In recent years, several authors have developed models that focus on the allocation of scarce marketing resources based on customer lifetime value (CLV). These approaches use CLV to develop a rank order of customers and recommend devoting more resources to customers with higher ranks. However, it has been discussed in the literature that a simple net present value analysis may not reflect the value of the flexibility to make such decisions. Therefore, some authors recommend the use of a real-options analysis in certain situations. Building on this stream of research and using the case of the option to abandon unprofitable customers, this article proposes an approach that combines real-options analysis and CLV; this approach explicitly values the seller's flexibility to abandon unprofitable customers. Using a combination of examples, empirical analysis, and Monte Carlo simulations, the authors provide evidence that the divergence between CLV that includes and CLV that excludes option value can be substantial and may not be the same for all customers. Therefore, the authors conclude that a distribution of customers based on CLV can change when option value is included. Thus, using CLV as a basis for marketing decisions but not including the value of the option to make such decisions a priori when calculating CLV can lead to an overall biased result.

Approximately 30 years ago, Kotler (1974, p. 24) defined long-term customer profitability as "the present value of the future profit stream expected over a given time horizon of transacting with the customer." Since then, customer lifetime value (CLV) has contributed to solving various problems, from mailing decisions in the catalog sales industry (Bitran and Mondschein 1996) to decisions related to mergers and acquisitions (Selden and Colvin 2003) to questions about company valuation (Gupta, Lehmann, and Stuart 2004). In recent years, several authors have developed models that focus on the allocation of scarce marketing resources based on CLV (e.g., Reinartz, Thomas, and Kumar 2005; Rust, Lemon, and Zeithaml 2004; Venkatesan and Kumar 2004). These approaches use CLV to develop a rank order of customers and recommend devoting more resources to customers with higher ranks. Some authors go so far as to state that customers with low ranks (especially when these show negative CLV) should be abandoned completely to increase overall profitability of the customer base (Zeithaml, Rust, and Lemon 2001).

However, it has been discussed in the literature that the simple net present value (NPV) analysis may not reflect the value of the flexibility to make such decisions. Therefore, some authors recommend the use of a real-options analysis in certain situations (e.g., Amram and Kulatilaka 1999; Dixit and Pindyck 1995; Pindyck 1991). To the best of our knowledge, real-options analysis has been investigated only twice in the context of CLV, namely, in a publication by Levett and colleagues (1999) and a working paper by Hogan and Hibbard (2002). Levett and colleagues propose that the relationship between a buyer and a seller can be expressed as a set of options. Because the customer has the option to choose between buying and not buying at any point in time, the authors determine CLV using a binomial option valuation model. Hogan and Hibbard develop a real-options-based framework for assessing the value of business relationships. They state that the value of any relationship is influenced by the core benefit value and the value of two options: a switching option (i.e., the flexibility to switch to another seller or buyer) and a growth option (i.e., the freedom to expand the relationship beyond its current scope). Thus, relationships can be categorized on the basis of a three-dimensional typology, depending on the magnitude of each of these value components.

In this article, we propose an alternative approach to combining real-options analysis and CLV that explicitly values the seller's flexibility to abandon unprofitable customers. Traditional CLV calculations assume that a company will continue to deal with unprofitable customers, and indeed, a company may do so, in which case regular CLV calculations are appropriate. However, when a firm discon-
tinutes unprofitable customers because it believes that the customer relationship cannot become profitable in future periods, we propose a modification of standard CLV calculations. In this sense, our work differs from that of Levet and colleagues (1999) because we focus on modeling the seller's options, whereas they concentrate on the buyer's options and build on the assumption that the seller will always sell the product when the buyer is willing to purchase it. Our article is also different from Hogan and Hibbard's (2002) work because we develop a specific metric (i.e., CLV) to quantify the value of a specific type of market-based asset (i.e., customer relationships), whereas their approach is conceptual and broader in scope. Because our results provide evidence that the divergence between CLV that includes and CLV that excludes option value can be substantial and may not be the same for all customers, we conclude that a distribution of customers based on CLV can change when option value is included. Therefore, using CLV as a basis for marketing decisions but not including the value of the option to make such decisions a priori when calculating CLV can lead to an overall biased result.

Take the mobile phone industry as an example. Because of the absence of indicators of customer quality at the time of acquisition, mobile phone operators usually attribute the same customer acquisition cost (mainly consisting of handset subsidies) to all customers acquired through the same acquisition channel. To prioritize acquisition channels in the allocation of a limited total budget, the CLV of an "average" customer acquired through any channel is calculated. Because customers expect to replace their handset at regular time intervals, this CLV includes the expected cost of future upgrades. In reality, however, companies upgrade customers only at the end of their acquisition contract if the revenues generated until that point indicate sufficiently good quality to justify a reinvestment in the relationship. At this point, revenue that the customer generates is a much better predictor of future customer quality than simply the information about the acquisition channel. Therefore, mobile phone operators keep the freedom to exercise their option not to upgrade when prior revenues are unfavorable. This usually equates to an abandonment decision because a customer who does not receive a desired upgrade is likely to leave. However, CLV does not account for this flexibility, because it assumes a fixed expected cost of future upgrades. In reality, however, companies upgrade customers only at the end of their acquisition contract if the revenues generated until that point indicate sufficiently good quality to justify a reinvestment in the relationship. At this point, revenue that the customer generates is a much better predictor of future customer quality than simply the information about the acquisition channel. Therefore, mobile phone operators keep the freedom to exercise their option not to upgrade when prior revenues are unfavorable. This usually equates to an abandonment decision because a customer who does not receive a desired upgrade is likely to leave. However, CLV does not account for this flexibility, because it assumes a fixed expected cost of future upgrades for all customers acquired through the same channel. Thus, the value of the flexibility not to upgrade low-value customers is not included in the calculation of CLV. This makes the traditional CLV measure inconsistent with actual behavior. Therefore, using CLV determined this way when allocating a limited total customer acquisition budget across acquisition channels is likely to lead to a bias.

The remainder of this article is structured as follows: In the next section, we introduce real-options theory by showing how this valuation technique can overcome some limitations of simple NPV analysis and how it can be applied in a marketing context. We then use two examples to illustrate how the value of the option to abandon unprofitable customers can be calculated. We go on to validate our approach empirically using data provided by the Direct Marketing Educational Foundation (DMEF) to estimate the divergence between CLV that includes and CLV that excludes option value. Subsequently, we conduct a Monte Carlo simulation to identify factors that influence this divergence. The article concludes with a discussion of the theoretical and managerial implications of our findings, as well as the limitations of our approach and areas of further research.

