, but typically a superior will hold one person responsible for the task. But what is the exact content of the expression *"holding someone responsible"*? And why is it considered to be necessary to hold someone responsible in so many instances? In this paper a simple theory is provided of what responsibility means in a multi-agent career concerns framework.

To illustrate the key idea consider a situation in which a manager of a certain department delegates a certain task or project to some of her subordinates. After having observed the outcome of the project, she will most certainly adapt her opinion on the talent of all individuals who have contributed to the task. If now this manager *believes* that the largest part of the work has been carried out by one employee, and the project has been a success, she will adapt her opinion on this employee's talent to a larger extent than her opinion on other employees. But in many instances a high opinion by a superior will pay off in the future. Due to those considerations, the employee who is believed to be contributing most, may then have indeed a higher incentive to contribute more than his colleagues. If the superior is not able to monitor the individual contributions to the task, her beliefs thus may become self-fulfilling. The larger she believes one agent's contribution to be, the stronger his reputation will be affected by the outcome as "*he will be held responsible*" to a larger extent for the success of task. But the stronger his reputation is affected, the stronger will be his incentives to work hard on the task.

When several agents can possibly contribute to the task, there will of course be a multiplicity of equilibria as the extent of each agent's contribution is essentially affected in equilibrium by the beliefs of the superior on the respective contributions. This leads to a natural notion of responsibility: If a superior holds a certain subordinate responsible for a task, this means that the superior believes that this subordinate contributes most to this task. But, the best response of this subordinate will then be indeed to contribute more to the task than others.

In his influential paper Holmström (1982) has first formally analyzed the incentive effects of career concerns. In his basic framework an agent's output is the sum of his effort, his initially unknown normally distributed ability and some random component. This output is observed by potential employers on a competitive labor market who bid for the agent's services. In the unique equilibrium an agent works hard in the beginning of his career to affect the market's beliefs on his ability and receive higher wage offers. Dewatripont et al. (1999b) generalize Holmström's model including the possibility that an agent's talent also determines his marginal product of effort.[1] In such a framework a multiplicity of equilibria arises in a single agent model.

We adopt the idea that an agent's imperfectly known talent determines his marginal product of effort. But whereas in the career concerns literature incentives are typically created by a competitive labor market paying an agent a wage proportional to his expected ability, in our model incentives arise due to an internal promotion decision such as for instance in the tournament literature (compare Lazear and Rosen (1981), Nalebuff and Stiglitz (1983) or Green and Stokey (1983)) or more specific in the literature on promotions as for instance in Waldman (1990), Prendergast (1993) or Fairburn and Malcomson (1994) and (2001). This seems a more natural assumption for modelling career concerns within an organization, as no visibility to the outside labor market is necessary. In our model, two agents with imperfectly known abilities contribute effort to a certain task. A principal observes the outcome of the task and can then promote one of the two agents. In contrast to the tournament literature, however, the principal cannot observe individual signals on the performance of each agent separately, but only learns the total outcome to which both agents can possibly contribute.

We will show that there are two types of equilibria in our model. On the one hand, under certain restrictions two *sole-responsibility*-equilibria exist in which exactly one of the two agents exerts a positive effort level and is held responsible for the task. On the other hand, there is always a *no-responsibility*-equilibrium in which no agent contributes effort but there is never an equilibrium where both agents contribute positive effort levels. This result in a sense formalizes the idea that in organizations it is commonly felt that one should assign sole responsibility to a task to a unique person.

Note that the choice of responsibility does not encompass the allocation of a formal decision right. The principal is not able to commit ex-ante on the consequences of the outcome as this seems to be an important feature of most decision procedures within organizations:[2] When a superior speaks out to some employee "You are responsible for this task" this is rather an informal act. No contract is signed which is enforced by third parties. In this model we try to capture this notion of responsibility. By announcing who should be held responsible for a certain task the principal creates a focal point and therefore selects one of several possible equilibria. Hence, the informal act of allocating responsibility does have economic consequences.

If the principal can select one of the possible equilibria, we have to ask which one she prefers. It is straightforward to see that this will never be the no-responsibility-equilibrium. Hence, in a next step, we compare the two sole responsibility equilibria if the agents differ in the principal's prior expectations on their respective abilities. As we will show, there are two aspects that should be considered when determining who should be held responsible for a certain task. On the one hand, we examine the *incentive aspect* of responsibility and characterize which agent works harder when being responsible and which one produces a higher expected output. But a second aspect is that by allocating responsibility, the principal can learn more about the talent of the responsible agent and this will pay off in the future. Hence, we also analyze who should be held responsible when this *learning aspect* is considered.[3]

The paper proceeds as follows. In section 2 we describe the simple model. In the subsequent section 3 we analyze how the promotion decision is made by the principal. After this, the possible equilibria of the game are characterized in section 4. Section 5 considers the incentive aspect of responsibility, section

6 the selection aspect.

