Analyzing the Interest Rate Risk of Banks Using Time Series of Accounting-Based Data: Evidence From Germany

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Abstract

This paper describes the first thorough analysis of the interest risk of German banks on an individual bank level. We develop a new method that is based on time series of accounting-based data to quantify the interest risk of banks and apply it to analyze the German banking system. We find evidence that our model yields a significantly better fit of banks’ internally quantified interest rate risk than a standard approach that relies on one-point-in-time data, and that the interest rate risk differs between banks of different size and banking group. Additionally, we find structural differences between trading book and non-trading book institutions.

Keywords: German financial institutions; interest rate risk; accounting-based approach; maturity transformation; banking supervision; model evaluation

JEL classification: G18, G21
1 Introduction

While over the past few years both banking supervisors and researchers have nearly exclusively focused their attention on banks’ credit and operational risk, the spotlight is now being turned again on interest rate risk. One reason for this is its threat to the stability of the financial system as a kind of systematic risk. In 2004, the Basel Committee on Banking Supervision (2004b) suggested ‘Principles for the Management and Supervision of Interest Rate Risk’ that go far beyond current practice. Within these principles the Committee states that “this [interest rate] risk is a normal part of banking and can be an important source of profitability” but stresses that it is “essential to the safety and soundness of banks” that interest rate risk is maintained within prudent levels. A historical example of a banking crisis where interest rate risk played an integral role is the ‘Savings and Loan Crisis’ which occurred in the US during the 1980s. Between 1980 and 1988, 563 of the approximately 4,000 then existing savings and loan institutions failed, while further failures were prevented by 333 supervisory mergers. The total costs of the crisis are estimated at USD 160 bn.¹

The German banking system has some features that make an investigation of interest rate risk in this market especially interesting. First, German banks still typically act as qualitative asset transformers which makes the German banking system quite particular in comparison to other banking systems like in the US or UK (Schmidt et al., 1999; Allen and Santomero, 2001). Interest rate risk hence still arises from the basic banking business. So far, there is no reliable analysis of the interest rate risk of most German banks and its determinants, but there are indications that the level of interest rate risk might be comparatively high: in 2006 the Deutsche Bundesbank conducted a stress test among a sample of 25 banks. For 10 medium-sized and smaller banks a 150 basis point interest rate shock caused an average loss of 15% of liable capital (Deutsche Bundesbank, 2006a). Via scaling, this is close to the Basel Committee’s definition of ‘outlier banks’.²

¹See Federal Deposit Insurance Corporation (1997) for a detailed analysis.
²The Basel Committee on Banking Supervision (2004a,b) recommends the supervisors to be particularly attentive to those banks (= outlier banks) whose interest rate risk in the banking book leads to an economic value decline of more than 20% of the sum of Tier 1 and Tier 2 capital (= liable capital) following a 200 basis point interest rate shock.
Second, the German universal banking system has a special structure: in addition to private commercial banks (‘Kreditbanken’), there are the state-owned savings banks (‘Sparkassen’) and the member-owned cooperative banks (‘Genossenschaftsbanken’). The banking groups differ in their business model (Schmidt and Tyrell, 2004) and the relevance of their interest bearing business (Deutsche Bundesbank, 2006b). Third, there is a large number of German universal banks, approximately 2,000 in 2005. This fact facilitates empirical statements.

Because there is still no standardized access to banks’ internally quantified interest rate risk, most models proposed in the literature and applied by banking supervisors rely on accounting-based data. These include Bennett et al. (1986), Planta (1989), Patnaik and Shah (2004), and the Federal Reserve’s Economic Value Model (EVM) presented by Houpt and Embersit (1991) and analyzed by Wright and Houpt (1996), Sierra (2004), and Sierra and Yeager (2004), as well as the ‘standardized framework’ suggested by the Basel Committee on Banking Supervision (2004b). The Net Portfolio Value Model applied by the Office of Thrift Supervision (OTS) is similar to these models but requires far more detailed information on the assets and liabilities of banks that is exclusively available to the OTS (Office of Thrift Supervision, 2000).

Accounting-based data typically shows the amount of positions within certain time bands: the total outstanding amount of a given position is distributed among a number of time bands according to remaining time to maturity (e.g. in the US) and/or initial maturity (e.g. in Germany). Given this information, the cash flow structure of a bank’s on-balance positions and its interest rate sensitivity can be estimated, but the results depend on the assumed distribution of maturities within these time bands. For example, the Federal Reserve’s EVM assumes a concentration in the middle of the time bands to estimate the interest rate risk of US commercial banks, which is also suggested by the Basel Committee on Banking Supervision (2004b). Bennett et al. (1986) assume a uniform distribution.

In this context, our contribution to the literature is threefold: first, we present a new method to derive a bank’s interest rate risk using time series of accounting-based data. This allows us

3There is an extensive literature on the interest rate sensitivity of stock returns of exchange-traded banks. See Staikouras (2003, 2006) for recent surveys. However, as most German banks are not listed, this approach for analyzing the interest rate risk is not applicable for the vast majority of German banks.
to estimate the distribution of maturities within the time bands. By doing so, we get greater precision with our estimates of the interest rate risk. The framework is a generalization of the aforementioned models such as the EVM and the proposal of the Basel Committee on Banking Supervision (2004b) and is flexible enough to capture different actual reporting practices, for example in the US and Germany, as well as the ‘reporting framework’ suggested by the Basel Committee on Banking Supervision (2004b). Additionally, it allows the integration of several data sources that are available to regulators and/or external analysts.


Third, we examine for the first time the interest rate risk of the German universal banking system on an individual bank level and analyze its determinants with respect to banks’ attributes. So far, the interest rate risk has only been analyzed for small samples of German banks using market values of equity (e.g. Oertmann et al., 2000) or bank internal data (e.g. Deutsche Bundesbank, 2006a).

The empirical evidence in this paper can be summarized as follows: first, our model is able to explain the cross-sectional variation of the interest rate risk of banks more accurately than a standard approach that relies on one-point-in-time data. Second, bigger banks have a higher level of interest rate risk than smaller banks and third, the interest rate risk of German banks differs between the banking groups. Savings banks and cooperative banks have a higher level of interest rate risk than private commercial banks. Additionally, there are structural differences in the interest rate risk of trading book and non-trading book institutions.

The remainder of the paper is organized as follows. Section 2 outlines our hypotheses. The method is presented in Section 3. Section 4 contains the description of the data sources, the model evaluation and the analysis of the interest rate risk of German banks. Section 5 concludes.
2 Hypotheses

The aforementioned models that aim to quantify the interest rate risk of banks using accounting-based information rely on one-point-in-time data. To derive the cash flow structure of a bank and its interest rate risk, these models use information on time bands of the maturities of the bank’s assets and liabilities and assume a certain distribution of maturities within the time bands. Wright and Houpt (1996) argue that the Federal Reserve’s EVM, acting as a representative of these simple models, yields results similar to the more complex Net Portfolio Value Model of the OTS. We in turn expect that the integration of time series information and different data sources – such as the breakdown by initial maturity – significantly improves our ability to explain the cross-sectional variation of banks’ interest rate risk even though our model relies on data that basically has the same simple structure as the input data of the EVM.