**Real-Options Theory**

Authors in the area of strategic investment decision making have noted that simple NPV analysis is associated with several implementation and conceptual problems. Although some of these problems can be cured by modifications of this approach that have been discussed in the literature (for a detailed list and explanation, see Adler 2000), a shortcoming remains (e.g., Amram and Kulatilaka 1999; Dixit and Pindyck 1995): NPV analysis assumes that one specific value for some kind of metric (e.g., cash flow) can be defined for each period within the time horizon of the analysis. Although it is possible to account for future uncertainty by specifying more than one state of nature per period, different values of this metric must be combined into one expected value before NPV can be calculated. Therefore, simple NPV analysis builds on the assumption that a project can be completely planned beforehand and ignores the project's potential to create future options that are not foreseeable based on the current stock of knowledge. Such options could include the freedom to abandon the project in case of unfavorable conditions or to extend its scope in case of favorable ones. However, it is a characteristic of options to show an asymmetric behavior in net payoffs because they always represent the right but not the obligation to carry out a future action. Thus, because NPV analysis ignores these options, it often underestimates the true value of an investment (e.g., Benninga and Tolkowsky 2002). The literature on the strategic management–finance interface that has emerged around the valuation of such (real) options is extensive, whereas to our knowledge, there are only four studies that discuss this approach in marketing (Finch, Becherer, and Casavant 1998; Hogan and Hibbard 2002; Levet et al. 1999; Slater, Reddy, and Zwirlein 1998). Nevertheless, the application of a real-options approach to customer acquisition/retention analysis using CLV seems to be particularly appropriate. These activities usually create future options because the decision to acquire or reinvest in a customer always creates the choice of further investment in the relationship in subsequent periods.

An essential step after identifying the existence of a real option is its valuation. For this task, there are three different approaches that can be used: Black and Scholes's (1973; hereinafter, the Black–Scholes model) model, contingent claims valuation (Cox, Ross, and Rubinstein 1979), and dynamic programming (for an application to real-options valuation, see Copeland and Tufano 2004). The Black–Scholes model has the advantage of providing a closed-form solution for option value. It has previously been applied in the context of real-options analysis (e.g., Luehrman 1998), but its drawback is that it is computationally demanding. Contingent claims valuation is much easier to apply than the Black–Scholes model and uses less strin-
gent assumptions, though it includes the Black–Scholes formula as a limiting case. It has also been used to value real options (e.g., Sachdeva and Vandenberg 1993), but it builds on the assumption of arbitrage-free markets and thus assumes that the real option is tradable. This assumption may be questionable in the context of CLV calculations. In addition, as Devinney and Stewart (1988) note, models that have their roots in financial markets cannot always be applied to product markets, because this may violate key assumptions underlying them. Therefore, we do not recommend either of these two approaches for valuing real options in a marketing context; rather, we suggest the use of a third method, the dynamic programming approach.

Dynamic programming is a class of solution methods for solving sequential decision problems. It builds on the assumption that “the optimum way of getting from a particular state to an end point is not a function of the path taken to that state” (Elton and Gruber 1971, p. 480). This is often also referred to as Bellman’s principle of optimality. Thus, the optimal decision at any time t can be identified by maximizing the sum of the paybacks from following a particular policy at time t plus the expected paybacks from following an optimal policy from time $t + 1$ onward. In the context of real-options analysis, dynamic programming helps examine options from more of a decision theory point of view. It is not based on the same assumptions underlying financial option valuation. Nevertheless, despite this fundamental difference, the contingent claim value of an investment can also be identified by the dynamic programming approach. However, this is true only after parameter calibration (Knudsen, Meister, and Zervos 1999) because the dynamic programming method assumes an exogenously determined discount rate, whereas contingent claims valuation determines a risk-adjusted discount rate endogenously.

In the following section, we use two examples to show how dynamic programming can be applied to valuing a seller’s abandonment option to terminate business with a customer who has proved to be unprofitable. We propose a real-options-based approach to CLV, which explicitly values the option to abandon unprofitable customers.

**Examples**

Imagine a marketing manager who must choose between implementing one of two alternative advertising campaigns (A and B). With Campaign A, the company can attract “good” customers with a probability of 30% and “bad” ones with a probability of 70%. A good client will generate a net profit of $1,250 in Period 1, whereas a bad one is assumed to result in a net loss of $-470 in the same period. For both types of clients, this net contribution can either improve or deteriorate by 20% in Period 2, whereas the likelihood of an improvement will be 5% for a good and 10% for a bad customer. In this context, the term improvement is viewed from a company perspective and represents either a profit increase or a loss decrease (see Figure 1). With Campaign B, only one type of customer, one that generates a constant net profit of $46 in Periods 1 and 2, can be acquired. For both campaigns, the average cost of recruiting the customer and setting up a new account is assumed to be $50, and the discount rate is 15%. Using an average lifetime of two periods, the CLV of a newly acquired customer based on a simple NPV analysis is approximately $-16 for Campaign A and $75 for Campaign B. Because the CLV of Campaign B is greater than that of Campaign A and greater than the acquisition cost, the decision would be in favor of Campaign B.1

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1Although the example is an artificial one, the key numbers can be considered realistic for the special case of the retail banking industry. The net contribution of a good and bad customer in Campaign A is determined on the basis of an expected revenue of $1,600 and $230 and the expected cost of $350 and $700, respec-
However, as we noted previously, NPV analysis does not account for the flexibility of the marketing manager after the end of Period 1. At that point, the marketing manager knows (e.g., on the basis of experience from comparable campaigns already carried out by the company) that a customer who proved to be bad in Period 1 will generate an expected loss in Period 2. Therefore, the marketing manager has the option to behave asymmetrically in Period 2; that is, in the case of a good customer, the client relationship will continue, and in the case of a bad one, it will be abandoned. This asymmetrical behavior protects the manager from the downside risk to realize the loss of a bad customer in Period 2. Simple NPV analysis does not account for this option because only one value for the expected net contribution of a customer in Period 2 needs to be determined. We assume that the marketing manager decides to terminate the client relationship with a bad customer at the end of Period 1 and that this abandonment does not cause additional cost. In this case, the lifetime value of a bad customer increases from ~$821 to ~$409, which increases the expected CLV from ~$16 to $273. On the basis of this increase of $289, which can be interpreted as the value of the abandonment option, Campaign A now seems far more attractive than Campaign B.

