Subsidizing Technological Innovations in the Presence of R&D Spillovers*

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Abstract

We analyze a situation where a principal wants to induce two firms to produce an output, e.g. electricity from renewable energy sources. Firms can undertake non-contractible investments to reduce production cost of the output. Part of these investments spills over and also reduces production cost of the other firm. Comparing a general price subsidy and an innovation tournament, we find that the principal’s expected cost of implementing a given expected output are always higher under the tournament, even though this scheme may lead to more innovation.

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1 Introduction

In this paper we analyze a situation where a principal wants to induce two firms to produce an output. The firms can undertake a costly investment to reduce production cost of the output. Part of this ‘innovation’ spills over and also reduces production cost of the other firm. We focus on the principal’s choice between subsidizing both firms or only the most successful innovator.

A topical problem that conforms to this general structure are recent measures to increase electricity production from renewable energy sources in order to combat climate change and to reduce dependency on fossil energy. Renewable energy is not competitive yet, but the hope is that innovations will bring down production costs (Manne and Richels 2004). Therefore, several countries like Germany, France and Spain have passed legislation by which all producers of renewable energy receive a fixed price for power sold to the grid that lies above the market price. The instrument went quite well in practice. For example, the share of renewables in the consumption of electricity increased in Germany from 4.6% in 1998 to 9.3% in 2004 (BMU 2005).

Nevertheless, some decision-makers have suggested that the subsidies should be focused on the most promising projects only. Therefore, we compare the general price subsidy to an innovation tournament, where only the winner receives an output price subsidy. This has an additional advantage. Firms disregard the beneficial effect that innovation spillovers have on other firms, resulting in under-investment. A tournament may strengthen innovation incentives since the firms try to outperform each other.

An innovation tournament has substantial similarities with the Non-Fossil Fuel Obligation (NFFO) in the UK (Cleirigh 2001). Under this scheme renewable energy production projects were awarded to the firm who asked the lowest price for producing a specified output. Intuitively, the firm which realized the better cost-reducing innovation should win the bidding competition. This is also the case in our tournament model, which is more simple, however, since it disregards the strategic interaction at the bidding stage. In practice, the NFFO had only limited success, and it has been replaced recently by a quota system.

We model firms’ choices as a two stage game. In the first stage, firms invest into an innovation that reduces the cost of producing output. In the second stage, stochastic innovations are observed and production takes place. While we assume that output can be contracted upon, contracts based on the value of innovation are not feasible. The reason is that even if the principal (i.e. the government) and the firms can evaluate the innovation, such information is usually difficult to verify by a court. Moreover, we assume that firms’ investments are not observable. Therefore, we have a moral hazard problem and the first best innovation/output profile will not be implementable if firms are wealth constrained.

\footnote{See the debate between the then German ministers for the economy and the environment (‘Clement sucht Konfrontation mit Trittin’, Frankfurter Allgemeine Zeitung, 02.09.2003, p. 11). Also the German chancellor Angela Merkel has criticized that “everyone has access to the subsidies” (‘Schwarz-gelber Mix’, DIE ZEIT, 02.06.2005, p. 24).}
The government minimizes the expected costs of achieving an output target, e.g., regarding electricity from renewable energies. We focus on two policy instruments: a general output price subsidy (GPS), and an innovation tournament such that only the winner receives an output price subsidy. This restriction to subsidize either both firms to the same extent or only one firm keeps the analysis tractable. Furthermore, these two schemes seem to be the most relevant, since guaranteeing firms different prices for electricity that has been generated from the same renewable energies would probably constitute illegal price discrimination.

A central feature of our model is that innovation is not completely appropriable due to technological spillovers, which may be substantial even in the presence of patent protection (Mansfield 1985). Reasons are (i) personnel movements between employers, (ii) formal and informal networks between researchers such as seminars, publications and casual encounters, as well as (iii) reverse engineering (see Geroski 1995). The first two channels relate to (input) spillovers that occur during the R&D process. The third channel relates to spillovers of the final R&D output. The formal analysis in our paper is restricted to the former. However, in the concluding remarks we will argue that spillovers of R&D output would further strengthen our main result.