**Hypothesis 1 (Model Quality)** *Our model is able to explain the cross-sectional variation of banks’ interest rate risk better than a standard approach that relies on one-point-in-time data.*

Maturity transformation is often seen as a specific function of banks (e.g. Niehans, 1978): customers tend to borrow long-term capital and to lend short-term capital. Additionally, banks may have an incentive to lend out for the long term and refinance for the short term since the slope of the term structure is usually positive (e.g. Bhattacharya and Thakor, 1993): on average, long-term interest rates (for assets) exceed the short-term interest rates (for liabilities). Thus, on average, the bank achieves a premium from maturity transformation. On the other hand, the resulting maturity mismatch between the assets and liabilities causes interest rate risk. While there is the incentive to bear interest rate risk due to the expected yield, there are several economic reasons why firms should manage and limit their risks (e.g. Allen and Santomero, 1998). These include, among others, costs of financial distress (Warner, 1977) and capital market imperfections (Froot et al., 1993; Froot and Stein, 1998). We expect that smaller banks have a higher incentive to keep their exposure to interest rate risk low than bigger banks for several reasons: first, bigger banks may be assumed to be “too big to fail” by their investors and other stakeholders and hence face lower financial distress costs (e.g. Saunders et al., 1990). Second, bigger banks are more diversified than smaller banks and hence have a lower level of
idiosyncratic risk. To obtain a certain level of total risk, these banks may bear more systematic (interest rate) risk (e.g. Demsetz and Strahan, 1997). Third, the information risk may be lower for investors of bigger banks and hence may be substituted by interest rate risk (e.g. Banz, 1981). Fourth, bigger banks may have more opportunities to trade their risk on the capital market and hence to alter their exposure to interest rate risk via off-balance activities quickly, once a stress situation occurs. Hence, we hypothesize for the German banking system:

**Hypothesis 2 (Bank Size)** *Bigger banks have a higher level of interest rate risk than smaller banks.*

We expect that the costs of bearing (interest rate) risk does not only depend on a bank’s size but also on its banking group. While German savings banks and cooperative banks rather merge within their respective banking group than default (e.g. Porath, 2006), German private commercial banks are faced with possible financial distress costs more directly. Hence, we expect that the latter have higher incentives to keep their exposure to interest rate risk low, and we hypothesize for the German banking system:

**Hypothesis 3 (Banking Group)** *The interest rate risk differs between banking groups. Savings banks and cooperative banks have a higher level of interest rate risk than private commercial banks.*

3 Model

3.1 Definition of Interest Rate Risk

In order to apply a comparable and widely accepted measure for the interest rate risk of banks, we follow the ‘standardized interest rate shock’ approach also proposed within the new Basel Capital Accord (Basel II) and the ‘Principles for the Management and Supervision of Interest Rate Risk’ that are published by the Basel Committee on Banking Supervision (2004a,b):

**Definition 1 (Interest Rate Risk)** *The interest rate risk (IRR) of a bank is given by the maximum absolute decline of its economic value caused by an upward and downward 200 basis point parallel interest rate shock in relation to its regulatory capital.*
Approximating the interest rate sensitivity by the duration, the interest rate risk of a bank in \( t_{\text{ref}} \) (‘reference date’) is measured by:

\[
IRR(t_{\text{ref}}) = 0.02 \left| \frac{\sum_{t_{\text{CF}}>t_{\text{ref}}} (t_{\text{CF}}-t_{\text{ref}}) \sum_{\text{pos} \in \text{POS}^A} \frac{\text{CF}(\text{pos},t_{\text{ref}},t_{\text{CF}})-\sum_{\text{pos} \in \text{POS}^L} \text{CF}(\text{pos},t_{\text{ref}},t_{\text{CF}})}{(1+R_{\text{ac}}(t_{\text{ref}},t_{\text{CF}}))^{t_{\text{CF}}-t_{\text{ref}}+1)}}{\sum_{\text{pos} \in \text{POS}^A} \text{CF}(\text{pos},t_{\text{ref}},t_{\text{CF}})} \right| . \tag{1}
\]

\( RC(t_{\text{ref}}) \) denotes the regulatory capital in \( t_{\text{ref}} \). \( \text{POS}^A \) is the set of all interest rate-sensitive asset positions, \( \text{POS}^L \) the set of all interest rate-sensitive liability positions and \( \{t_{\text{CF}}\} \) the set of all points in time when cash flows are due. \( \text{CF}(\text{pos},t_{\text{ref}},t_{\text{CF}}) \) denotes the cash flow of position \( \text{pos} \) in \( t_{\text{CF}} > t_{\text{ref}} \) from the perspective of \( t_{\text{ref}} \).\(^5\) The set of all \( \text{CF}(\text{pos},t_{\text{ref}},t_{\text{CF}}) \) will be referred to as ‘cash flow structure’. Finally, \( R_{\text{ac}}(t_{\text{ref}},t_{\text{CF}}) \) represents the annually compounded spot rate in \( t_{\text{ref}} \) for the date \( t_{\text{CF}} \). In line with the earlier accounting-based models (See Section 1) we only capture here the interest rate risk of the net portfolio value, excluding other components such as the exposure of the going concern value (See Samuelson, 1945).

The key to the analysis is determining the detailed cash flow structure in (1) that is usually unknown to regulators, external analysts and a bank’s stakeholders. In the following, we present a new method to derive this cash flow structure using accounting-based data. Since the distinctive feature of our model is the ability to integrate time series information, we refer to the model as ‘Time Series Accounting-Based Model’ (TAM). To highlight the underlying idea, we start with a simplified example before we present the general framework.

### 3.2 Introductory Example

The bank is assumed to contract only one type of interest rate-sensitive, default-free business in each year with initial maturities of 1, 2, 3, 4, and/or 5 years. To keep the example as simple as possible, we omit here payments other than those due to the face value at maturity. Thus,

\(^4\)It is well known that the duration of a defaultable cash flow usually differs from the duration of a corresponding default-free cash flow. Jacoby and Roberts (2003) provide a recent literature overview. In line with the approaches proposed in the literature (See Section 1) we assume default-free cash flows and apply the effective duration (See Ho, 1990).

\(^5\)The cash flow due in \( t_{\text{CF}} = t_{\text{ref}} \) does not influence the bank’s interest rate risk and is hence omitted.
the cash flow structure is completely determined by the face values of contracted business and the corresponding maturity dates. For the reference date $t_{ref} = 0$, the bank structure, i.e. the composition of the portfolio the bank holds, is shown in the top of Figure 1, where

$$X_{t_{beg},t_{end}} \geq 0 \quad \forall t_{beg}, t_{end}$$

(2)

denotes the face value of business that has been contracted in $t_{beg}$ and that matures in $t_{end}$. The cash flow in $t_{CF} > t_{ref}$ is the sum of all business with maturity date $t_{CF}$ that has been contracted until $t_{ref}$:

$$CF(t_{ref}, t_{CF}) = \sum_{i \leq t_{ref}} X_{i,t_{CF}}.$$  

(3)

If each $X_{t_{beg},t_{end}}$ in (3) or each cash flow $CF(t_{ref}, t_{CF})$ were known to the regulators or external analysts, they could easily calculate the interest rate risk in $t_{ref}$ via (1). However, banks do not usually report the $X_{t_{beg},t_{end}}$ nor the detailed cash flows. But they regularly report the outstanding amount of the business within certain time bands according to the remaining time to maturity and/or the initial maturity.

Assume that the bank’s annual report in $t$ gives the outstanding amount of the business with a remaining time to maturity of up to 1 year, denoted by $RTM_{0,1}^t$, and of more than 1 year and up to 3 years, denoted by $RTM_{1,3}^t$. If regulators only take $RTM_{0,1}^t$ and $RTM_{1,3}^t$ into account, they know in $t = 0$ the cash flow that is due in one year but cannot distinguish between the cash flows in 2 and 3 years, because only its sum $RTM_{0,3}^0$ is reported. However, they can gain further information if they incorporate past reports: since the remaining time to maturity of a certain business decreases from date to date, the business migrates between the time bands over time. This effect can be used to restrict the estimates of future cash flows.