The previous example builds on a simple situation and attempts to explain the basic idea behind our approach in more detail. Now, we apply our approach to a more complex and, thus, more realistic situation.

Imagine that a marketing manager is faced with the decision to acquire a new customer, Mrs. Smith. Again, we assume that this acquisition costs $50. In every period after acquisition, Mrs. Smith can purchase a high-margin product, a low-margin product, or no product at all. We assume that a high-margin product generates a revenue of $200 and that a low-margin product generates a revenue of $100. In every period, the bank invests marketing costs into the relationship with Mrs. Smith. In the first period after acquisition, these costs are assumed to be $100. In every following period, they depend on Mrs. Smith’s purchase in the previous period. If she purchases a high-margin product, marketing costs will be $100; if she purchases a low-margin product, marketing costs will drop to $50; and if she makes no purchase, marketing costs will be $25. After every period, the marketing manager has the choice to terminate the relationship with Mrs. Smith. However, we assume that this decision is associated with an abandonment cost of $25. On the basis of previous experience with customers similar to Mrs. Smith in terms of a set of observable characteristics (e.g., acquisition channel, demographics, socioeconomic status), the marketing manager expects a purchasing behav-

ior for the three periods after acquisition (see Figure 2). The discount rate assumed for the analysis is 15%.

Using this information, Mrs. Smith’s CLV is $46.51 when option value is excluded. Because this is less than the acquisition cost of $50, the decision would be not to acquire Mrs. Smith. However, this CLV does not account for the option to behave asymmetrically in future periods. After every period, the marketing manager can terminate the relationship with Mrs. Smith depending on the future net contribution streams expected from this relationship. If we apply a dynamic programming approach to this example, there are three points at which abandonment is economically sensible: (1) when Mrs. Smith purchases a high-margin product in the period after acquisition, (2) when she purchases a low-margin product in the period after acquisition and a high-margin product in the next period, and (3) when she makes no purchase in the period after acquisition and purchases a low-margin product in the next period. Including the value of the option to abandon Mrs. Smith at these three points in time leads to a CLV of $52.97, which is greater than the acquisition cost. Thus, when option value is accounted for, the acquisition of Mrs. Smith is a worthwhile strategy.

Abandoning Mrs. Smith at these three points makes sense because of a combination of two factors: First, there is a relatively high probability of no purchase in the third period after acquisition, especially when purchases have been carried out in previous periods. This can be explained either by Mrs. Smith’s saturation in terms of product consumption or by budget constraints she might be facing. For example, after Mrs. Smith purchases a high-margin product in Periods 1 and 2 after acquisition, the probability that she will not purchase in the following period is 80%. Second, marketing expenditures depend on previous purchasing patterns. This makes a no-purchase scenario highly unattractive when, for example, it is preceded by a high-margin product purchase. There are also other situations in which Mrs. Smith generates a loss; however, in such cases, this loss is less than the abandonment cost of $25, so the abandonment is not a recommendable strategy (given the time frame chosen for analysis). This example shows that in a more complex and realistic setting, the value of the abandonment option requires calculations that can no longer be easily carried out using simple spreadsheet methods. Thus, there is a need to apply more advanced option-pricing approaches.

At this point, it is important to note how real-options analysis differs from two other techniques that have been discussed in the literature: Bayesian updating and elementary decision tree analysis. Bayesian updating is closely linked to real-options analysis in that both techniques deal with the revision of prior beliefs in light of new information. However, there are some specific characteristics of real options, for example, the inherent asymmetry included in their analysis or that the value of a real option increases, with both reduced and increased uncertainty. Thus, it is more appropriate to view Bayesian updating as a routine component of real-options analysis than to view both techniques as identical (for a more detailed discussion, see Herath and Park 2001; Miller and Park 2005). With respect
FIGURE 2
Expected Future Purchasing Probabilities of Mrs. Smith (Example 2)
Empirical Validation

The aforementioned examples, which are built on hypothetical figures, should be viewed only as an illustration of our basic idea. In this section, we focus on empirically validating our approach and on obtaining an estimate of the divergence between CLV that includes and CLV that excludes option value. We use empirical data provided by the DMEF (academic data set three), which consists of 12 years of sanitized purchasing data for more than 100,000 customers of a specialty catalog company. Our empirical validation consists of four steps: First, we estimate the future purchasing behavior (i.e., probability of purchase and dollar amount expected to be spent) for a set of customer segments using the recency, frequency, and monetary value (RFM) approach. Second, we determine costs generated per customer per year. Third, we use these two inputs to estimate the expected future revenue and profit contribution for each customer segment. Fourth, we calculate CLV that includes versus CLV that excludes option value and compare these values on customer base level to obtain an estimate of the divergence between them (for computational details regarding our approach, see the Appendix).

The first step, the estimation of the future purchasing behavior for a set of customer segments, is carried out with the RFM approach. The RFM approach, which Cullinan (1978) first presented and Bauer (1988) subsequently extended, assumes that a customer's future purchase behavior is determined by three purchase history variables: recency, frequency, and monetary value. Recency is the time elapsed since the previous purchase, frequency is the number of purchases made in a specified time period, and monetary value is the dollar sales value of prior purchases (Bauer 1988). Although other, more advanced techniques have been proposed in the literature to model future customer behavior, such as the Pareto/NBD model (Schmittlein, Morrison, and Colombo 1987) and the beta-geometric/NBD framework (Fader, Hardie, and Lee 2005a), we decided to apply the RFM approach in our context. Because the focus of our empirical validation lies in comparing two approaches to determining CLV given an assumed future purchasing behavior, and not in evaluating the predictive accuracy of these methods or in proposing a better method to forecast customer activity, the RFM approach, which is widely known and easy to use, is preferable.