2 The Model

We consider the following simple model. A principal employs two agents $i = 1, 2$. Each agent has an initially unknown ability a_i. The principal and the agents share a common prior belief on each agent's ability and the abilities independently normally distributed

$$a_i \sim N\left(m_i, \sigma_i^2\right) \text{ for } i = 1, 2$$

with both $m_i > 0$. In a first period both agents can exert effort e_i on a task. They have an identical convex cost function $c(e)$, with

$$c(0) = 0 \text{ and } c'(0) = 0.$$

The output of the task accrues to the principal and is given by

$$\pi = a_1 e_1 + a_2 e_2.$$

Hence, the ability of an agent determines the marginal productivity of his effort. The higher his ability, the more productive is a given amount of effort exerted by an agent. The principal only observes the total output and not the individual contributions.

In the second period one of the two agents gets promoted to a better job by the principal. The principal cares for having the better agent promoted.[4] For simplicity we assume that the principal's payoff from the promotion decision is a linear function of the ability of the agent who is promoted, i.e. in expected terms her second period output is $E[R \cdot a_i | \pi]$ where R is a given constant. The agent who is promoted receives a private benefit B from promotion, which may simply be the wage differential between the old and the new job. We do not consider any endogenous wage setting.

3 The Promotion Decision

We look for pure strategy perfect Bayesian equilibria of the model described above. First, the optimal promotion decision in period 2 is analyzed given the agents' equilibrium effort levels e_1 and e_2. Of course, the principal wants to promote the agent with the higher expected ability, i.e. agent i is always promoted if

$$E[a_i|\pi] > E[a_j|\pi]$$

after the principal has observed output π. The principal is indifferent, whom to promote when $E[a_i|\pi] = E[a_j|\pi]$. Due to the normality assumption the expected ability of agent i is given by[5]

$$E[a_i|a_i e_i + a_j e_j] = m_i + \frac{e_i \sigma_i^2}{e_i^2 \sigma_i^2 + e_j^2 \sigma_j^2} (\pi - E[\pi]). \qquad (1)$$

for strategies e_1 and e_2 chosen by the agents. Note that in a pure strategy equilibrium the principal knows these equilibrium effort levels. The principal adapts her beliefs on the ability of each agent after she has observed the outcome of the task π by taking into account the equilibrium strategies played by the agents in the first period. If the task is completed successfully and the outcome is higher than the expected outcome given the principal's beliefs on the agents' contributions (i.e. $\pi > E[\pi]$), the reputation of both agents raises above the prior level, otherwise it decreases.

The strength of the adaptation depends on the respective effort contributions of both agents and the precision of the knowledge on their abilities. Comparing both agents, note that if $e_i \sigma_i^2 > e_j \sigma_j^2$ then the beliefs on the ability of agent i are adapted to a stronger extent than those on agent j. In particular, if the initial beliefs on the abilities are identical the adaptation is stronger for the agent with the larger presumed contribution.

Applying those considerations we can derive the principal's optimal second period promotion decision given the agent's equilibrium actions.

Lemma 1 *Suppose that in equilibrium $\sigma_i^2 e_i > \sigma_j^2 e_j$. In that case agent i is promoted if the realized profit exceeds a certain performance standard π_T, which is given by*

$$\pi_T = E[\pi] - \frac{e_i^2 \sigma_i^2 + e_j^2 \sigma_j^2}{e_i \sigma_i^2 - e_j \sigma_j^2} (m_i - m_j).$$

Furthermore, if the initial beliefs on the agent's abilities are identical, agent i will be promoted if profits are larger than the initially expected profits.
If however, in equilibrium $\sigma_i^2 e_i = \sigma_j^2 e_j$ the principal will always promote the agent with the higher initially expected ability.

Proof: See Appendix.

If initial beliefs on both agents are identical – for instance because both are new to the organization – the agent with the higher equilibrium effort is promoted if and only if profits are higher than expected profits. As we have seen above, both agents' reputation is always adapted in the same direction if they both contribute positive effort levels. A higher than expected outcome increases the principal's beliefs on both agents' abilities, a lower than expected outcomes reduces those beliefs. But in addition, the adaptation of beliefs is stronger for the agent with the higher presumed contribution. Hence, it is him who gets promoted if the task is a success ($\pi > E[\pi]$) and his colleague in case of a failure ($\pi < E[\pi]$).

If, however, agent i is initially believed to be more able than his colleague (i.e. $m_i > m_j$), his profit target is lower than the initially expected profit. It takes a comparatively worse result to edge him out and this "discount" is larger, the larger the initial ability difference.