For illustration, we restrict our example to two reporting dates: $t = 0$ and $t = -1$. As $RTM_{0,1}^0$ denotes the business in $t = 0$ that has a remaining time to maturity of 1 year it consists of business that had a remaining time to maturity of 2 years in the previous date $t = -1$ and new business that is contracted in $t = 0$ and that matures in 1 year. Hence, we know that $CF(-1,1) \leq RTM_{0,1}^0$. This relation also provides information on $CF(-1,2)$, as the sum of both cash flows equals the report item $RTM_{1,3}^{-1} = CF(-1,1) + CF(-1,2)$. Some minor rearrangements yield $CF(-1,2) \geq RTM_{1,3}^{-1} - RTM_{0,1}^0$. Additionally, $CF(-1,2)$ cannot exceed
$CF(0, 2)$, since the latter consists of the former plus the business contracted in $t = 0$ that matures in $t = 2$. Hence, we can conclude that

$$CF(0, 2) \geq RTM_{-1}^{1,3} - RTM_{0}^{0,1}. \tag{4}$$

This relationship restricts possible values of $CF(0, 3)$, too, as the sum of both equals the amount reported in the time band $RTM_{0}^{1,3}$:

$$CF(0, 3) \leq RTM_{0}^{1,3} - (RTM_{-1}^{1,3} - RTM_{0}^{0,1}). \tag{5}$$

This simple example shows that former reports can indeed add information on today’s cash flow structure by restricting the possible estimates and, hence, can improve the estimation of a bank’s interest rate risk. When we model a realistic bank with a lot of different positions, time bands and reporting dates, the system of equations becomes rather complex and unhandy. However, the representation can be conceptually simplified by using the above-defined variables $X_{t_{beg}}^{t_{end}}$ to express each single report item as a function of these variables.

[Figure 1 about here.]

For instance, Figure 1 shows the bank structure in $t = 0$ (top) and $t = -1$ (bottom). The outstanding amounts with a remaining time to maturity of up to 1 year and those of more than 1 year and up to 3 years, respectively, are marked on the left-hand side. Since the reported amounts equal the sum of the business within the respective time bands, the following equations must hold:

$$X^{-4,1} + X^{-3,1} + X^{-2,1} + X^{-1,1} + X^{0,1} = RTM_{0}^{0,1}, \tag{6a}$$

$$X^{-5,0} + X^{-4,0} + X^{-3,0} + X^{-2,0} + X^{-1,0} = RTM_{-1}^{0,1}, \tag{6b}$$

$$X^{-3,2} + X^{-2,2} + X^{-1,2} + X^{0,2} + X^{-2,3} + X^{-1,3} + X^{0,3} = RTM_{0}^{1,3}, \tag{6c}$$

$$X^{-4,1} + X^{-3,1} + X^{-2,1} + X^{-1,1} + X^{-3,2} + X^{-2,2} + X^{-1,2} = RTM_{-1}^{1,3}. \tag{6d}$$

This formulation additionally allows us to easily integrate further sources of relevant information. In Germany for instance, also time bands according to the initial maturity are available to the regulators. Assume that the bank reports in $t$ the amount of business with an initial
maturity of up to 1 year and of more than 1 year and up to 3 years, denoted as \( ITM_{t}^{0,1} \) and \( ITM_{t}^{1,3} \), respectively. The corresponding variables \( X^{t_{\text{beg}} t_{\text{end}}} \) are marked on the right-hand side of Figure 1 and we have:

\[
\begin{align*}
X^{0,1} &= ITM_{0}^{0,1}, \\
X^{-1,0} &= ITM_{-1}^{0,1}, \\
X^{-2,1} + X^{-1,1} + X^{-1,2} + X^{0,2} + X^{0,3} &= ITM_{0}^{1,3}, \\
X^{-3,0} + X^{-2,0} + X^{-2,1} + X^{-1,1} + X^{-1,2} &= ITM_{-1}^{1,3}.
\end{align*}
\]

The approaches proposed in the literature only use the information of (6a) and (6c) and omit the remaining information. Equations (6a) to (7d) ((6a) to (6d) if reports on the initial maturity are not available) and the non-negativity restriction (2) form a system of linear equations that restricts the possible bank structure, and hence via (3) and (1) the bank’s interest rate risk.

However, in general the system of equations is not uniquely determined. Instead, there exists a space of solutions of bank structures that are all consistent with the reported data.\(^6\) To identify an economically sensible solution, further assumptions are necessary. For illustration, we here assume that the cash flow structure of the bank is as constant as possible over the two dates \( t = 0 \) and \( t = -1 \). This can be expressed by optimizing a function \( F \) on the space of solutions: we select that solution that minimizes the sum of the quadratic deviations between \( t = 0 \) and \( t = -1 \) of the cash flows due in 1, 2, ..., 5 years in relation to the sum of all cash flows from the perspective of the respective date. Formalizing this procedure, we obtain:

\[
\text{minimize} \quad F(X) = d_1^2 + d_2^2 + d_3^2 + d_4^2 + d_5^2
\]

subject to

(1) Deviation Variables

\[
d_s = \frac{CF(0, s)}{\sum_{t_{CF}} CF(0, t_{CF})} - \frac{CF(-1, s - 1)}{\sum_{t_{CF}} CF(-1, t_{CF})} \quad \forall \ s \in \{1, ..., 5\}
\]

(2) Restrictions

(6a), (6b), (6c), (6d), (7a), (7b), (7c), (7d), (2),

\(^6\)Note that an ambiguous bank structure does not necessarily imply an ambiguous level of interest rate risk.
where $X$ denotes the set of all variables $X^{t_{\text{beg}},t_{\text{end}}}$. We go on formalizing the ideas presented above in a general framework.

### 3.3 General Framework

Analogous to the previous example, we start by describing the bank structure. We choose a discrete setting that allows us to capture the bank structure by a finite number of variables. Due to the discrete granularity of the available data, no information is omitted.

**Model Component 1 (Bank Structure)** *The bank contracts interest rate-sensitive default-free business in certain time points with a fixed time to maturity and a fixed, periodically paid coupon. Once contracted, the business is on-balance with a constant amount until its maturity. The cash flow implied by each business consists of the repayment of the face value and coupon payments.*

The assumption of a fixed, periodically paid coupon as well as the omission of amortization payments and default of assets can be relaxed easily. However, calculations on a subsample of banks have shown that these assumptions do not affect our later results but yield a reduction of calculation burden. A possible bias implied by these or the remaining assumptions is compensated as shown in Section 4. We go on formalizing Model Component 1.

The set of points in time when business is contracted and/or cash flows are due is given by $T = \{t_i\}_{i \in \mathbb{Z}}$ where we set $t_0 = 0$ for simplicity. In each $t_{\text{beg}} \in T$ the bank contracts certain amounts of business of a position $pos \in POS$ (such as customer loans on the asset side or interbank liabilities on the liability side) that mature in $t_{\text{end}} > t_{\text{beg}}$. The amount is denoted $X^{pos,t_{\text{beg}},t_{\text{end}}}$ and assumed to be non-negative:

$$X^{pos,t_{\text{beg}},t_{\text{end}}} \geq 0 \quad \forall \ pos, t_{\text{beg}}, t_{\text{end}}.$$  \hfill (8)

We will refer to these variables as ‘business items’. The set of all business items is referred to as ‘bank structure’. The corresponding coupons are denoted $c^{pos,t_{\text{beg}},t_{\text{end}}}$.

---

7Daily maturing business is not included, since – applied for the German banking system – the model cannot distinguish between daily and monthly maturing business due to the monthly granularity of the reports.
Under these assumptions, the cash flow in $t_{CF} > t_{ref}$ related to position $pos$ is given by:

$$CF(pos, t_{ref}, t_{CF}) = \sum_{t_i \leq t_{ref}} X_{pos,t_i,t_{CF}} + \sum_{t_i < t_{ref}, t_j \geq t_{CF}} X_{pos,t_i,t_j} c_{pos,t_i,t_j}.$$  \hspace{1cm} (9)

The first part of the right-hand side of (9) denotes the payments due for the repayment of the face values of all business items that are on-balance in $t_{ref}$ and that mature $t_{CF}$. The second part represents the corresponding coupon payments.