The analysis focuses on two related issues: innovation investments and the government’s cost of implementing a targeted output level. Investments may be higher under the tournament than under the GPS if the stakes are such that firms are highly motivated to win the innovation contest. However, research spillovers dilute this motivation since they reduce the effect of own research efforts on the chances of winning. Furthermore, a firm’s ex-ante expected output is higher under the GPS, which increases the incentive to invest in cost-reducing innovations under this scheme. In summary, it turns out that with perfect spillovers the GPS always induces more innovation investments, while the comparison is ambiguous if spillovers are low.

However, even in those cases where the tournament induces more innovation, it always leads to higher expected costs of implementing a targeted output level. One reason is our assumption that marginal production cost increase in the level of output, which favors the GPS where both firms produce. Our model allows this effect to be arbitrarily small, but in this case the GPS turns out to produce the better innovation.

Our basic setup is related to the large industrial organization literature on innovation spillovers. A seminal contribution to this literature is d’Aspremont and Jacquemin (1988), who also consider the interaction among firms that invest in cost-reducing innovations. These are not completely appropriable due to spillovers, leading to underinvestment in R&D.²

Our paper differs from d’Aspremont and Jacquemin (1988) and most of the related literature in several important respects. First, there is no problem of

imperfect competition in our framework. Under the policy instruments that we consider the government commits ex-ante to a price that it pays for renewable energies. From the perspective of the firms this price is exogenously given.

Second, there is an active regulator who can use his budget to provide incentives for innovation investments and output production. Hinloopen (1997) also considers an active government, but he focuses on subsidies for R&D, which is non-contractible in our framework.

In particular, firms’ innovation investments are usually non-observable to external parties and their outcome is subject to uncertainty. To reflect this, we model innovation as a stochastic process. Consequently, related investments are non-contractible, leading to moral hazard. In our paper, investments improve the distribution of the stochastic innovation and, thereby, reduce expected production cost. Fullerton and McAfee (1999) analyze a research tournament with a similar innovation technology. In contrast to our model, they consider a broader class of probability distributions, but do not allow for research spillovers.

Early contributions to the analysis of innovative activity under uncertainty include Loury (1979), Lee and Wilde (1980), and Dasgupta and Stiglitz (1980). These authors analyze innovation races, i.e., contests in which the winner is the party that first discovers an innovation with ex-ante specified characteristics. By contrast, an innovation tournament, as considered in, e.g., Taylor (1995), Fullerton and McAfee (1999), Fullerton et al. (1999), and the present paper, rewards the party with the best (random) innovation on a prespecified date. As in our model, Martin (2002) and Miyagiwa and Ohno (2002) also incorporate spillovers into a stochastic R&D process. The former paper analyzes a patent racing model of cost-saving innovation in a quantity-setting duopoly with input and output spillovers. Miyagiwa and Ohno (2002) compare different cooperative R&D regimes in an innovation race with output spillovers.

There is also a substantial environmental economics literature on the stimulation of technological innovation. However, most of this literature analyzes firms’ decisions to adopt a known technology under different instruments such as permits, taxes and standards (e.g., Requate and Unold 2003; Jung, Krutilla, and Boyd 1996). Nevertheless, some notable exceptions exist. Fisher, Parry, and Pizer (2003) analyze endogenous innovation and also allow for research spillovers. However, in their model only one firm is an innovator, the innovation process is deterministic and they analyze different policy instruments than we do, namely taxes and permits. Biglaiser and Horowitz (1995) consider binary choices whether to undertake research into a technology that reduces the emission intensity of production. In Tsur and Zemel (2002), a regulator auctions the procurement of an environmental project to an individual firm, and conditions transfers to this firm on the project completion time.

Finally, our paper is related to the literature that compares relative and absolute reward schemes regarding their efficiency in the provision of incentives. In our model, one of the main reasons why the research tournament performs badly

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is that a firm’s own investment in R&D also improves the rival’s expected performance. Focussing on multi-tasking in team production, Lazear (1989) and Drago and Garvey (1998) make a similar point. They show that the negligence of tasks with positive externalities on team members is a drawback of relative incentive schemes.

The paper proceeds as follows. Section 2 introduces the model. Sections 3 and 4 analyze the GPS and the tournament, respectively. Section 5 compares these two policy instruments, and section 6 concludes.