4 Responsibility and Incentives

Given the optimal promotion decision by the principal, we can now analyze the optimal effort levels exerted by the agents. The agent's decisions must be best responses given the principal's beliefs on the effort contributions by the agents and her promotion strategy in period 2. We denote those beliefs on the agent's actions by $\hat{e}_i$ and $\hat{e}_j$. First, suppose that the principal's beliefs are such that for one agent i condition $\hat{e}_i \sigma_i^2 > \hat{e}_j \sigma_j^2$ holds. Agent j will then be promoted only if profits are smaller than a certain cut-off value. Hence, the best he can do is to exert no effort at all. A good outcome of the task will always increase his colleague's reputation to a larger extent than his own. Hence, it is straightforward that in any equilibrium with $\hat{e}_i \sigma_i^2 > \hat{e}_j \sigma_j^2$ we must have that $e_j = 0$.

In that case, agent i bears *sole responsibility* for the task. It is only him, who contributes to the task. Hence, only the beliefs on his talent are adapted according to the outcome of the task. According to Lemma 1, he will then be promoted if the outcome of the task exceeds

$$\pi_T = E[\pi] - \hat{e}_i (m_i - m_j) = \hat{e}_i m_j.$$

Hence, the agent has to beat a performance standard, determined by the product of his own conjectured contribution and his colleague's expected ability. To understand the value of the standard, just think of the principal asking the following hypothetical question: "Given my initial beliefs, which profit would agent j attain in expected terms if he exerted the same effort level that I think agent i has exerted?". If the profit generated by i exceeds this value, then agent i will be promoted otherwise j is preferred.

In period 1 agent i then maximizes the following expression, given the principal's equilibrium beliefs:

$$\max_{e_i} \Pr(a_i e_i > \hat{e}_i m_j) B - c(e_i). \tag{2}$$

If there is a unique solution to this problem with a positive $e_i = \hat{e}_i$ we have an equilibrium. Indeed, if the uncertainty on agent i's ability is not too small, such an equilibrium exists as is shown in the following result:[6]

Proposition 1 *For each agent i, if σ_i is sufficiently large there exist a "sole responsibility" equilibrium, in which agent i contributes a strictly positive effort level e_i^*, determined by*

$$\frac{m_j B}{\sigma_i} \phi\left(\frac{m_j - m_i}{\sigma_i}\right) = c'(e_i^*) e_i^*. \tag{3}$$

and the other agent supplies no effort at all ($e_j^* = 0$). The principal will promote agent i if the outcome exceeds the expected outcome when j instead of i exerted effort level e_i^*, that is, i is promoted if

$$\pi \geq e_i^* m_j.$$

Proof: See Appendix.

To understand why we can show the existence of such an equilibrium only if the variance of the agent's ability is large just note that if it is zero the principal knows the agent's abilities perfectly and the outcome of the task will not affect her beliefs. There must be some uncertainty on the agent's abilities such that they indeed have an interest to demonstrate their respective abilities by exerting a high effort level.

Now suppose that we have an equilibrium where $e_i\sigma_i^2 = e_j\sigma_j^2$. In that case the principal cannot learn from the realized profit. We know from Lemma 1 that if in that case the initially expected abilities differ, the agent with the higher expected ability will always be promoted. But then neither the more, nor the less able agent has any incentive to exert effort. Hence, for if expected abilities differ we must have that $e_i = e_j = 0$ whenever $e_i\sigma_i^2 = e_j\sigma_j^2$. If the initially expected abilities coincide the principal is indifferent between promoting each of the agents. If the principal promotes an agent i with any given probability, again, both agents have no incentives to exert effort and we must have that $e_i = e_j = 0$.

Furthermore, suppose that there is an equilibrium in which e_i and e_j are both strictly positive. From the considerations before Proposition 1 we know that if $\hat{e}_i\sigma_i^2 > \hat{e}_j\sigma_j^2$ this cannot be the case as e_j will always be 0. But, as we have seen, if $e_i\sigma_i^2 = e_j\sigma_j^2$ we must have that $e_i = e_j = 0$. Hence, there cannot be an equilibrium in which both agents exert positive effort levels. We can summarize those considerations in the following result:

Proposition 2 *There is always a "no responsibility" equilibrium in which no agent exerts any effort. If the expected abilities of the agents differ, the agent with the higher expected ability is always promoted. Furthermore, there is no equilibrium in which both agents exert positive effort levels.*

Hence, there is either a situation in which exactly one of the agents is responsible for the task or a situation without any responsibility. But in the latter case no agent contributes to the success of the task, as no one can gain anything relative to his colleague in the promotion tournament.

From now on we assume that both σ_i and σ_j are sufficiently large such that the two sole responsibility equilibria indeed exist as we want to compare the two possible equilibria.[7] Supposing that the principal can choose among the mentioned equilibria by holding one of the agents responsible for the task, it is now interesting to examine which agent this should be.