**Model Component 2 (Reports on the Bank Structure)** At certain points in time the bank reveals information on its structure by reporting time bands of the outstanding amount for each position broken down by remaining time to maturity and/or initial maturity, respectively.

This assumption captures current reporting practice. In each reporting date $t \in T_{obs} \subseteq T$ the bank reports the sum of all business (classified by position) within specified ranges of remaining time to maturity and/or initial maturity, respectively. We assume that for each $pos \in POS$ in each $t \in T_{itm}^{obs} \subseteq T_{obs}$ there are $|N^{pos}|$ report items characterized by the business’ remaining time to maturity: $RTM_{t}^{pos,n}$ with $n \in N^{pos}$ denotes the amount of $pos$ in $t$ with a remaining time to maturity $t_{end} - t$ within the time band $(h_{lower}^{pos,n}, h_{upper}^{pos,n}]$. Analogously, in each $t \in T_{itm}^{obs} \subseteq T_{obs}$ there are $|M^{pos}|$ items characterized by the business’ initial maturity: $ITM_{t}^{pos,m}$ with $m \in M^{pos}$ denotes the amount of $pos$ in $t$ with an initial maturity $t_{end} - t_{beg}$ within the time band $(h_{lower}^{pos,m}, h_{upper}^{pos,m}]$.

Since the amount within a certain time band equals the sum of the relevant business items, the former can be expressed as a linear function of the latter. Thus we obtain one function (restriction) for each reported time band and each reporting date, leading to a system of linear equations:

$$\sum_{h_{lower}^{pos,n} < t_j - t \leq h_{upper}^{pos,n}, t_i \leq t} X_{pos,t_i,t_j} = RTM_{t}^{pos,n} \quad \forall t \in T_{obs}^{itm}, n \in N^{pos},$$  \hspace{1cm} (10)

$$\sum_{h_{lower}^{pos,m} < t_j - t \leq h_{upper}^{pos,m}, t_i \leq t, t < t_j} X_{pos,t_i,t_j} = ITM_{t}^{pos,m} \quad \forall t \in T_{obs}^{itm}, m \in M^{pos},$$  \hspace{1cm} (11)

\footnote{According to German reporting practice, we model left open intervals.}
where (10) captures the restrictions due to reports on the remaining time to maturity and (11) those on the initial maturity.

Model Component 3 (Objective Function) An economically sensible solution is obtained by the optimization of a function on the space of solutions.

The system of equations given by (10), (11) and (8) restricts the values of the business items $X_{pos,t,\text{beg}}^{t,\text{end}}$ and hence via (9) and (1) the bank’s interest rate risk. However, it is clear that the system is in general not uniquely determined, i.e. it is not possible to infer the bank structure and the bank’s interest rate risk unambiguously using accounting-based data. Thus, analogously to the example, further assumptions on the bank structure are necessary in order to identify an economically sensible solution. These can again be formulated in terms of the optimization of a function $F$ (‘objective function’\(^9\)) on the space of solutions, and the optimization problem can be expressed as follows:

\[
\begin{align*}
\text{optimize} & \quad F(X) \\
\text{subject to} & \quad (10), (11), (8),
\end{align*}
\]

where $X$ denotes the set of all business items $X_{pos,t,\text{beg}}^{t,\text{end}}$.

4 Empirical Analysis

4.1 Data

The analysis is based on German universal banks as of 31 December 2005. Table 1 shows the composition of our sample. Banks for which data are incomplete are excluded from our analysis, which results in a total of 1,785 banks, accounting for 92.4% of all German universal banks. We make use of the following major regulatory data sources: the data schedule pursuant to the auditor’s report (‘Sonderdatenkatalog’) and the monthly balance sheet statistics (‘Monatliche Bilanzstatistik’) for estimating the bank structure, and an interest rate risk survey for evaluating our model. Due to structural breaks within the data sources at the end of 1998, we restrict our

\(^9\)Note that we present a description model here, thus ‘objective function’ does not refer to the bank’s objective.
sample to the period from January 1999 to December 2005. All data is provided by the Deutsche Bundesbank.

[Table 1 about here.]

The data schedule pursuant to the auditor’s report contains asset and liability positions broken down by remaining time to maturity for all banks on a yearly basis.\textsuperscript{10} In contrast, the monthly balance sheet statistics contain on a monthly basis a respective breakdown by initial maturity.\textsuperscript{11} We consider 3 interest rate-sensitive asset and 4 interest rate-sensitive liability positions. The former include ‘interbank loans’, ‘customer loans’, and ‘debt securities held’. The latter consist of ‘interbank liabilities’, ‘customer liabilities’, ‘debt securities issued’, and ‘non-maturing deposits’. The positions and the reported time bands are shown in Table 2.

[Table 2 about here.]

Additionally, we use the Bundesbank’s survey of lending and deposit rates (‘Erhebung über Soll- und Habenzinsen’) and the MFI interest rate statistics, which replaced the former in 2003.\textsuperscript{12} Both statistics contain monthly information on the coupons charged and paid by banks on new business classified by time bands. For mid- and long-term default-free interest rates, we use the term structure estimated by the Deutsche Bundesbank from German governmental bonds according to Svensson (1994), and for the short-term rates we use the money market rates reported by Frankfurt banks (‘Geldmarktsätze am Frankfurter Bankplatz’). Regulatory capital (own funds) is taken from the reports of Principle I.\textsuperscript{13}

To evaluate our model results for the German banking system we use a non-published interest rate risk survey (IRRS) that was carried out by the Deutsche Bundesbank and the German Federal Financial Supervisory Authority (BaFin). All German banks were asked to report the losses in their banking book (in relation to regulatory capital) as a consequence of, among others,

\textsuperscript{10}See Section 68 of the German Auditor’s Report Regulation (‘Prüfungsberichtsverordnung’) for detailed information.

\textsuperscript{11}See Deutsche Bundesbank (2003a) for detailed information.

\textsuperscript{12}See European Central Bank (2003) and the respective report forms for detailed information.

\textsuperscript{13}According to Sections 10 and 53 of the German Banking Act (‘Kreditwesengesetz’).
a 200 basis point shift in the term structure according to their internal risk-management system. The participation rate was nearly 60%. For these banks we have bank internal interest rate risk reports for the reference date of 30 September 2005 that are consistent with our definition of interest rate risk.

4.2 Model Specification

In order to test our hypotheses we estimate the interest rate risk of the German banking system for the reference date of 31 December 2005 on the basis of the TAM using the data from January 1999 to December 2005. We have to specify the system of equations (10) and (11) and the coupons of the respective business items to calculate the cashflow structure via (9). In this context, we need to decide how to deal with non-maturing deposits as there is no reliable information available on the economic maturity structure or time bands of non-maturing deposits. Additionally, we have to define an objective function $F$ to select an economically sensible solution from the space of solutions.