2 The model

There are two ex-ante identical firms indexed alternatively by \(i, j = 1, 2\). In the first stage, the government commits to a mechanism, i.e. either a general price subsidy (GPS) or a tournament.

In the second stage, each firm undertakes a non-observable investment \(x_i \geq 0\) into the development of a process innovation that reduces production cost. The uncertain and non-verifiable innovation output of this investment is \(y_i\). Specifically, consider a twice continuously differentiable function \(g_i(x_i, x_j) \geq 0\) which, as will be specified in a moment, reflects the effects of innovation investments on the innovation output. We make the following assumption:

(i) If \(g_i(x_i, x_j) = 0\), then \(y_i = 0\).

(ii) If \(g_i(x_i, x_j) > 0\), then \(y_i\) is a random variable with range \((0, 1]\) and cumulative distribution function \(F_i(y_i|x_i, x_j) = y_i^{g_i(x_i, x_j)}\). The corresponding density function is denoted \(f_i(y_i|x_i, x_j)\).

Given \(x_i\) and \(x_j\), the two random variables \(y_i\) and \(y_j\) are independently distributed. Since firms are identical ex ante, \(g_i(a, b) = g_j(a, b)\) for all \(a, b \geq 0\). If firms do not invest, the innovation output will be zero, i.e. \(g(0, 0) = 0\). Furthermore, we assume that

\[
\frac{\partial g_i(x_i, x_j)}{\partial x_i} > 0 \quad \text{and} \quad \frac{\partial g_i(x_i, x_j)}{\partial x_j} \geq 0, \tag{A1}
\]

\[
\frac{\partial g_i(x_i, x_j)}{\partial x_i} \geq \frac{\partial g_i(x_j, x_i)}{\partial x_i}, \tag{A2}
\]

\[
\frac{\partial^2 g_i(x_i, x_j)}{\partial x_i^2} \leq 0 \quad \text{and} \quad \frac{\partial^2 g_i(x_j, x_i)}{\partial x_i^2} \leq 0 \tag{A3}
\]

for all \(x_i, x_j \geq 0\) and \(i, j = 1, 2\), \(i \neq j\).

The first inequality in assumption (A1) says that \(g_i(x_i, x_j)\) is strictly increasing in \(x_i\), implying that \(\frac{\partial F_i}{\partial x_i} < 0\) for \(y_i < 1\). That is, a higher investment of firm \(i\) improves the distribution of its innovation in the sense of first-order stochastic dominance and, therefore, reduces expected production costs. Furthermore,

\[4\]Later, we consider the specific example \(F_i(y_i) = y_i^{x_i + x_j}\), where \(0 \leq z \leq 1\). If \(x_i = x_j = x\), the innovation process can be interpreted as each firm taking \(x\) identical, independent draws from the distribution \(F_i(y_i) = y_i^{1+z}\).
$g_i(x_i, x_j)$ may also increase in $x_j$, which means that there are spillovers of R&D inputs. However, by assumption (A2), a firm’s own investments have a (weakly) more beneficial effect on the distribution of its innovation than foreign investments. This assumption will guarantee that firms are willing to make strictly positive investments in the research tournament. It reflects that spillovers are usually incomplete and that knowledge acquired from rivals may not fit exactly with a firm’s existing knowledge base (see Hinloopen 2003).

Finally, assumption (A3) will ensure that a firm’s optimization problem satisfies certain regularity requirements. This assumption also entails that $\frac{\partial^2 F_i}{\partial x_i^2}, \frac{\partial^2 F_i}{\partial x_j^2} \geq 0$. Consequently, the probability that a firm’s innovation exceeds some given level increases at a decreasing rate with investments.\footnote{See Amir (1996), who introduces such a convexity assumption for distribution functions to analyze a game where two players invest in a jointly owned productive asset.}

In the third stage, each firm observes its innovation and produces the verifiable output $q_i, q_j \geq 0$. We assume that the model focuses on innovation which is ‘essential’ in the sense that no output can be produced in the absence of innovation. By the assumptions $y_i = 0$ if $g_i(x_i, x_j) = 0$, and $g_i(0, 0) = 0$, this case occurs whenever firms do not invest. For $y_i > 0$, firm $i$’s total production cost after accounting for the process innovation are given by

$$c(x_i, y_i, q_i) = \frac{q_i^{1+s}}{(1+s)y_i} + x_i, \quad s, r > 0. \quad (4)$$