5 Who works harder?

To answer this question we have to compare the two *sole responsibility* equilibria. First, we suppose that both agents have different prior expected ability levels, but that the information on both agents is equally precise (i.e. $\sigma_i^2 = \sigma_j^2$). Without loss of generality we assume that i is the agent with the higher expected ability, i.e. $m_i > m_j$. The difference in abilities is denoted by $\Delta m = m_i - m_j$.

As $\phi(.)$ is symmetric it can be seen from equation (3) that for a given difference in abilities Δm the effort of the responsible agent is the higher the higher his colleague's ability. Hence, the agent with the lower ability works harder if he is held responsible than his colleague with a higher expected ability in the same situation! However, it is less clear how the total output is affected as the more able agent has a higher marginal product of output. This is examined in the following Proposition:

Proposition 3 *Suppose that the information on both agents' abilities is equally precise. Then the agent with the lower expected ability contributes more effort when he is responsible than the agent with the higher expected ability. However, the total expected outcome is higher when the more able agent is responsible.*

Proof: See Appendix.

To understand the result that the agent with the lower expected ability will supply a higher effort level, note that the optimal promotion rule by the principal yields a threshold value for the generated profits such that the responsible agent is promoted if and only if profits are higher than this value. But the threshold value is determined by the expected ability of the agent who is not responsible. For the more able agent it is easier to attain this value than for the less able agent as his marginal product of effort is higher. Hence, the latter has to work harder to achieve a promotion. However, the more able agent has a higher marginal product of effort and therefore produces a higher expected total output. As the result shows this effect dominates the incentive effect. Hence, for equal precisions the principal prefers that the more able agent is held responsible and works for the task.

After having considered the case where the expectations on the agents' abilities differ but are equally precise, it seems interesting to examine the polar case, where the expected abilities coincide, but the precisions differ. For $\Delta m = 0$ equation (3) yields

$$\frac{m_j B}{\sigma_i} \phi(0) = c'(e_i^*) e_i^*.$$

The left hand side is decreasing in the variance. Hence, it can directly be seen that the agent with the more precise signal will exert a higher equilibrium effort if he is held responsible for the task. As the expected marginal product is the same for both agents this directly implies that the expected output must be higher if this agent is held responsible.

To understand this observation note that the marginal product of effort in equilibrium (at $e_i = \hat{e}_i$) is larger for an agent with a smaller variance: To see that, consider equation (2) and note that for $m_i = m_j$ the marginal product of effort is determined by the density of a normal distribution at its mean at $e_i = \hat{e}_i$. But the density at the mean is higher the lower the variance. On a less technical level, when less is known on the agent's ability and therefore on his marginal product of effort, there is a higher variance of the possible outcome for a given effort level. Hence, the impact of effort on the outcome of the tournament is weak relative to the impact of chance. Therefore exerting effort is less attractive for an agent who is less sure on his ability than for an agent who knows his productivity more precisely. This result is related to a typical result from the tournament literature where for a given prize spread the equilibrium effort is higher the lower the variance of the noise terms.

We can summarize those considerations in the following result:

Proposition 4 *If both agents have the same expected ability but differ in the precision with which the respective abilities are known, the agent with the more precisely known ability exerts more effort and produces a higher output.*

So far we have only considered the impact of responsibility on the current output. But contrary to typical career concerns models where future returns accrue to the agent due to a perfectly competitive labor market, in our model the firm also cares for promoting the better agent. As we will see in the next section this selection purpose is also important to figure out who should best be held responsible for carrying out the task.

6 How to learn most?

If an agent's career goes on within the same organization, it will of course be important to promote the agent with the higher expected ability. Recall the assumption, that when promoting an agent with expected ability $E[a_i|\pi]$ the principal receives a payoff of $R \cdot E[a_i|\pi]$. First, we consider a situation where the principal only cares for this second period payoff to isolate the learning effects from the incentive effects of allocation responsibility.

Note that the firm learns nothing on the agent who is not responsible. Hence, the interesting question is from which agent the firm can learn more to make an appropriate promotion decision. Suppose that agent i was responsible for carrying out the task and a profit of π_i has been realized. As we have seen, the firm will promote agent i if $E[a_i|\pi_i] > m_j$. In that case, the firm's return on the second stage is $R \cdot \max\{E[a_i|\pi_i], m_j\}$. The prior expectation of the firm's profits on the second stage when holding agent i responsible for the task are therefore given by

$$R \cdot E[\max\{E[a_i|\pi_i], m_j\}].$$

This can be reformulated using equation (1), the expressions for π and $E[\pi]$ and the fact that $e_j = 0$ if agent i is held responsible for carrying out the task:

$$E\left[\max\left\{m_i + \frac{1}{\hat{e}_i}(\pi - E[\pi]), m_j\right\}\right] = E[\max\{a_i, m_j\}]$$

If the firm only cares for second period profits, she will assign responsibility to agent i instead of agent j if

$$E[\max\{a_i, m_j\}] \geq E[\max\{a_j, m_i\}].$$

By computing those two expressions and comparing them, we obtain the following clear-cut result:

Proposition 5 *If the principal only cares to promote the better agent, she should allocate responsibility to the agent whose ability is less certain regardless of the agents' expected abilities.*

Proof: See Appendix.