The system of equations is based on the positions and the time bands that are reported within the data schedule pursuant to the auditor’s report and the monthly balance sheet statistics (See Table 2). Within our sample period, 7 yearly observations of the former and 84 monthly observations of the latter report are available. Identifying the reference date December 2005 with $t_{ref} = t_0 = 0$, we obtain $T_{\text{obs}}^{\text{m}} = \{-6, -5, ..., 0\}$ and $T_{\text{obs}}^{\text{m}} = \{-83/12, -82/12, ..., 0\}$. To reduce calculation burden the initial maturity of business $X_{\text{pos}, t_{\text{beg}}, t_{\text{end}}}$ is restricted to $t_{\text{end}} - t_{\text{beg}} \in \{1/12, ..., 6/12, 9/12, ..., 24/12, 30/12, ..., 60/12, 72/12, ..., 120/12\}$. This results in 21,864 variables and 1,750 linear equations plus the non-negativity restriction for each variable.\(^{14}\)

We assume that coupons of positions that are contracted with banks or the capital market equal the default-free market interest rate. For the remaining positions we take the coupons from the statistics presented in Section 4.1 and assume that they refer to the middle of the reported time bands. The coupons for the remaining maturities are obtained by interpolating the spreads

\(^{14}\)The system of equations turned out to be not solvable for a small number of smaller banks. For these banks our assumptions were not consistent with the reported data. To obtain a solution we selected that bank structure that best fitted the system of equations.
to the default-free capital market rates linearly between the coupons that are given.

To select an economically sensible solution we assume a stable bank structure over the time period 1999 to 2005.\textsuperscript{15} ‘Stability’ of the asset positions means here that the fraction of the outstanding amount of all asset positions with a certain initial and/or remaining time to maturity in relation to the complete outstanding amount of all asset positions of the respective date remains as constant as possible over time. ‘Stability’ of the liability positions is interpreted analogously. We implement this by minimizing the sum of the quadratic deviations of the relative outstanding amount with a certain remaining time to maturity and initial maturity, respectively, over all possible maturities and all observation dates. See Appendix A for details.

Non-maturing deposits can be interpreted as short-term liabilities. However, it is well known that they can exhibit a high level of interest rate risk (e.g. Hutchison and Pennacchi, 1996, Jarrow and van Deventer, 1998, and O'Brien, 2000). This can to a large extent be explained by the fact that the development of their volume and deposit rates turns out to be sticky, yielding a duration or, equivalently, an economic maturity that is higher than that of other short-term products. For estimating the most ‘reasonable’ economic maturity of non-maturing deposits, we make use of the IRRS above. For different assumptions about the economic maturity, we calculate the TAM and estimate the following linear regression on the IRRS-subsample of banks (IRRS-banks):

\[
RM_i^{IRRS} = \alpha + \beta RM_i^{TAM} + \varepsilon_i, \tag{12}
\]

where \(RM_i^{IRRS}\) and \(RM_i^{TAM}\) denote the interest rate risk for bank \(i\) derived from the IRRS and the TAM, respectively. We choose the economic maturity which yields the highest \(R^2\). The best

\textsuperscript{15}Deshmukh et al. (1983) show in a simple framework that banks may have an incentive to alter their exposure to interest rate risk depending on the magnitude and volatility of interest rates. Since the term structure of interest rates was quite stable during the sample period, we maintain the assumption of a stable bank structure. The actual specification of the objective function was selected in a simulation-based analysis. We modeled several synthetic banks with reasonable business strategies such as constant or varying maturity transformation strategies. We then created reports that the respective banks would have had submitted to the Deutsche Bundesbank and applied several specifications of the objective function to compare the respective model results with the theoretical ones. Based on several measures of fit like the sum of the absolute or quadratic deviations between the model implied and the real cash flow structure at the reference date, we identified the actual specification as superior.
fit is obtained when we assume that the economic maturity of non-maturing deposits equals the legal maturity of 3 months.

Interestingly, when we apply the bank-individual assumption to the economic maturity of non-maturing deposits that is also reported in the IRRS, we obtain a rather poor fit. This suggests that a number of banks did not include their internally assumed exposure of non-maturing deposits when reporting the interest rate risk, which implies that the evaluation of our model in the following is quite conservative (i.e. we underestimate the model quality using the IRRS), since we cannot distinguish between the respective banks.

4.3 Model Evaluation

To benchmark our model to standard approaches and hence to test Hypothesis 1, we additionally estimate a ‘Benchmark Model’ (BM) that relies on one-point-in-time data. We consider the same asset and liability positions as for the TAM, assume also an economic maturity of 3 months for the deposits and use the breakdown by initial maturity on 31 December 2005 for the remaining positions,\textsuperscript{16} assuming that the business is concentrated in the middle of the reported time bands.\textsuperscript{17} Based on this, we assume that the business has been contracted continuously over time and has a coupon of 5%. This allows us to derive a (continuous) cash flow structure for each position and to estimate the interest rate risk analogously to (1).

For the evaluation we run regression (12) for the TAM and the analogue regression for the BM on the subsample of IRRS-banks. The part of the cross-sectional variation of interest rate risk explained by the respective models is measured by the coefficients of determination $R^2$, 27.33% for the TAM and 19.15% for the BM, respectively (See Table 3). In Appendix B we develop a procedure similar to Morgan (1939) that can tell whether the $R^2$ differ significantly. Table 3

\textsuperscript{16}We first estimated a model based on the remaining time to maturity in line with the standardized framework proposed by the Basel Committee on Banking Supervision (2004b) which, however, yielded a rather poor fit for the IRRS. We therefore specify the BM based on the initial time to maturity, whereas all other assumptions are in line with the Basel Committee on Banking Supervision (2004b).

\textsuperscript{17}For the time bands $(2; \infty)$ and $(5; \infty)$, we assume a concentration at 4 and 6, respectively.
shows the results. We find strong evidence that the $R^2$ of the TAM exceeds that of the BM and conclude:

**Result 1** *There is strong evidence that the Time Series Accounting-Based Model is able to explain the cross-sectional variation of banks’ interest rate risk better than the Benchmark Model.*

Table 4 shows the $R^2$ for some subsamples of IRRS-banks. We stress two distinctive features here: first, the TAM better fits the internal data of those banks that have not merged during the sample period. Mergers are dealt with by creating an artificial bank before the date of the merger, aggregating the balance sheet positions. As the assumption of a stable structure over time is quite strong for merged banks, our model provides better quality for banks that did not merge during the sample period. However, in both samples, the TAM fits the banks’ internal data on the interest rate risk better than the BM.

[Table 4 about here.]

Second, as reported in Section 4.2, we find indications that a number of banks did not include their internally assumed exposure of non-maturing deposits when reporting the interest rate risk in the IRRS. If some banks include this exposure, we underestimate the model quality using the IRRS as a benchmark. However, this problem should be less severe for banks that assume a low economic maturity for non-maturing deposits. In fact, Table 4 shows that we obtain much better results for these banks. In the first quintile of the reported bank internal assumption about the economic maturity of non-maturing deposits, the coefficient of determination yields more than 50%, and for non-merged banks even more than 66%.

A detailed analysis (not reported here) shows that both the TAM and the BM overestimate the true interest rate risk on average – assuming that the results of the IRRS represent the banks’ true interest rate risk. This is also reflected in the regression results in Table 3. If the models produce unbiased estimates of the true interest rate risk then the respective constants $\alpha$ should be zero and the slopes $\beta$ should be one. A positive (negative) $\alpha$ and a $\beta$ bigger (smaller) than one means that the respective model tends to underestimate (overestimate) the interest rate risk. The slope for both models is positive but clearly smaller than one. As the constants are comparatively
small, this corresponds to the fact that both the TAM and the BM overestimate the interest rate risk of banks. This bias was to be expected as it can be explained by our assumptions: we exclude provisions and premature redemptions on the asset side and prolongation options on the liability side and assume a very small economic maturity of non-maturing deposits, i.e. we tend to overestimate the interest rate sensitivity of the asset side and underestimate the interest rate sensitivity of the liability side. As banks usually have a positive duration gap (e.g. Deutsche Bundesbank, 2003b), these assumptions hence cause higher estimates of the level of interest rate risk. Additionally, we do not consider off-balance positions, such as interest derivatives, that on average can be expected to be used to reduce interest rate risk by the banks (e.g. Schrand, 1997). Similar biases are also reported by Wright and Houpt (1996), Sierra (2004) and Sierra and Yeager (2004) when evaluating the EVM. Since the coefficient of determination is invariant to any linear transformation our analysis of the model quality is not affected by the bias.