To use the RFM approach, we first split the 12 years of historical purchase data available in our data set into two groups, the first one covering two-thirds (8 years, from t - 12 to t - 5) and the second group covering one-third (4 years, from t - 4 to t - 1). We then determine recency (operationalized as the number of years since the last order) and frequency (measured as the total number of years in which at least one purchase occurred) as of t0 = t - 4. This leads to eight values for recency and frequency. From the total number of 106,284 customers in our data set, 47,617 could be assigned to one of the resulting 36 distinct recency-frequency cells. The remaining 58,667 customers did not make any purchases between t - 5 and t - 12 and were discarded from the analysis.

For each of these 36 customer segments, we subsequently calculate conditional purchase frequencies and average dollar amounts spent for all possible purchasing patterns between years t - 4 and t - 1 (for a similar approach, see Berger, Weinberg, and Hanna 2003). Figure 3 visualizes the purchasing pattern of one segment, namely, customers with a recency and frequency of 1 (i.e., customers who last purchased in t - 5 and conducted exactly one purchase between t - 5 and t - 12). For ease of notation, we refer to this segment as "Segment 1" in the following discussion. In this graph, a purchase in one year is represented by an upward move and a white cell, whereas no purchase is represented by a downward move and gray-shaded cell. From the total of 8345 customers in this segment, 2715 conducted a purchase in t - 4, and 5630 did not. Thus, the conditional purchase frequency for a customer to carry out a purchase in t - 4 is 32.53% (2715/8345). Because these 2715 customers spent a total of $229,088, the associated average dollar amount spent is $84.38 per customer ($229,088/2715). In total, there are 15 conditional purchase frequencies and average amounts spent (corresponding to the 15 white cells) per customer segment that can be determined this way. The resulting 15 pairs of values serve as a proxy for future purchasing behavior in t to t + 3 for all consumers who belong to this specific segment.

The second step consists of the estimation of costs generated per customer per year. Specifically, we include two types of costs in our model: the cost of goods sold and direct marketing expenditures. We assume that the cost of goods sold represents a certain percentage of the average dollar amount spent per year, and we estimate this percentage to be approximately 55% on the basis of data available in our data set. For direct marketing expenditures, we face the problem that the data set does not provide sufficient information for their estimation. Therefore, we build on the work of Gönil and Shi (1998, p. 1256), who assume an average cost per mailing of $2.50. On the basis of the assumption that our specialty catalog company might send out between zero and four catalogs per year (i.e., at a maximum of one catalog per season), we assume direct marketing expenditures to vary between $0 and $10 per customer per year. Because there is no way to determine which of these values is more likely than the others, we calculate 100 different scenarios by increasing cost in steps of $1.0, beginning at $0 and continuing to $10.

---

2 We allow marketing expenditures to vary in $.10 increments instead of assuming five distinct values ($0.00, $2.50, $5.00, $7.50,
FIGURE 3
Purchasing Pattern of Customer Segment 1 in Years t – 4 to t – 1

| Total Number of Customers (Dollar Amount Spent by Customers in Respective Year) |
|----------------------------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| t - 5                            | t - 4           | t - 3           | t - 2           | t - 1           |
| 8345                             | 2715 ($229,088) | 1180 ($113,023) | 5630            | 384 ($36,500)   |
|                                  |                 |                 |                 |                 |
| 1535                             | 362 ($25,937)   | 229             | 123 ($10,482)   |                 |
|                                  |                 |                 |                 |                 |
| 808 ($56,624)                    | 321 ($26,841)   | 198             |                 |                 |
|                                  |                 |                 |                 |                 |
| 5630                             | 475 ($33,982)   | 147 ($11,918)   |                 |                 |
|                                  |                 |                 |                 |                 |
| 4822                             | 4347            | 4095            |                 |                 |

Unprofitable Customers and Customer Lifetime Value / 11
In the third step, we use these two inputs to estimate the expected future revenue and profit contribution of an average customer in each segment. We first calculate expected revenue as the average amount spent given a certain purchasing pattern multiplied by the probability that this purchasing pattern can be observed. We then calculate profit contribution as expected revenue less the cost of goods sold less marketing expenditures. Marketing expenditures are weighted with the probability of observing last year’s purchasing pattern because we assume that they are paid out at the beginning of each year. Figure 4 shows the resulting profit contributions for an average customer in Segment 1, determined on the basis of the information provided in Figure 3 and the assumption that marketing expenditures are equal to $5 per customer per year.

In the fourth step, we calculate CLV that includes and CLV that excludes option value. We discount profit contributions back to the beginning of year t and sum them up, which results in a CLV of an average customer in Segment N that excludes option value $\text{CLV}^N_{\text{EOV}}$. We make the simplifying assumption that profit contributions are realized in the middle of each year because expected revenue can occur at any time, whereas we assume that marketing expenditures are paid out at the beginning of each year. Using a discount rate of 20% and the example of Segment 1, CLV$^1_{\text{EOV}}$ is equal to $\text{CLV}^1_{\text{EOV}}$. To calculate CLV that includes option value $\text{CLV}^N_{\text{IOV}}$, we must first determine the activity status of each customer conditional on his or her purchasing pattern. To do this, we make two assumptions: (1) that a customer should be abandoned in year T, whenever the sum of expected future profit contributions from T onward discounted back to T is less than the cost associated with the abandonment, and (2) that an abandoned customer becomes inactive for all years after abandonment. Then, it is straightforward to define a variable that describes the activity level of each customer conditional on his or her purchasing pattern, which takes the value of 1 in the case of the customer being active and 0 in the case of abandonment. In the specific case of Segment 1, this rule results in the decision that a customer who did not conduct a purchase during time t should be abandoned. Thus, $\text{CLV}^N_{\text{IOV}}$ can be defined as the discounted sum of expected profit contributions multiplied by the corresponding activity level. On the basis of a discount rate of 20% and the assumption that abandonment is not associated with any cost, CLV$^1_{\text{IOV}}$ is equal to $13.65$ for the example of Segment 1. To obtain an estimate of the divergence between these two values of CLV on customer base level, we multiply $\text{CLV}^N_{\text{EOV}}$ and $\text{CLV}^N_{\text{IOV}}$ by the total number of customers in Segment N and sum them over N. Although this is not strictly necessary—a comparison could also be carried out between $\text{CLV}^N_{\text{EOV}}$ and $\text{CLV}^N_{\text{IOV}}$ directly—we carry out this calculation to obtain a weighted average of these two values over all segments.