Hence, if she only cares for learning about the agents' abilities the principal should hold the agent responsible on whose ability less is known, whatever the initially expected abilities of both. It is this agent, on which there is more to learn. To understand this result suppose that agent i was held responsible for the task. Note that after having observed the outcome the principal can always promote agent j and receive an expected return of $R \cdot m_j$. She will promote agent i only if his posterior expected ability exceeds m_j. If the uncertainty on i's ability is higher, so will be the prior variance of $E[a_i|\pi_i]$. But then the "option value" of such an agent is higher for the principal as there is more weight in the upper tail of the distribution.[8]

In a certain sense, it is beneficial to assign responsibility to the less precisely known agent simply as there is more to learn on him than on his colleague. Note that this result holds whatever the prior expected abilities are.

Now consider the case where both agents have equal prior expected abilities but differ in the precision with which those abilities are known. Note that in this case the result of Proposition 5 contrasts that of Proposition 4 as when considering the incentive aspect the agent with the more precisely known ability should be promoted, but when only taking the selection aspect into account the contrary is true. A direct implication is the following result:

Corollary 1 *If the agents' expected abilities are identical, the principal should assign responsibility to the agent with the more precisely known ability if the importance of the future task R is sufficiently small, otherwise she should assign responsibility to the agent with the less precisely known ability.*

Hence, if it is sufficiently important to promote the better qualified agent, then the principal should assign responsibility to the agent whose ability is less well known.

7 Conclusion

We have provided a simple model to clarify the notion of responsibility in a multiagent career concerns framework. We have shown that two types of equilibria may exist. Either no agent bears responsibility and, hence, no agent contributes effort to the task or there are sole responsibility equilibria in which exactly one agent contributes effort to the task and hence, the principal's beliefs on this agent's ability is adapted according to the outcome of the task.

The principal is of course always better off with a sole responsibility equilibrium, as one agent has an incentive to exert effort on the task. This yields a simple formal explanation for the commonly felt principle in practice that responsibility for a certain task should be allocated to exactly one member of an organization even though others can possibly contribute to the task.

We have then analyzed, which agent should be responsible for a given task if the agents differ in their abilities or the precision with which those abilities are known. Two aspects have been identified in our model, which are important for assigning responsibility. On the one hand, the principal has to consider which agent works harder, when being responsible for the task and which agent attains a higher expected output. On the other hand, she has to bear in mind that the assignment of responsibility generates information as she learns more about the ability of the responsible agent and this in turn is valuable as it helps to improve the promotion decision.

8 Appendix

Calculation of expression (1):

We know that for normally distributed random variables the following equation holds:[9]

$$E[Y|X] = E[Y] + \frac{Cov[X,Y]}{Var[X]}(X - E[X]).$$

Applying this, we obtain

$$\begin{aligned} E[a_i|a_ie_i + a_je_j] &= m_i + \frac{Cov[a_i, a_ie_i + a_je_j]}{Var[a_ie_i + a_je_j]}(a_ie_i + a_je_j - E[a_ie_i + a_je_j]) \\ &= m_i + \frac{e_i\sigma_i^2}{e_i^2\sigma_i^2 + e_j^2\sigma_j^2}(\pi - E[\pi]). \end{aligned}$$

Proof of Lemma 1:

The principal will promote agent i iff $E[a_i|a_ie_i + a_je_j] > E[a_j|a_ie_i + a_je_j]$ or

$$\begin{aligned} m_i + \frac{e_i\sigma_i^2}{e_i^2\sigma_i^2 + e_j^2\sigma_j^2}(\pi - E[\pi]) &> m_j + \frac{e_j\sigma_j^2}{e_i^2\sigma_i^2 + e_j^2\sigma_j^2}(\pi - E[\pi]) \Leftrightarrow \\ \frac{e_i\sigma_i^2 - e_j\sigma_j^2}{e_i^2\sigma_i^2 + e_j^2\sigma_j^2}(\pi - E[\pi]) &> m_j - m_i. \end{aligned}$$

If $e_i\sigma_i^2 > e_j\sigma_j^2$ this is equivalent to

$$\pi > E[\pi] - \frac{e_i^2\sigma_i^2 + e_j^2\sigma_j^2}{e_i\sigma_i^2 - e_j\sigma_j^2}(m_i - m_j).$$