4.4 The Interest Rate Risk of German Banks

The next step is testing the Hypotheses 2 and 3. The previous Section 4.3 showed that our estimates of the interest rate risk of banks are biased. Since the bias could depend on the banks' characteristics, we un-bias our estimates for size and banking group. In addition, we include a control dummy variable for trading book institutions.

We estimate the following regression for the banks that took part in the IRRS:

\[
RM_i^{IRRS} = \alpha + \beta_1 RM_i^{TAM} + \beta_2 \log(Assets_i) + \delta^{SB} D_i^{SB} + \delta^{CB} D_i^{CB} + \delta^{TB} D_i^{TB} + \varepsilon_i, \quad (13)
\]

where $Assets_i$ denotes the sum of assets of bank $i$ and $D_i^{SB}$ ($D_i^{CB}$, $D_i^{TB}$) is a dummy for savings banks (cooperative banks, trading book institutions). Table 5 shows the results.

[Table 5 about here.]

The coefficients imply that the bias is independent of the bank size but depends on the banking group and whether the bank has a trading book. The positive sign of the dummies for the banking group suggests that – as we overestimate the interest rate risk for all investigated subgroups on average (not reported here) – the overestimation is less severe for savings banks
and cooperative banks than for private commercial banks. This implies that the effect of the simplifying assumptions, discussed in Section 4.3, is more severe for the latter banks. One specific reason could be off-balance activities. The more a bank uses derivatives to decrease (increase) the interest rate risk, the more (less) we overestimate the risk as the TAM does not consider derivatives. So our results could indicate that private commercial banks use interest derivatives for hedging purposes to a higher extent than the other banks. Analogously, the positive sign of the trading book dummy implies that the overestimation is less pronounced for trading book institutions. However, we would have expected a negative sign of the dummy, as our approach includes the on-balance positions of the banking and the trading book, whereas the IRRS only captures the banking book (including off-balance positions). On the other hand, trading book institutions may assume a higher level of interest rate risk via derivatives, because once a stress situation occurs they can more quickly alter their exposure to interest rate risk via capital market transactions. When doing so, there are regulatory incentives to take the interest rate risk rather in the banking book than in the trading book (e.g. Jones, 2000). This could be done by ‘hedging’ the on-balance positions on the liability side of the banking book via derivatives. The resulting effect would be captured by the IRRS but not by the TAM which would reduce the overestimation and hence would explain the sign of the dummy.

With the coefficients from regression (13) we calculate for each bank in the whole data sample the unbiased measure of interest rate risk (e.g. Greene, 2003)

$$\hat{RM}_{i}^{TAM} = \hat{\alpha} + \hat{\beta}_1 RM_{i}^{TAM} + \hat{\beta}_2 \log(\text{Assets}_i) + \hat{\delta}_S^{SB} D_{i}^{SB} + \hat{\delta}_C^{CB} D_{i}^{CB} + \hat{\delta}_T^{TB} D_{i}^{TB}$$ (14)

and run the regression

$$\hat{RM}_{i}^{TAM} = \alpha + \beta \log(\text{Assets}_i) + \gamma_S^{SB} D_{i}^{SB} + \gamma_C^{CB} D_{i}^{CB} + \gamma_T^{TB} D_{i}^{TB} + \varepsilon_i$$ (15)

on the whole data sample to test Hypotheses 2 and 3. Table 6 shows the results. The coefficient of size turns out to be significantly positive, i.e. we find evidence that there is a positive correlation between size and interest rate risk which supports Hypothesis 2.

**Result 2** There is strong evidence that bigger banks have a higher level of interest rate risk than smaller banks.
Interestingly, the trading book dummy is also positive. This is consistent with the consideration from above and from Section 2 that banks that can alter their exposure more quickly through capital market transactions tend to assume a higher level of interest rate risk.

Result 2 seems to contradict the results of the stress test among German banks, where—based on a sample of 25 banks—medium-sized and smaller banks turn out to have a higher level of interest rate risk than commercial banks and central institutions (See Deutsche Bundesbank, 2006a). However, we find evidence that the difference of interest rate risk quantified within the stress test is rather attributable to the banking group than to size as the analyzed subgroup ‘medium-sized and smaller banks’ mainly consists of savings banks and cooperative banks: The coefficients of the savings banks and cooperative banks in our empirical analysis are both highly significantly positive. This means that the interest rate risk of theses banks is higher than that of the remaining banks which supports Hypothesis 3.

Result 3 There is strong evidence that the interest rate risk differs between banks of different banking groups and that the interest rate risk of savings banks and cooperative banks is higher than that of the remaining banks.

5 Conclusion

In this paper we developed a model to quantify the interest rate risk of banks using time series of accounting-based data. The framework is flexible enough to include different data sources and to capture the standardized reporting framework suggested by the Basel Committee on Banking Supervision (2004b) and different actual reporting practices like in the US and in Germany. Further, we examined— for the first time— the interest rate risk of the German universal banking system on an individual bank level and analyzed its determinants.

Based on a subsample of German banks to whose internally quantified interest rate risk we have access via a unique interest rate risk survey conducted by the Deutsche Bundesbank and BaFin, we could evaluate our model results. In line with earlier models and studies we found that our accounting-based approach produces biased estimates of the interest rate risk of banks,
which is attributed to some simplifying assumptions. However, we found convincing evidence that our model is able to explain the cross-sectional variation of banks’ interest rate risk better than a benchmark model that relies on assumptions similar to models proposed in the literature and applied by banking supervisors. We controlled for the bias and analyzed factors that affect the level of interest rate risk of German banks. We found strong evidence that bigger banks have a higher level of interest rate risk than smaller banks. Further, we found good evidence that the interest rate risk differs among banks of different banking groups. Savings banks and cooperative banks have a higher level of interest rate risk than private commercial banks. Lastly, there are structural differences between trading book and non-trading book institutions.

Banks may also have incentives to alter their exposure depending on the magnitude and volatility of interest rates. Within our sample period, the term structure of interest rates was quite stable in Germany. As a structural break in the data at the end of 1998 bars us from going further back in time, an analysis of the time variability of interest rate risk in the German banking system must be left for future research.
Appendix A  Specification of the Objective Function

The objective function that is to be minimized $F() = F_A() + F_L()$ consists of two functions $F_A()$ and $F_L()$ that are related to the asset and the liability side, respectively. Since the asset and the liability sides are treated analogously, we only present the definition of $F_A()$ here:

$$F_A(X) = \sum_{t \in T_{obs}} \sum_{rtm \in RTM} d_{\text{assets},t,rtm}^2 + \sum_{t \in T_{obs}} \sum_{itm \in ITM} d_{\text{assets},t,itm}^2$$

with the deviational variables

$$d_{\text{assets},t,rtm} = \frac{\sum_{pos \in POS^A} \sum_{j \geq 0} \begin{cases} X_{\text{pos},-j/12,rtm} \quad \text{with } \frac{j}{12} \in ITM \\ X_{\text{pos},-j,12,i/j} \quad \text{with } \frac{i+j}{12} \in ITM \end{cases} \sum_{pos \in POS^A} \sum_{j \geq 0} \sum_{i \geq 1} \begin{cases} X_{\text{pos},t-j,12,i/12} \quad \text{with } \frac{i+j}{12} \in ITM \\ X_{\text{pos},t-j,12,t+rtm} \quad \forall \ t \in T_{obs}, rtm \in RTM, \forall \ \frac{i+j}{12} \in ITM \end{cases}}{\sum_{pos \in POS^A} \sum_{j \geq 0} \sum_{i \geq 1} \begin{cases} X_{\text{pos},0,itm} \quad \text{with } \frac{i+j}{12} \in ITM \\ X_{\text{pos},-j,12,i/j} \quad \text{with } \frac{i+j}{12} \in ITM \end{cases}} \sum_{pos \in POS^A} \sum_{j \geq 0} \sum_{i \geq 1} \begin{cases} X_{\text{pos},t-j,12,i/12} \quad \text{with } \frac{i+j}{12} \in ITM \\ X_{\text{pos},t-j,12,t+itm} \quad \forall \ t \in T_{obs}, itm \in ITM. \end{cases}$$