Figure 5 shows the divergence (calculated as $\text{CLV}^N_{\text{IOV}}/\text{CLV}^N_{\text{EOV}} - 1$) between these two values as a function of marketing expenditures per customer per year. The result supports our hypothesis that CLV that excludes option value underestimates customer value. For marketing expenditures equal to $5$ per customer per year, the divergence is ~1.5% on average, and for marketing expenditures greater than $6.50$, it consistently exceeds 5%. For any given marketing expenditure, the divergence at the customer segment level differs greatly from one segment to another. For example, in the case of marketing expenditures of $7.50$ per customer per year, segment-specific divergence varies from 0% to 70%. This latter point shows that there may be factors other than cost that determine the divergence. Therefore, we conduct a Monte Carlo simulation to investigate this point further.

Monte Carlo Simulation

The purpose of our Monte Carlo simulation is to gain a better understanding of the influence of factors other than cost on the divergence between CLV that includes and CLV that excludes option value. Specifically, we focus on the potential impact of a change in expected revenues, which we modify by changing the standard deviations and average values for the probability of purchase and dollar amount expected to be spent. The simulation consists of two steps: First, we analytically identify which kind of influence these factors can be expected to have on the divergence. Second, we conduct a set of Monte Carlo simulations to determine the order of magnitude of this influence.

For the first step, the analytical identification of the kind of influence to be expected, we build on the equations that Black and Scholes (1973) derive to value financial options. The Black–Scholes model has the advantage of providing a closed-form solution for the value of a (real) option, which is not given either for contingent claims valuation or for dynamic programming. Although we applied dynamic programming, not the Black–Scholes model, as a valuation technique in our empirical validation and the following Monte Carlo simulation, we are comfortable with this choice in the context of the analytical verification. The Black–Scholes model is included as a limiting case in contingent claims valuation (Cox, Ross, and Rubinstein 1979), and contingent claims valuation and dynamic programming are equivalent modulo parameter calibrations (Knudsen, Meister, and Zervos 1999).

Black and Scholes (1973) show (not building on dynamic programming but on the solution of heat-transfer equations in physics) that the value of a put option $v_{RO,P}$ (i.e., the right to sell a stock) can be calculated using the following three equations:

3 We also conduct a similar analysis in which cost of goods sold represent 50% and 60% of revenue. The results show that an increase (decrease) in cost of goods sold simply results in an upward (downward) shift of the graph without substantially influencing variance in segment-specific divergence.

4 We use the equations that describe a put versus a call option because the real option of abandonment is more similar to a put option in financial terms (Sachdeva and Vanden Berg 1993).
 FIGURE 4
Profit Contribution of an Average Customer in Segment 1

<table>
<thead>
<tr>
<th>t - 1</th>
<th>t</th>
<th>t + 1</th>
<th>t + 2</th>
<th>t + 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>$7.35</td>
<td>$4.47</td>
<td>$2.75</td>
<td>$1.57</td>
<td></td>
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<tr>
<td>$7.35</td>
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In this expression, \( x \) represents the stock price, \( c \) represents the exercise price, \( r \) represents the risk-free interest rate, \( \sigma^2 \) represents the variance rate of the return on the stock, and \( t \) represents the time to expiry. The notation \( \ln(a) \) is used to describe the natural logarithm, and \( N(a) \) is used to represent the cumulative normal density function. In the context of real-options analysis, this interpretation changes slightly. As Luehrman (1998) discusses, \( x \) can then be viewed as the present value, \( \sigma^2 \) can be viewed as the riskiness of expected cash flows, and \( c \) can be interpreted as the present value of fixed costs. As we stated previously, the focus of our Monte Carlo simulation is to investigate the potential impact of a change in the standard deviations and average values assumed for the probabilities of purchase and dollar amounts expected to be spent. Because expected revenues can be determined as the product of these two...
parameters, this is equal to investigating a change in the standard deviations and average values of expected revenues. Using the terminology of the Black–Scholes model, we focus on the potential impact of changes in \( \sigma^2 \) and \( x \) on real-options value.

To derive the impact of these two variables on option value analytically, we calculate the partial derivatives of \( \text{VRoP} \) with respect to \( \sigma \) and \( x \). Keeping in mind that

\[
\frac{\partial N(a)}{\partial \sigma} = f(a) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}a^2},
\]

where \( f(a) \) is the normal density function, it follows that

\[
\frac{\partial \text{VRoP}}{\partial \sigma} = -\frac{\partial t}{\partial \sigma} \left[ e^{-t}f(-d_2) - x f(-d_1) \right] + e^{-t}f(-d_2) \sqrt{t}
\]

and that

\[
\frac{\partial \text{VRoP}}{\partial x} = -\frac{\partial t}{\partial x} \left[ x f(-d_1) - e^{-t}f(-d_2) \right].
\]

Because \( d_2 = d_1 - \sqrt{t} \) and

\[
f(-d_2) = f(-d_1) e^{\sigma^2 t},
\]

this can be simplified to

\[
\frac{\partial \text{VRoP}}{\partial \sigma} = ce^{-t} \sqrt{t} f(-d_2) \quad \text{and}
\]

\[
\frac{\partial \text{VRoP}}{\partial x} = -N(-d_1).
\]

In this simple form, the partial derivative of \( \text{VRoP} \) with respect to \( \sigma \) has a positive sign, and the one with respect to \( x \) has a negative sign. Thus, \( \text{VRoP} \) increases with increasing \( \sigma \) and decreasing \( x \). In other words, the value of the real option of abandonment increases with increasing uncertainty and decreasing present value of expected cash flows. Translating this back to the factors we investigated in the context of the Monte Carlo simulation, this implies that the difference between \( \text{CLV}_{\text{IOV}} \) and \( \text{CLV}_{\text{EOV}} \) can be expected to increase with increasing standard deviations and decreasing average values of future purchase probabilities and dollar amounts expected to be spent.