Accordingly, production costs are increasing and convex in output $q$. Since we assume that firms receive a fixed price for their output, the convexity property will assure that firms’ profit functions are concave. Production cost decrease in the innovation output $y_i$. The parameters $s$ and $r$ describe how responsive production costs are to changes in $q_i$ and $y_i$, respectively. In particular, $1 + s$ is the elasticity of production cost with respect to output $q_i$, and $-r$ is the elasticity of production cost with respect to the innovation level $y_i$.

All parties are risk neutral, and firms’ reservation utility is zero. Furthermore, firms are wealth constrained so that they cannot pay entry fees for participation in the tournament or for being entitled to receive subsidies. For parsimony, we assume that firms receive payments for their output only from the principal. Accordingly, under the tournament scheme the losing firm, which receives no subsidy, will not produce output. This reflects that without subsidies renewable energies are not competitive yet due to higher production costs than conventional energies.

We suppose that the government’s objective function increases in the environmental benefits that are associated with a higher level of renewable energies and decreases in the expected costs for implementing this level. Maximization of the objective function would require a specification of the government’s monetarized benefits from producing electricity with renewable rather than ‘conventional’ energy sources. Doing so would be challenging since these benefits are essentially not known. Therefore, we minimize the government’s expected cost for implementing any arbitrarily fixed expected overall output, which is a necessary condition for
maximizing the objective function. This approach is convenient since our results on the comparison between the two mechanisms do not depend on the implemented output level. Therefore, to decide on the superiority of one mechanism, it is not necessary to determine the output level that maximizes the government’s objective function.

3 General price subsidy

The game is solved by backwards induction, and we first consider the GPS. In the last stage, given innovation \( y_i > 0 \), firm \( i \) chooses output \( q_i \) to maximize earnings less production cost:

\[
\max_{q_i} pq_i - \frac{q_i^{1+s}}{(1+s)\bar{y}_i^r}.
\]

(5)

From the first-order condition, output is chosen according to

\[
q_i(p, y_i) = \left( py_i^r \right)^{1/s}.
\]

(6)

In the investment stage, anticipating \( q_i(\cdot) \) and given firm \( j \)'s investment \( x_j \), firm \( i \) solves

\[
\max_{x_i} E \left[ p \left( py_i^r \right)^{1/s} - \frac{\left( py_i^r \right)^{1+s}}{(1+s)\bar{y}_i^r} \right] _{x_i, x_j} - x_i = \max_{x_i} \frac{p^\sigma}{\sigma} E[y_i^r | x_i, x_j] - x_i,
\]

(7)

where \( \sigma := (1 + s)/s, \nu := r/s, \) and

\[
E[y_i^r | x_i, x_j] = \int_0^1 y_i^r f_i(y_i | x_i, x_j) dy_i = \int_0^1 y_i^r g_i(x_i, x_j) g_i(x_i, x_j)^{-1} dy_i
\]

(8)

\[
= \frac{g_i(x_i, x_j)}{g_i(x_i, x_j) + \nu}.
\]

(9)

Assuming that \( p \) is large enough to induce positive investments, the first-order conditions for an equilibrium in the investment stage are\(^6\)

\[
\frac{p^\sigma}{\sigma} \left[ \frac{\nu \partial g_i}{g_i(x_i, x_j) + \nu} \right]_i - 1 = 0, \quad i, j = 1, 2, \quad i \neq j.
\]

(10)

\(^6\)Specifically, the equilibrium investment level is strictly positive whenever the l.h.s. of (10) is positive at \( x_i = x_j = 0 \), i.e.

\[
\frac{p^\sigma}{\nu} \left. \frac{\partial g_i}{\partial x_i} \right|_{x_i = x_j = 0} - 1 > 0.
\]

The second order conditions holds since, using assumptions (A1) and (A3), it is straightforward to verify that (7) is concave in \( x_i \).
Since firms are identical ex ante, we focus on a symmetric equilibrium \( x_i = x_j = x_a \) in the investment stage.\(^7\) Hence a firm’s investment \( x_a \) is given by

\[
p^\sigma \nu \frac{\partial g_i}{\partial x_i} \bigg|_{x_a} (g_a + \nu)^2 - 1 = 0,
\]

(11)

where \( g_a := g_i(x_a, x_a) \).\(^8\) Implicit differentiation of (11) shows that investments under the GPS increase in the output price. Intuitively, for any given innovation \( y_i \), a higher price induces more output (see 6). This makes cost reducing investments more beneficial.