The deviational variables $d_{\text{assets},t,rtm}$ and $d_{\text{assets},t,itm}$ correspond to the remaining time to maturity and the initial maturity, respectively. The first part of $d_{\text{assets},t,rtm}$ ($d_{\text{assets},t,itm}$) denotes the outstanding amount with a remaining time to maturity $rtm$ (initial maturity $itm$) at the reference date 0 in relation to the complete outstanding amount. The second part denotes the respective fraction at the observation dates $t \in T_{obs} = \{-83/12, -82/12, ..., 0\}$. As described in Section 4.2, the initial maturity $t_{end} - t_{beg}$ of business $X_{\text{pos},t_{beg},t_{end}}$ is restricted to
$ITM = \{1/12, \ldots, 6/12, 9/12, \ldots, 24/12, 30/12, \ldots, 60/12, 72/12, \ldots, 120/12\}$. To improve numerical efficiency of the optimization procedure, we additionally reduce the number of deviational variables: we do not include the deviational variable $d_{assets,t,rtm}$ for any possible remaining time to maturity $rtm$, but only consider $rtm \in RTM = \{1/12, 2/12, \ldots, 6/12, 18/12, \ldots, 120/12\}$ in the objective function.\footnote{Several robustness checks showed that this simplification does not affect our results.}
Appendix B  Comparison of the Models’ Accuracy

Let $\varepsilon^1$ and $\varepsilon^2$ with $E(\varepsilon^j) = 0, j = 1, 2$, be the error terms of two regressions with the same dependent variable $y$. Let $R^2_j$ denote the respective coefficient of determination and $\sigma_j$ the standard deviation of the respective error term. Since we have by definition $R^2_j = 1 - \frac{\sigma^2_j}{\text{var}(y)}$, testing the equality of the $R^2_j$ is equivalent to testing the equality of the $\sigma_j$.

The error terms can, but need not be, uncorrelated. The covariance between the two errors is given by $\sigma_{12}$. Note the following regression:

$$\varepsilon^1 = w(\varepsilon^1 - \varepsilon^2) + \eta. \quad (B.1)$$

As the $\varepsilon^j$ have an expectation of zero, there is no intercept in the regression. The regression coefficient $w$ is calculated as

$$w = \frac{\text{cov}(\varepsilon^1, \varepsilon^1 - \varepsilon^2)}{\text{var}(\varepsilon^1 - \varepsilon^2)} = \frac{\sigma^2_1 - \sigma_{12}}{\sigma^2_1 + \sigma^2_2 - 2\sigma_{12}}.$$

Minor manipulation shows that the following relation holds:

$$w \gtrless \frac{1}{2} \iff \sigma_1 \gtrless \sigma_2.$$ 

Therefore, testing the equality of the coefficients of determination is equivalent to running the OLS regression (B.1) and then carrying out the $t$–test if $w$ equals $\frac{1}{2}$. 


References


This figure shows the bank structure in \( t = 0 \) (top) and \( t = -1 \) (bottom), respectively. The left-hand side shows the business items with a remaining time to maturity of up to 1 year (light shading) and those of more than 1 year and up to 3 years (dark shading). The right-hand side shows the items with an initial maturity within these ranges, respectively. The rows refer to the time of contract \( (t_{\text{beg}}) \), whereas the columns refer to the maturity date \( (t_{\text{end}}) \).

\( X^{t_{\text{beg}}, t_{\text{end}}} \) denotes the amount of business that is contracted in \( t_{\text{beg}} \) and that matures in \( t_{\text{end}} \).

Figure 1: Bank structure and bank reports in \( t = 0 \) and \( t = -1 \).
### Table 1: German universal banks

<table>
<thead>
<tr>
<th>Category</th>
<th>All Banks</th>
<th>Sample</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Number</td>
<td>Fraction</td>
</tr>
<tr>
<td>Total Number</td>
<td>1,932</td>
<td>1,785</td>
</tr>
<tr>
<td>Commercial Banks</td>
<td>175</td>
<td>9.06%</td>
</tr>
<tr>
<td>Savings Banks</td>
<td>463</td>
<td>23.96%</td>
</tr>
<tr>
<td>Cooperative Banks</td>
<td>1,294</td>
<td>66.98%</td>
</tr>
</tbody>
</table>

This table shows the breakdown of banks according to their banking group as of 31 December 2005, both for all German universal banks and for the sample analyzed in this paper. 'Trading Book' refers to banks that have a trading book and 'Merged' refers to banks that took part in a merger during the sample period (1999 to 2005). 'Number' refers to the absolute number of banks, whereas 'Fraction' refers to the relative share. Branches of foreign banks are not included. Commercial banks include 'Landesbanks' and cooperative central banks as they show more similarities to commercial banks than to savings banks or cooperative banks. Deutsche Bundesbank (2006a) uses a similar classification.
<table>
<thead>
<tr>
<th>Asset positions (POS)</th>
<th>(0;3/12)</th>
<th>(3/12;1]</th>
<th>(1;2]</th>
<th>(2;5]</th>
<th>(5;∞)</th>
</tr>
</thead>
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<tr>
<td>interbank loans</td>
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<td>SON01.355</td>
<td>SON01.356</td>
<td>SON01.357</td>
</tr>
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<td>A1.100/03</td>
<td>A1.100/04</td>
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<tr>
<td>customer loans</td>
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<td>SON01.358</td>
<td>SON01.359</td>
<td>SON01.360</td>
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<td>debt securities held</td>
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<tr>
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</table>

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<th>(2;5]</th>
<th>(5;∞)</th>
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<tr>
<td></td>
<td>ITM</td>
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<td>customer liabilities</td>
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<tr>
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<tr>
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<td></td>
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<td>F1.100/02 + F2.400/02</td>
<td>F1.100/03 + F2.400/03</td>
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<tr>
<td>non-maturing deposits</td>
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<td>HV21.221</td>
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</table>

This table shows the positions we use in our analysis and the time bands that are available for the German banking system. The rows refer to the balance sheet positions POS. RTM denotes the time bands according to remaining time to maturity (included in the data schedule pursuant to the auditor's report). ITM denotes the time bands according to initial maturity (included in the monthly balance sheet statistics). The columns refer to the range of the respective bands (in years). The cells refer to the report items, that are denoted in line with Deutsche Bundesbank, i.e. [form].[item] and [form].[row].[column] for the data schedule according to the auditor's report and the monthly balance sheet statistics, respectively. The report items from the data schedule according to the auditor's report are available on a yearly basis, those from the monthly balance sheet statistics on a monthly basis.

The amounts in the data schedule according to the auditor's report do not fit the corresponding data in the monthly balance sheet statistics for a small number of banks. To obtain consistent data, we scale the former to fit the latter for these banks.

^# : The amount of business within the time band is bigger than the report item.
† : The complete outstanding amount of the position 'debt securities issued' is given on a monthly basis by $F1.100/01 + F2.400/01 + F1.100/02 + F2.400/02 + F1.100/03 + F2.400/03 + HV21.224$.
‡ : On a yearly basis, the outstanding amount of the position 'debt securities issued' with a remaining time to maturity of up to 1 year (i.e. $RTM_{t}^{0,1}$) is reported by $SON01.380 + SON01.374 + SON01.375 - HV21.234$. 