In the second step, we then carry out a set of Monte Carlo simulations to get an idea of the order of magnitude of these influences. For this, we build on the always-a-share CLV example that Dwyer (1989) proposes. Dwyer’s approach is essentially equivalent to the one we used in the empirical validation; it is characterized by the following two assumptions: First, purchase probabilities, dollar amounts expected to be spent, and marketing expenditures
are assumed to depend on the recency cell of each customer (see Table 1). Second, cost of goods sold is assumed to represent 60% of revenue. Dwyer’s example is more realistic than our empirical validation in some dimensions (i.e., if we assume that marketing expenditures depend on each customer’s recency rather than that they are constant, as in our case) and less in others (i.e., if we assume that purchase probabilities and dollar amounts expected to be spent depend only on recency rather than that purchase probabilities and dollar amounts expected to be spent depend on recency and frequency, as in our analysis). However, because the focus of our Monte Carlo simulation is to investigate how the divergence between CLV that includes and CLV that excludes option value changes with a variation in model parameters, it is not essential that the simulation is completely identical to our empirical validation. It is necessary only to ensure that the assumptions underlying it change in a controlled manner among different scenarios. However, it should be kept in mind that the absolute values of divergence are not comparable. In total, we carried out 1.8 million simulations, split into nine scenarios with 200,000 runs each, to investigate how the divergence changes when we change (1) the standard variation in future purchase probabilities and dollar amounts expected to be spent and (2) the average values assumed for these two parameters.

To address the first objective, the investigation of the change in the divergence associated with a change in standard deviations, we assume that the eight values for purchase probabilities and dollar amounts expected to be spent (one probability and one dollar amount for each of the four recency cells) behave according to a normal distribution with average values as assumed by Dwyer (1989; see Table 1) and standard deviations that represent 5% of these average values. For modeling purchase probabilities, we used a truncated normal distribution to avoid probabilities that were inferior to 0 and larger than 1. We then conducted the following three-step procedure: First, we simulated 20,000 customers by calculating probabilities and dollar amounts expected to be spent on the basis of these distributional assumptions. Second, we determined the resulting divergence between CLV that includes and CLV that excludes option value for each of them. We excluded all customers for which CLV_{EOV} < 0 because this resulted in CLV_{JIV} = 0 and a divergence of -100%. Third, we calculated an overall average divergence for all 20,000 customers as the median of all customer-level divergences. We decided on the median rather than the arithmetic mean to obtain a conservative estimate and to avoid biases caused by outliers as far as possible. We subsequently increased the standard deviations in 5% increments up to 50% of the average values and followed the same three-step procedure described previously. The results of these 200,000 runs (10 different standard deviations with 20,000 runs each) appear in Figure 6, in which we plotted the resulting overall average divergence as a function of the standard deviation.

Figure 6 shows two interesting properties: First, there appears to be a quadratic relationship between the average divergence and the standard deviation. This is also confirmed by an associated analysis we conducted using SPSS. Regressing overall average divergence on standard deviation results in an adjusted R-square of .735, an F value of 26.01, and a t-value of the dependent variable of 5.10, whereas regressing it on the squared standard deviation (i.e., variance) results in an adjusted R-square of .909, an F value of 91.35, and a t-value of 9.56. This relationship is intuitive because increasing variance is associated with increasing probabilities for unprofitable customer relationships and, thus, increasing occurrence (and value) of abandonment decisions. Second, the absolute impact of standard deviation on average divergence seems to be marginal. Although standard deviation increases tenfold from 5% to 50%, the average divergence increases only slightly (from 0.85% to 1.21%). In summary, this analysis confirms our theoretical finding that average divergence increases with increasing standard deviation. Nevertheless, this increase seems to be of limited size.

With respect to the second objective, the analysis of a potential influence of a change in average values on a change in overall average divergence, we again use Dwyer’s (1989) assumptions as a starting point. We subsequently modify the average values for purchase probabilities and dollar amounts spent by either doubling or halving the values. This leads to seven other scenarios; for each scenario, we conducted the same analysis described previously (i.e., increasing the standard deviation in 5% increments from 5% to 50% and determining an overall average divergence), the results of which appear in Table 2. The overall average divergence increases with decreasing purchase probabilities and dollar amounts spent, which is in line with our theoretical expectations. For the range of values we investigated in our simulation, this effect seems to be substantially larger than the impact of standard deviation on average diver-

---

3This result was also confirmed in the seven other scenarios we calculated. Because of space constraints, we show only one graph, but this additional information can be obtained from the first author on request.

---

### TABLE 1

**Assumptions of Always-a-Share CLV Example as Proposed by Dwyer (1989)**

<table>
<thead>
<tr>
<th>Recency Cell</th>
<th>Description</th>
<th>Probability of Purchase in Period t (%)</th>
<th>Amount Expected to Be Spent in Period t ($)</th>
<th>Marketing Expenditures in Period t ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Customer last purchased in t - 1</td>
<td>30</td>
<td>100</td>
<td>3.60</td>
</tr>
<tr>
<td>2</td>
<td>Customer last purchased in t - 2</td>
<td>20</td>
<td>80</td>
<td>3.10</td>
</tr>
<tr>
<td>3</td>
<td>Customer last purchased in t - 3</td>
<td>15</td>
<td>60</td>
<td>1.80</td>
</tr>
<tr>
<td>4</td>
<td>Customer last purchased in t - 4</td>
<td>5</td>
<td>40</td>
<td>1.30</td>
</tr>
</tbody>
</table>

Unprofitable Customers and Customer Lifetime Value / 15
FIGURE 6
Divergence Between Customer Base Values Determined by Including Versus Excluding Option Value
(Simulation, Average Values as Noted in Table 1)

![Graph showing the divergence between customer base values determined by including versus excluding option value.](image)

TABLE 2
Divergence Between Customer Base Values Determined by Including Versus Excluding Option Value
(Simulation)

<table>
<thead>
<tr>
<th>Purchase Probability</th>
<th>Dollar Amount Expected to Be Spent</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Halved</td>
</tr>
<tr>
<td>Halved</td>
<td>11.52%</td>
</tr>
<tr>
<td>Unchanged</td>
<td>5.99%</td>
</tr>
<tr>
<td>Doubled</td>
<td>.25%</td>
</tr>
</tbody>
</table>

Notes: "Halved," "unchanged," and "doubled" refer to the average values in Table 1. We did not calculate an overall average divergence for the scenario in which both purchase probabilities and dollar amounts spent were halved, because in this case, for all standard deviations, at least 80% of all simulated customers resulted in CLV < 0.