Using (6) and (9), expected overall output is

\[
g_a(p) := E[q_i + q_j | x_a] = 2p^{\frac{1}{\sigma}} E[y_i|x_a] = 2p^{\frac{1}{\sigma}} \frac{g_a}{g_a + \nu}.
\]

(12)

The effect of spillovers on investments and output depends on the characteristics of the function \( g_i(x_i, x_j) \). For example, consider again \( g_i(x_i, x_j) = x_i + zx_j \), \( 0 \leq z \leq 1 \). Implicit differentiation of (11) then shows that

\[
\frac{dx_a}{dz} = -\frac{x_a}{1 + z},
\]

(13)

i.e. investments under the GPS decrease in input spillovers. Intuitively, given that own and foreign investments are substitutes, a firm’s incentive to invest decreases as it can absorb more of the other firm’s innovation investments. However, this need not be the case if investments are complements, i.e. if the effect of own investments on \( g_i \) increases as R&D spillovers from the other firm increase.

Turning to expected output, this is increasing in \( g_a \) (from 12). Denoting R&D spillovers by \( z_i(x_i, x_j) := \frac{\partial g_i(x_i, x_j)}{\partial x_j} \) we obtain

\[
\frac{d}{dz_i} g_i (x_a(z_i), x_a(z_i); z_i) = \frac{\partial g_i}{\partial x_i} \frac{dx_i}{dz_i} + \frac{\partial g_i}{\partial z_i}.
\]

(14)

While the second term is positive, we have just argued that the first term may be negative if investments are substitutes. This is the case for the example \( g_i = x_i + zx_j \), for which the two effects just cancel out so that expected output is independent of R&D spillovers.

\(^7\) Depending on the particular form of \( g_i(x_i, x_j) \), asymmetric equilibria may exist. For example, suppose that \( g_i(x_i, x_j) = x_i + zx_j \), \( 0 \leq z \leq 1 \). It is easily verified that, in this case, the equilibrium is unique and symmetric if \( z < 1 \). However, if spillovers are perfect (\( z = 1 \)) so that only the overall amount of investments matters, there is a continuum of equilibria.

\(^8\) Participation constraints hold under both mechanisms since investing \( x_i = 0 \) leads to an expected payoff of at least zero, so that the expected payoff under the optimal investment must be nonnegative.
4 Tournament

Under the general price subsidy firms disregard the positive effect that R&D spillovers have on the production cost of other firms. In order to stimulate innovation investments, the government may consider research tournaments. Under this scheme only the winner, i.e. the firm with the better innovation, receives the price subsidy. Accordingly, firms have an additional investment incentive since they want to outperform each other.

The sequence of moves is the same as in the previous section: In stage 1, the government commits to a price subsidy for the tournament winner. In stage 2, firms invest and the winner is determined. Ties are solved by flipping a fair coin. Since they occur with probability zero, they are henceforth neglected. In stage 3, the winner produces output. By assumption, the losing firm will not find it profitable to produce.

The innovation of the tournament winner is $\hat{y} := \max\{y_i, y_j\}$. Its cumulative distribution function is $F(\hat{y}|x_i, x_j) = F_i(\hat{y}|x_i, x_j)F_j(\hat{y}|x_i, x_j)$, and the corresponding density takes the form

$$f(\hat{y}|x_i, x_j) = f_i(\hat{y}|x_i, x_j)F_j(\hat{y}|x_i, x_j) + F_i(\hat{y}|x_i, x_j)f_j(\hat{y}|x_i, x_j) \quad (15)$$