Table 2: Positions and time bands
Table 3: Evaluation of the model quality

<table>
<thead>
<tr>
<th>TAM</th>
<th>BM</th>
<th>p-Value</th>
<th>N</th>
</tr>
</thead>
<tbody>
<tr>
<td>Intercept</td>
<td>Slope</td>
<td>( R^2 )</td>
<td>Intercept</td>
</tr>
<tr>
<td><strong>IRRS-banks</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.0365</td>
<td>0.2554</td>
<td>0.2733</td>
<td>0.0283</td>
</tr>
<tr>
<td>(3.88)***</td>
<td>(19.82)***</td>
<td>(2.32)**</td>
<td>(15.73)***</td>
</tr>
</tbody>
</table>

This table shows the results of the regression \( RMI_{IRRS} = \alpha_{Model} + \beta_{Model} RMI_{Model} + \epsilon \) for the subsample of banks that took part in the IRRS (IRRS-banks) and for which both the TAM and the BM could be calculated. \( RMI_{IRRS} \) denotes the interest rate risk of bank \( i \) reported in the IRRS and \( RMI_{Model} \) represents the risk measure of the TAM and the BM, respectively. The \( p \)-values are based on \( t \)-statistics (See Appendix B) and correspond to the null hypothesis \( H_0: R^2_{TAM} = R^2_{BM} \). N denotes the number of observations. Significance at the 10%/5%/1% level is marked ***/**/*.

Table 4: Evaluation of the model quality on sub samples of IRRS-banks

<table>
<thead>
<tr>
<th></th>
<th>TAM</th>
<th>BM</th>
<th>p-Value</th>
<th>N</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total Sample (IRRS-banks)</td>
<td>0.2733</td>
<td>0.1915</td>
<td>0.0020</td>
<td>1,047</td>
</tr>
<tr>
<td>By Banks' Assumption on the Economic Maturity of Non-Maturing Deposits</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1st Quintile (shortest maturity)</td>
<td>0.3018</td>
<td>0.2386</td>
<td>0.0000</td>
<td>198</td>
</tr>
<tr>
<td>2nd Quintile</td>
<td>0.2959</td>
<td>0.2315</td>
<td>0.2264</td>
<td>212</td>
</tr>
<tr>
<td>3rd Quintile</td>
<td>0.2300</td>
<td>0.1517</td>
<td>0.3375</td>
<td>211</td>
</tr>
<tr>
<td>4th Quintile</td>
<td>0.1900</td>
<td>0.1509</td>
<td>0.3599</td>
<td>207</td>
</tr>
<tr>
<td>5th Quintile (longest maturity)</td>
<td>0.2493</td>
<td>0.2389</td>
<td>0.7518</td>
<td>208</td>
</tr>
<tr>
<td>Non-Merged Banks (IRRS-banks)</td>
<td>0.3089</td>
<td>0.1782</td>
<td>0.0001</td>
<td>585</td>
</tr>
<tr>
<td>By Banks' Assumption on the Economic Maturity of Non-Maturing Deposits</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1st Quintile (shortest maturity)</td>
<td>0.6689</td>
<td>0.2756</td>
<td>0.0000</td>
<td>103</td>
</tr>
<tr>
<td>2nd Quintile</td>
<td>0.2089</td>
<td>0.1489</td>
<td>0.0492</td>
<td>121</td>
</tr>
<tr>
<td>3rd Quintile</td>
<td>0.2462</td>
<td>0.1650</td>
<td>0.0057</td>
<td>121</td>
</tr>
<tr>
<td>4th Quintile</td>
<td>0.2254</td>
<td>0.1863</td>
<td>0.4545</td>
<td>116</td>
</tr>
<tr>
<td>5th Quintile (longest maturity)</td>
<td>0.2315</td>
<td>0.2400</td>
<td>0.8505</td>
<td>118</td>
</tr>
</tbody>
</table>

This table shows the results of the regression \( RMI_{IRRS} = \alpha_{Model} + \beta_{Model} RMI_{Model} + \epsilon \) for subsamples of banks that took part in the IRRS (IRRS-banks) and for which both the TAM and the BM could be calculated. \( RMI_{IRRS} \) denotes the interest rate risk of bank \( i \) reported in the IRRS and \( RMI_{Model} \) the risk measure of the TAM and of the BM, respectively. The \( p \)-values are based on \( t \)-statistics (See Appendix B) and correspond to the null hypothesis \( H_0: R^2_{TAM} = R^2_{BM} \). The quintile analysis is based on the bank internal assumptions on the (economic) maturity of non-maturing deposits as reported in the IRRS. Banks that did not report their internal assumptions are excluded from the quintile analysis. 'Non-merged banks' refers to those banks that did not merge during the sample period (1999 to 2005). N denotes the number of observations.
Table 5: Analysis of the TAM bias

<table>
<thead>
<tr>
<th></th>
<th>Intercept</th>
<th>Slope 1 $RM^{TAM}$</th>
<th>Slope 2 $\log(\text{Assets})$</th>
<th>DSB</th>
<th>DCB</th>
<th>DTB</th>
<th>Adj. $R^2$</th>
<th>N</th>
</tr>
</thead>
<tbody>
<tr>
<td>IRRS-banks</td>
<td>0.1046</td>
<td>0.2326</td>
<td>-0.0033</td>
<td>0.0376</td>
<td>0.0132</td>
<td>0.0270</td>
<td>0.2747</td>
<td>1.077</td>
</tr>
</tbody>
</table>

This table shows the results of the multiple regression $RM^{IRR}_i = \alpha + \beta_1 RM^{TAM}_i + \beta_2 \log(\text{Assets}_i) + \delta_{SB} D_{SB} + \delta_{CB} D_{CB} + \delta_{TB} D_{TB} + \epsilon_i$ in the subsample of banks that took part in the IRRS (IRR-banks) and for which the TAM could be calculated. $RM^{IRR}_i$ denotes the interest rate risk of bank $i$ reported in the IRRS and $RM^{TAM}_i$ the risk measure of the TAM. $\text{Assets}_i$ denotes the sum of assets of bank $i$. $D_{SB}$ ($D_{CB}$, $D_{TB}$) is a dummy for savings banks (cooperative banks, trading book institutions). The basis scenario refers to a commercial bank without a trading book. $t$-ratios are in parentheses. $N$ denotes the number of observations. Significance at the 10%/5%/1% level is marked */**/***.

Table 6: Determinants of the interest rate risk

<table>
<thead>
<tr>
<th></th>
<th>Log(Assets)</th>
<th>DSB</th>
<th>DCB</th>
<th>DTB</th>
<th>Adj. $R^2$</th>
<th>N</th>
</tr>
</thead>
<tbody>
<tr>
<td>Complete Sample</td>
<td>0.0059</td>
<td>0.1163</td>
<td>0.1460</td>
<td>0.0426</td>
<td>0.3181</td>
<td>1.785</td>
</tr>
</tbody>
</table>

This table shows the results of the regression $\hat{RM}^{TAM}_i = \alpha + \beta \log(\text{Assets}_i) + \gamma_{SB} D_{SB} + \gamma_{CB} D_{CB} + \gamma_{TB} D_{TB} + \epsilon_i$ on the whole data sample for which the TAM could be calculated. $\hat{RM}^{TAM}_i$ denotes the unbiased TAM-risk measure (14) of bank $i$. $D_{SB}$ ($D_{CB}$, $D_{TB}$) is a dummy for savings banks (cooperative banks, trading book institutions). We do not report the value of the intercept since the level of interest rate risk for the German banking system is handled confidentially by the Deutsche Bundesbank. The basis scenario refers to a commercial bank without a trading book. $t$-ratios are in parentheses. $N$ denotes the number of observations. Significance at the 10%/5%/1% level is marked */**/***.