In summary, several authors have used CLV to develop models that focus on the allocation of scarce marketing resources (e.g., Reinartz, Thomas, and Kumar 2005; Rust, Lemon, and Zeithaml 2004; Venkatesan and Kumar 2004). However, it has been discussed in the literature that simple NPV analysis may not reflect the value of the flexibility to make such decisions. Therefore, some authors recommend the use of real-options analysis in certain situations (e.g., Amram and Kulatilaka 1999; Dixit and Pindyck 1995; Pindyck 1991). Building on this stream of research, we proposed a real-options approach for customer valuation. Using the case of the option to abandon unprofitable customers, we first illustrated through example the basic idea behind real-options analysis and its application to determine CLV. We then used data provided by the DMEF to validate...
our approach empirically and to obtain an estimate of the divergence between CLV that includes and CLV that excludes option value. Finally, we conducted a set of Monte Carlo simulations to obtain a better understanding of factors that influence this divergence. We showed both analytically and through our simulation that the divergence increases with (1) decreasing future purchase probabilities, (2) decreasing dollar amounts expected to be spent in future purchases, and (2) increasing variance in these two parameters. Regarding the order of magnitude of these influences, our simulations provide evidence that the impact of a change in variance is marginal compared with a change in average values. In addition, a change in average purchase probabilities has a larger impact than a change in average dollar amounts spent.

From a theoretical perspective, our approach contributes to research in the area of CLV by proposing a methodology that can be used to incorporate the value of the seller’s flexibility into CLV. In this sense, it adds to the literature in three ways: First, it complements Levett and colleagues’ (1999) publication and adds to Rust, Lemon, and Zeithaml’s (2004) work, both of which explicitly model the customer’s flexibility when determining CLV. Second, it addresses two research directions that Hogan and Hibbard (2002) propose; specifically, by using the example of the option to abandon unprofitable customers, we developed a metric to quantify the value of business relationships using the real-options-based framework by explicitly modeling the decision to “fire a customer.” Finally, the current study can be viewed in the context of Hogan, Lemon, and Libai’s (2003) article because we provide evidence that customers who appear to be unprofitable from an NPV-based CLV perspective may not be as unattractive as they seem.

Therefore, on the basis of our findings, we recommend including option value in the calculation of CLV. Our results show that neglecting option value can lead to a substantial underestimation of true customer value. Because this underestimation is not uniform across the whole customer base, it is likely to affect decisions made on the basis of a rank order of customers developed using CLV. The results of our Monte Carlo simulation show that customers with low probabilities of future purchase and low dollar amount expected to be spent tend to be underestimated more often than customers with more favorable values along these two dimensions. Thus, a rank order of customers based on CLV may be different when CLV is calculated including option value than when it is calculated excluding option value. Most methods for the allocation of scarce marketing resources recommended in the literature (e.g., Reinartz, Thomas, and Kumar 2005; Rust, Lemon, and Zeithaml 2004; Venkatesan and Kumar 2004) build on such a rank order of customers and recommend the allocation of more resources to customers with higher ranks. However, our results lead to the hypothesis that this allocation could be substantially different if option value were included.

With respect to the implementability of our approach, we showed in our example and our empirical validation that the calculation of CLV that includes option value does not require additional data compared with the determination of CLV that excludes option value. Many companies already classify customers into a set of distinct RFM cells. Thus, the basic information to determine option value is most likely already available in any given company. If it is not available, it can easily be generated if information about the purchase history of each individual customer is known. Building on this allocation of customers into distinct RFM cells, it is possible to determine option value using standard spreadsheet software. We conducted all calculations in this article using Excel 2002.

Regarding the practicability of abandoning unprofitable customers, approaches have been discussed in the literature for a long time under the label “selective demarketing” (Kotler and Levy 1971). With the increasing importance of CLV, the topic has recently received the attention of a wider range of researchers (e.g., Rosenblum, Tomlinson, and Scott 2003; Selden and Colvin 2003; Zeithaml, Rust, and Lemon 2001). Techniques for dealing with unprofitable customers that have been proposed in this context can be grouped into “hard” and “soft” approaches. Hard approaches imply telling the customer directly that he or she is abandoned. For example, a recent article in The Seattle Times (Freed 2004) speaks about the Massachusetts-based fashion retailer Filene’s Basement. This company banned two sisters, who claimed to have been loyal customers for years, from all 21 of its stores after deciding that they returned too many items after wearing them for an evening and complained too often about service. In contrast to this strategy, soft approaches summarize all actions that can be taken to make the customer leave on his or her own. Among others, this includes not sending any promotional materials to such customers, introducing additional fees (e.g., chargeable customer service calls), and stopping certain services (Selden and Colvin 2003).

**Limitations and Areas of Further Research**

Our study has two limitations that further research could address: First, our findings apply only to the specific type of option we analyzed, namely, the abandonment option. Thus, the degree of divergence we determined during our empirical validation and the factors that influence it should not be generalized to other types of real options. We chose this specific option type as an illustration because some authors argue that the flexibility that lies at the core of the real-options approach requires the possibility of abandoning investment initiatives. This perspective makes abandonment essential for limiting downside risk (Adner and Levinthal 2004). It also differentiates abandonment options from other option types that have been discussed in the literature (i.e., defer, learning, growth, and switching options). However, our approach can be modified easily to reflect these types of options. Nevertheless, further analysis is needed to obtain a better understanding of this type and other types of options.

Second, our empirical analysis and Monte Carlo simulation both assume that abandoning a customer is not associ-
ated with any cost. However, this is unlikely to be the case. Although it may be possible to cease business with an unprofitable customer without creating additional direct cost, abandonment carries the risk of creating an image of unfairness and discrimination in the market, which could lead to substantial indirect cost (Cullwick 1975). For example, previous research has shown that companies that behave unfairly may be punished in the long run, even when this results in cost to their customers (Kahneman, Knetsch, and Thaler 1986). One such punishment could be the spread of negative word of mouth, the effects of which can be substantial if the number of customers who experience dissatisfaction is high enough (Richins 1983). Such indirect abandonment costs should be considered, perhaps by an approach similar to the one that Hogan, Lemon, and Libai (2004) use to quantify positive word of mouth.