Consequently, we obtain

$$E[\hat{y}^\nu|x_i, x_j] = \int_0^1 \hat{y}^\nu \, f(\hat{y}|x_i, x_j) d\hat{y} \quad (17)$$

$$= \frac{g_i(x_i, x_j) + g_j(x_j, x_i)}{g_i(x_i, x_j) + g_j(x_j, x_i) + \nu} \quad (18)$$

Comparing (9) and (18), for given investments $x_i = x_j$ the expected innovation level of the tournament winner is larger than the average innovation level under the GPS. This reflects that the tournament selects the most successful innovator. However, the chance of winning is only 50 percent. Hence for a given price $p$ and investments $x_i = x_j$, each firm’s ex-ante expected output is lower under the tournament than under the GPS, i.e. $\frac{1}{2} p^2 E[\hat{y}^\nu] < p^2 E[y_i^\nu]$.

We now turn to the firms’ optimal investment decisions. In the last stage, the tournament winner’s problem of maximizing profit for a given innovation is equivalent to the GPS, leading to output as given in (6). Thus, in the investment stage firm $i$ maximizes expected profits, taking into account that the effect of investments $x_i$ on innovations $y_i$ is stochastic:

$$\max_{x_i} \int_0^1 \left[ \int_{y_i}^1 \left( p(p y_i^\nu)^{1/s} - \frac{(p y_i^\nu)^{1+s}}{(1 + s) y_i^s} \right) f_i(y_i) dy_i \right] f_j(y_j) dy_j - x_i, \quad (19)$$

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9This requires that realized innovations are observable by the government, but they need not be verifiable to a third party.
where the term in square brackets reflects that firm $i$ will only produce output if it has achieved the better innovation $y_i > y_j$. After substitution for the density functions and some transformations, the problem can be restated as

$$\max_{x_i} p^\sigma \frac{g_i(x_i, x_j)}{\sigma (g_i(x_i, x_j) + g_j(x_j, x_i) + \nu)} - x_i.$$  

(20)

As under the GPS, the equilibrium investment level will be positive if the output price $p$ is sufficiently large, which we assume to be the case.\(^\text{10}\) Furthermore, we assume that firm $i$’s objective function (20) is quasi-concave in $x_i$ for every $x_j$.\(^\text{11}\)

Again, we restrict attention to a symmetric equilibrium, which we denote by $x_i = x_j =: x_t$.\(^\text{12}\) From the first-order conditions, $x_t$ is given by

$$\frac{p^\sigma \frac{\partial g_i}{\partial x_i}}{\sigma} \bigg|_{x_t} (g_t + \nu) - g_t \frac{\partial g_i}{\partial x_i} \bigg|_{x_t} = 1 = 0,$$

where $g_t := g_i(x_t, x_t) = g_j(x_t, x_t)$.

From (6) and (18), expected output under the tournament is

$$q_t(p) := p^{1/s} E [y^\nu | x_t] = p^{1/s} \frac{2g_t}{2g_t + \nu}.$$  

(22)

To analyze the effect of spillovers on innovation investments, consider again the example $g_i(x_i, x_j) = x_i + zx_j, 0 \leq z \leq 1$. Implicit differentiation of (21) then yields

$$\frac{dx_t}{dz} = \frac{2zx_t[2(1 + z)x_t + \nu] + 4x_t[(1 - z^2)x_t + \nu]}{(1 - z^4)[2(1 + z)x_t + \nu] - 4(1 + z)[(1 - z^2)x_t + \nu]} - \frac{x_t}{1 + z},$$

(23)

since the denominator is negative by the second order condition. Comparing this with (13), R&D spillovers have a more detrimental effect on investments under the tournament than under the GPS. Intuitively, the higher spillovers, the lower the effect that own investments have on the chances of winning the tournament.

We now turn to a more thorough comparison of investments and of the government’s cost of implementing a targeted output level under the two schemes.

\(^{\text{10}}\)Using assumption (A2), it is easily verified that $\frac{m}{g_i + g_j + \nu}$ strictly increases in $x_i$ at $x_i = 0$ for all $x_j$. Thus, there are always prices under which investments are positive. Indeed, it is straightforward to see that the condition for positive investment levels is identical to that under the GPS (see footnote 6).