If, however, $e_i\sigma_i^2 = e_j\sigma_j^2$ this is equivalent to

$$m_i > m_j.$$

$\blacksquare$

Proof of Proposition 1:

Agent i's expected utility is determined by the probability of being promoted:

$$\Pr\left(a_i e_i \geq \hat{e}_i m_j\right) B - c\left(e_i\right). \tag{4}$$

If $e_i = 0$ and $\hat{e}_i > 0$ this probability of being promoted is zero. The agent gets zero utility. If $e_i > 0$, we can reformulate (4) and obtain:

$$\max_{e_i} \Pr\left(\frac{a_i}{m_j\hat{e}_i} \geq \frac{1}{e_i}\right) B - c\left(e_i\right). \tag{5}$$

where

$$\frac{a_i}{m_j\hat{e}_i} \sim N\left(\frac{m_i}{m_j\hat{e}_i}, \frac{\sigma_i^2}{\left(m_j\hat{e}_i\right)^2}\right).$$

Hence,

$$\Pr\left(\frac{a_i}{m_j\hat{e}_i} \geq \frac{1}{e_i}\right) = 1 - \Phi\left(\frac{\frac{1}{e_i} - \frac{m_i}{m_j\hat{e}_i}}{\frac{\sigma_i}{m_j\hat{e}_i}}\right),$$

and the maximization problem is given by:

$$\max_{e_i} \Phi\left(\frac{1}{\sigma_i}\left(m_i - m_j\frac{\hat{e}_i}{e_i}\right)\right) B - c\left(e_i\right). \tag{6}$$

The first order condition is

$$\phi\left(\frac{1}{\sigma_i}\left(m_i - m_j\frac{\hat{e}_i}{e_i}\right)\right)\frac{m_j\hat{e}_i}{\sigma_i e_i^2} B - c'\left(e_i\right) = 0.$$

Hence, if a pure strategy equilibrium with $e_i^* > 0$ exists we must have that

$$\phi\left(\frac{\Delta m}{\sigma_i}\right)\frac{m_j}{\sigma_i} B - e_i^* c'\left(e_i^*\right) = 0, \tag{7}$$

where $\Delta m = m_j - m_i$. This equation yields a unique strictly positive value for e_i as a possible equilibrium strategy. Note that the first order condition is necessary but not sufficient as the objective function (6) will typically be not concave. Hence, we have to check that choosing $\hat{e}$ is indeed a global maximum for given beliefs $\hat{e}$. First, we compute the second derivative of the expected payoff from promotion, which is

$$\left[\left(\frac{\hat{e}_i}{e_i}m_j - m_i\right)\frac{m_j\hat{e}_i}{\sigma_i^2 e_i} - 2\right]\frac{m_j\hat{e}_i}{\sigma_i e_i^3}\phi\left(\frac{m_j\hat{e}_i}{\sigma_i e_i} - \frac{m_i}{\sigma_i}\right)B.$$

Note that there is a unique inflection point at:

$$e_{IP} = \frac{m_j\hat{e}_i}{4\sigma_i^2}\left(\sqrt{(m_i^2 + 8\sigma_i^2)} - m_i\right).$$

The payoff from promotion is strictly convex below e_{IP} and strictly concave above. At $\hat{e}_i$ the second derivative is negative iff $(m_j - m_i)\frac{m_j}{\sigma_i^2} - 2 < 0$ which is always the case if σ_i is sufficiently large. Hence, for large values of the variance, $\hat{e}_i > e_{IP}$ and $\hat{e}_i$ is the unique *local* maximum above e_{IP}.

If the global maximum would to be the left of $\hat{e}_i$ the agents payoff in such a global maximum is bounded from above by the gross payoff from promotion at the inflection point (as the winning probability is increasing in e)

$$\Phi\left(\frac{m_i}{\sigma_i} - \frac{4\sigma_i}{\left(\sqrt{(m_i^2 + 8\sigma_i^2)} - m_i\right)}\right)B. \tag{8}$$

A sufficient condition for the existence of an equilibrium is therefore that the agent's utility at $\hat{e}_i = e_i^*$ which is given by

$$\Phi\left(\frac{m_i - m_j}{\sigma_i}\right)B - c\left(e_i^*\right) \tag{9}$$

exceeds (8). Both functions are continuous in σ_i. Note from (7) that e_i^* is strictly decreasing in σ_i if σ_i is large enough as[10]

$$\frac{\partial\left(\frac{m_j B}{\sigma_i}\phi\left(\frac{\Delta m}{\sigma_i}\right)\right)}{\partial\sigma_i} < 0 \Leftrightarrow \sigma_i^2 > \Delta m^2.$$

and it converges to zero for $\sigma_i \to \infty$. Hence, expression (9) tends to $\frac{B}{2}$ as σ_i approaches infinity. But the limit of the upper boundary (8) can be computed as

$$\lim_{\sigma_i \to \infty} \Phi\left(\frac{m_i}{\sigma_i} - \frac{4\sigma_i}{\left(\sqrt{(m_i^2 + 8\sigma_i^2)} - m_i\right)}\right) B = \Phi\left(-\sqrt{2}\right) B$$

which is smaller than $\frac{B}{2}$. Hence, for sufficiently large σ_i the unique global maximizer is $\hat{e}_i$ and the equilibrium exists. ■