In addition to these two areas of further research, we believe that studies calculating real-options value by making use of more sophisticated approaches to model future customer behavior, such as the Pareto/NBD (Schmittlein, Morrison, and Colombo 1987) or the beta-geometric/NBD (Fader, Hardie, and Lee 2005a) framework, would be worthwhile. Such methods could help account for heterogeneity among different customers more effectively than the RFM approach we used in our empirical validation, which could contribute to a better understanding of customer-specific factors that influence option value. Furthermore, this line of research would be an initial step in marrying our approach with the work done regarding the calculation of CLV based on these approaches (e.g., Fader, Hardie, and Lee 2005b; Reinaert and Kumar 2000, 2003). Finally, we believe that combining our work with publications focused on the integration of the buyer's flexibility into CLV (e.g., Levett et al. 1999; Rust, Lemon, and Zeithaml 2004) to come up with an integrated model that simultaneously values the buyer's and seller's options would be a promising area of further research. Such a model could be helpful in two ways: First, it could further improve the understanding of the “true value of a lost customer” (Hogan, Lemon, and Libai 2003) and, consequently, the quality of decisions regarding customer selectivity by better distinguishing between good and bad customers. Second, it could contribute to firms making better recommendations about the allocation of scarce marketing resources based on a more profound understanding of “return on marketing” (Rust, Lemon, and Zeithaml 2004).

Appendix

In the following notation, let $X_{ijkl}$ describe a variable, $X$, which applies to an average customer of Segment $N$ ($N = 1, 2, ..., 36$) with purchasing pattern $ijkl$. Here, $i(j, k, l)$ takes the value of 1 in the case of a purchase having taken place in year $t$ ($t + 1, t + 2, t + 3$) and 0 otherwise. Furthermore, let COGS represent cost of goods sold as a percentage of sales and ME stand for fixed marketing expenditures per customer per year that are assumed to be paid at the beginning of each year. Finally, let $f_{ijkl}$ and $AA_{ijkl}$ describe the purchase frequencies and average amounts spent determined on the basis of the RFM approach.

Using these notations, we can express the expected revenue, $ER_{ijkl}$, and profit contribution, $PC_{ijkl}$, as follows:

(A1) $PC_{ijkl} = ER_{ijkl} \times (1 - COGS) - ME$, with $ER_{ijkl} = f_{ijkl} \times AA_{ijkl}$;

(A2) $PC_{ijl} = ER_{ijl} \times (1 - COGS) - f_{ijl} \times ME$, with $ER_{ijl} = f_{ijl} \times AA_{ijl}$;

(A3) $PC_{ij} = ER_{ij} \times (1 - COGS) - f_{ij} \times f_{ijl} \times ME$, with $ER_{ij} = f_{ij} \times f_{ijl} \times AA_{ijl}$; and

(A4) $PC_{jkl} = ER_{jkl} \times (1 - COGS) - f_{jkl} \times f_{ijkl} \times f_{ijl} \times ME$, with $ER_{jkl} = f_{jkl} \times f_{ijkl} \times f_{ijl} \times AA_{jkl}$.

On the basis of Equations A1–A4, it is now straightforward to calculate CLV that excludes option value. If we take a discount rate of $d\%$ per year and, for the sake of simplicity, assume that the profit contribution of each customer is paid out in the middle of each year (expected revenue can occur at any time, whereas marketing expenditures are assumed to be paid out at the beginning of each year), we can calculate the CLV for an average customer in Segment $N$ based on a lifetime of four years ($t$ to $t + 3$), $CLV_{N_{EOV}}$ as follows:

(A5) $CLV_{N_{EOV}} = \sum_{ijkl} PC_{ijkl} \sum_{ijl} PC_{ijl} \sum_{ij} PC_{ij} \sum_{jkl} PC_{jkl}$.

To determine CLV that includes option value, it is first necessary to define a decision rule that provides information about when a customer should be abandoned. Let $DV_{ijkl}$ be the value of an average customer of Segment $N$ at the beginning of year $T$ ($T = t + 1, t + 2, t + 3$), which we can calculate as the sum of expected future profit contributions from $T$ onward, discounted back to $T$, according to the following three formulas:

(A6) $DV^{N_{ijkl}}_{1} = \frac{\sum_{ijkl} PC_{ijkl}}{(1 + d)^{2.5}} + \frac{\sum_{ijl} PC_{ijl}}{(1 + d)^{4}} + \frac{\sum_{ij} PC_{ij}}{(1 + d)^{5}} + \frac{PC_{ijkl}}{(1 + d)^{5}}$;

(A7) $DV^{N_{ijkl}}_{ij} = f_{ijl} \times f_{ijkl} \times \frac{\sum_{ijkl} PC_{ijkl}}{(1 + d)^{2.5}} + \frac{PC_{ijl}}{(1 + d)^{4.5}}$; and

(A8) $DV^{N_{ijkl}}_{ijl} = f_{ij} \times f_{ijkl} \times f_{ijl} \times \frac{PC_{ijkl}}{(1 + d)^{4.5}}$.

A customer should be abandoned at the beginning of year $T$ whenever $DV^{N_{ijkl}}_{ijkl}$ is less than the cost associated with this abandonment $AC$. Let $AB_{ijkl}$ be a variable that takes the value of 1 when the customer should be abandoned at the beginning of year $T$ and 0 otherwise. Using Equations A5–A8, we can determine $AB_{ijkl}$ as follows:
Taking into account that a customer, after he or she is abandoned, turns from an active to an inactive status for all future years, we can define a variable, $S_{N,t}^T$, which defines the activity status of a customer at year $T$ and takes either the value of 1 when the customer is active or the value of 0 when the customer is inactive, as follows:

\[
\begin{align*}
S_{N,t}^T &= 1 - \min(S_{N,t}^T + 1, 1), \\
S_{N,t}^T &= 1 - \min(S_{N,t}^T + 1, 1), \\
S_{N,t}^T &= 1 - \min(S_{N,t}^T + 1, 1), \\
S_{N,t}^T &= 1 - \min(S_{N,t}^T + 1, 1).
\end{align*}
\]

Finally, building on Equations A5 and A13–A16, we can now easily calculate CLV that includes option value as follows:

\[
\text{CLV}_{\text{NTV}} = \frac{\sum_{i \neq k} S_{N,t}^T + 3 \text{PC}_{N,t}^N + \sum_{j} S_{N,t}^T + 2 \text{PC}_{N,t}^N + \sum_{l} S_{N,t}^T + \text{PC}_{N,t}^N}{(1 + d)^{3.5}} + \frac{\sum_{j} S_{N,t}^T + \text{PC}_{N,t}^N}{(1 + d)^{2.5}} + \frac{\sum_{l} S_{N,t}^T + \text{PC}_{N,t}^N}{(1 + d)^{1.5}}.
\]

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