Proof of Proposition 3:

The expected profit if agent i (j) is held responsible is $e_i^* m_i$ ($e_j^* m_j$). We know that for $\sigma_i = \sigma_j = \sigma$,

$$\begin{aligned} \frac{m_j B}{\sigma} \phi\left(\frac{\Delta m}{\sigma}\right) - e_i^* c'\left(e_i^*\right) &= 0, \\ \frac{m_i B}{\sigma} \phi\left(\frac{\Delta m}{\sigma}\right) - e_j^* c'\left(e_j^*\right) &= 0, \end{aligned}$$

which are equivalent to

$$\begin{aligned} e_i^* m_i &= \frac{\frac{m_j B}{\sigma} \phi\left(\frac{\Delta m}{\sigma}\right)}{c'\left(e_i^*\right)} m_i \\ e_j^* m_j &= \frac{\frac{m_i B}{\sigma} \phi\left(\frac{\Delta m}{\sigma}\right)}{c'\left(e_j^*\right)} m_j \end{aligned}$$

We get that $e_i^* m_i > e_j^* m_j$ iff

$$\begin{aligned} \frac{\frac{m_j B}{\sigma} \phi\left(\frac{\Delta m}{\sigma}\right)}{c'\left(e_i^*\right)} m_i &> \frac{\frac{m_i B}{\sigma} \phi\left(\frac{\Delta m}{\sigma}\right)}{c'\left(e_j^*\right)} m_j \Leftrightarrow \\ c'\left(e_j^*\right) &> c'\left(e_i^*\right) \Leftrightarrow e_j^* > e_i^*. \end{aligned}$$

As we have seen this is the case iff $m_i > m_j$. ■

Proof of Proposition 5:

By adapting a result from Gourieroux and Monfort (1989) (p. 529), we first get that the expected value of the maximum of normally distributed random variables is given by

$$E\left[\max\{X, Y\}\right] = m_X + \sqrt{\sigma_X^2 + \sigma_Y^2}\, \phi\left(\frac{m_X - m_Y}{\sqrt{\sigma_X^2 + \sigma_Y^2}}\right) - (m_X - m_Y)\, \Phi\left(\frac{m_Y - m_X}{\sqrt{\sigma_X^2 + \sigma_Y^2}}\right).$$

Let again $\Delta m = m_i - m_j$. We obtain

$$E\left[\max\left\{a_i, m_j\right\}\right] = m_i + \sqrt{\sigma_i^2}\phi\left(\frac{\Delta m}{\sigma_i}\right) - \Delta m \Phi\left(\frac{-\Delta m}{\sigma_i}\right).$$

The principal prefers to promote agent i if

$$\begin{aligned}
\Delta m + \sigma_i \phi\left(\frac{\Delta m}{\sigma_i}\right) - \Delta m \Phi\left(-\frac{\Delta m}{\sigma_i}\right) &\geq \sigma_j \phi\left(\frac{\Delta m}{\sigma_j}\right) + \Delta m \Phi\left(\frac{\Delta m}{\sigma_j}\right) \Leftrightarrow \\
\Delta m + \sigma_i \phi\left(\frac{\Delta m}{\sigma_i}\right) - \Delta m\left(1 - \Phi\left(\frac{\Delta m}{\sigma_i}\right)\right) &\geq \sigma_j \phi\left(\frac{\Delta m}{\sigma_j}\right) + \Delta m \Phi\left(\frac{\Delta m}{\sigma_j}\right) \Leftrightarrow \\
\sigma_i \phi\left(\frac{\Delta m}{\sigma_i}\right) + \Delta m \Phi\left(\frac{\Delta m}{\sigma_i}\right) &\geq \sigma_j \phi\left(\frac{\Delta m}{\sigma_j}\right) + \Delta m \Phi\left(\frac{\Delta m}{\sigma_j}\right).
\end{aligned}$$

Note that for $\sigma_i = \sigma_j$ both sides are equal and the principal is indifferent whom to promote whatever the value of Δm. To see that she prefers to promote agent i iff $\sigma_i \geq \sigma_j$ check that $\sigma\phi\left(\frac{\Delta m}{\sigma}\right) + \Delta m \Phi\left(\frac{\Delta m}{\sigma}\right)$ is strictly increasing in σ whatever the sign of Δm:

$$\frac{\partial}{\partial \sigma}\left(\sigma\phi\left(\frac{\Delta m}{\sigma}\right) + \Delta m \Phi\left(\frac{\Delta m}{\sigma}\right)\right) = \phi\left(\frac{\Delta m}{\sigma}\right) - \frac{\Delta m}{\sigma}\phi'\left(\frac{\Delta m}{\sigma}\right) - \frac{\Delta m^2}{\sigma^2}\phi\left(\frac{\Delta m}{\sigma}\right).$$

Using that $\phi'(x) = -x\phi(x)$ this expression is simply equal to $\phi\left(\frac{\Delta m}{\sigma}\right) > 0$. ■

References

Aghion, P. and Tirole, J. (1997): Formal and Real Authority in Organizations. Journal of Political Economy, 105, pp. 1–29.

Baker, G., Gibbons, R. and Murphy, K. J. (1999): Informal Authority in Organizations. Journal of Law, Economics and Organization, 15, pp. 56–73.

Bhattacharya, S. and Guasch, J. L. (1988): Heterogeneity, Tournaments, and Hierarchies. Journal of Political Economy, 96, pp. 867–881.

Dewatripont, M., Jewitt, I. and Tirole, J. (1999a): The Economics of Career Concerns, Part I: Comparing Information Structures. Review of Economic Studies, 66, pp. 183–198.

Dewatripont, M., Jewitt, I. and Tirole, J. (1999b): The Economics of Career Concerns, Part II: Application to Missions and Accountability of Government Agencies. Review of Economic Studies, 66, pp. 199–207.

Fairburn, J. and Malcomson, J. (1994): Rewarding Performance by Promotion to a Different Job. European Economic Review, 38, pp. 683–690.

Fairburn, J. and Malcomson, J. (2001): Performance, Promotion, and the Peter Principle. Review of Economic Studies, 68, pp. 45–66.

Gourieroux, C. and Monfort, A. (1989): Statistique et Modèles Économétriques, Vol. 2. Economica, Paris.

Green, J. R. and Stokey, N. L. (1983): A Comparison of Tournaments and Contracts. Journal of Political Economy, 91, pp. 349–364.

Holmström, B. (1982): Managerial Incentive Problems - A Dynamic Perspective. In: Essays in Economics and Management in Honor of Lars Wahlbeck, Helsinki.

Lazear, E. P. and Rosen, S. (1981): Rank-Order Tournaments as Optimum Labor Contracts. Journal of Political Economy, 89, pp. 841–864.

Meyer, M. (1994): The Dynamics of Learning with Team Production: Implications for Task Assignment. Quarterly Journal of Economics, 109, pp. 1157–1184.

Nalebuff, B. J. and Stiglitz, J. E. (1983): Prizes and Incentives: Towards a General Theory of Compensation and Competition. Bell Journal of Economics, 3, pp. 21–43.

Prendergast, C. J. (1993): The Role of Promotion in Inducing Specific Human Capital Acquisition. Quarterly Journal of Economics, 108, pp. 523–34.

Prendergast, C. J. (1995): A Theory of Responsibility in Organizations. Journal of Labor Economics, 13, pp. 387–400.

Sliwka, D. (2001): On the Costs and Benefits of Delegation in Organizations. Journal of Institutional and Theoretical Economics, 157, pp. 568–590.

Waldman, M. (1990): Up-or-Out Contracts: A Signaling Perspective. Journal of Labor Economics, 8, pp. 230–50.

Notes

[1]Dewatripont et al. (1999a) provide more general results for a single-agent career concerns model for instance by dropping specific assumptions on distribution functions.

[2]For the relationship between informal and formal authority when a principal can allocate formal decision rights compare Aghion and Tirole (1997). Baker et al. (1999) show that informal commitment may be feasible and beneficial if an infinitely repeated game is considered.

[3]In the latter respect the paper is related to Meyer (1994) who studies how the necessity of learning agents' abilities affects the task assignment policy of a firm in a team production framework without incentive problems.

[4]Note that unlike in Prendergast (1995), we do not consider the case where the principal herself competes with the agents for future rents.

[5]The computation of this expression is shown in the appendix.

[6]Here, $\phi(.)$ denotes the density of a standard normal distribution.

[7]Note that a similar assumption typically is required for the existence of a pure strategy equilibrium in most tournament models (compare for instance Lazear and Rosen (1981) p.845 or Bhattacharya and Guasch (1988) p.871).

[8]Sliwka (2001) shows that a principal will sometimes delegate a decision to an agent with imperfectly known ability instead of making it herself even

if the agent has a lower expected ability as it may turn out that the agent is more talented.

[9]Compare for instance Gourieroux and Monfort (1989) p. 524.

[10]Note that $\phi'(x) = -x\phi(x)$ for the density of the standard normal distribution.