\(^{\text{11}}\)Quasi-concavity and sufficiency of the first-order condition are ensured as follows. The price $p$ is chosen such that there is an investment level $x_i = \hat{x} > 0$ at which the first derivative of (20) is zero. A sufficient condition for quasi-concavity of (20) is that, if the first derivative is zero at $x_i$, then the second derivative is strictly negative at $x_i$. This condition also implies that $\hat{x}$ is the global maximizer (and not a saddle point). It is straightforward to show that it is satisfied if $\frac{\partial^2 g_i}{\partial x_i^2}(y_j + \nu) - \frac{\partial^2 g_j}{\partial x_i^2}g_i \leq 0$. This inequality always holds if $g_i(x_i, x_j)$ is linear in $x_i$ and $x_j$. If $g_i(x_i, x_j)$ is strictly concave, the inequality is valid if, for all $x_i$ and $x_j$, $\frac{\partial^2 g_i}{\partial x_i^2}g_i$ is bounded from below and $\nu$ is sufficiently large.

\(^{\text{12}}\)In the example $g_i(x_i, x_j) = x_i + zx_j$, equivalently to the GPS, there is a unique and symmetric equilibrium if $z < 1$ and a continuum of equilibria if $z = 1$. 

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5 Comparison of the two schemes

In the previous section we have discussed three effects that determine differences in innovation investments under the GPS and the tournament. First, for a given price \( p \) and identical investments \( x_i, x_j \), each firm’s ex-ante expected output is lower under the tournament. This weakens incentives to invest in cost reducing innovations under this scheme. Second, the tournament rewards the firm that achieves the best innovation. This strengthens incentives to invest. Third, the latter effect is diluted through spillovers of research inputs. If they are perfect, own investments have no effect on the chances of winning the tournament anymore.

The comparison of the expected innovation level under the two schemes depends on the relative strength of these effects. With perfect spillovers the GPS always induces more innovation. As spillovers are reduced, the comparison becomes ambiguous and it may happen that the tournament performs better. In particular, with no spillovers the tournament leads to more innovation investments if the expected innovation of the tournament winner – and therefore \( E[\hat{y}^\nu | x_i, x_j] \) and the expected payoff – is large, since this implies a high incentive to win the tournament. Noting from (18) that \( E[\hat{y}^\nu | x_i, x_j] \) increases in \( g_i \) and falls in \( \nu = \frac{r}{s} \), the following proposition summarizes these considerations.

**Proposition 1** Suppose that the output price \( p \) is large enough to induce positive investments. With perfect input spillovers \( x_a > x_t \). With no input spillovers, \( x_t > x_a \) if and only if \( s g_a > 0.5 r (1 + \sqrt{5}) \).

**Proof.** Given \( p \), from (11), (21), and quasi-concavity of (20), it follows that \( x_a > x_t \) if and only if

\[
\frac{\sigma}{p^2} = \frac{\nu \frac{\partial g_a}{\partial x_i} |_{x_a}}{(g_a + \nu)^2} > \frac{\frac{\partial g_a}{\partial x_i} |_{x_a} (g_a + \nu) - g_a \frac{\partial g_j}{\partial x_i} |_{x_a}}{(2g_a + \nu)^2},
\]

where the r.h.s. has been obtained from evaluating (21) at \( x_t = x_a \). With perfect spillovers, \( \frac{\partial g_a}{\partial x_i} = \frac{\partial g_j}{\partial x_i} \), the numerators are the same on both sides of the inequality sign, and the first statement follows straightforwardly. With no input spillovers, (24) simplifies to

\[
\frac{\nu}{(g_a + \nu)^2} > \frac{g_a + \nu}{(2g_a + \nu)^2}.
\]

Given that \( x_a > 0 \) and thus \( g_a > 0 \), the above inequality holds if and only if \( g_a < \frac{\nu}{2} (1 + \sqrt{5}) \). \(\square\)

Obviously, investments under the two schemes are crucial for the government’s cost of implementing a targeted output level. According to Proposition 1, this will favor the GPS more often. In addition, given our assumption of convex production cost the GPS has the advantage that both firms produce output, although this effect is small as \( s \) approaches 0. On the other hand, the tournament enables the government to concentrate subsidies on the firm that has been most successful in reducing its production costs. As the following results show, it turns out that the effects which favor the GPS always dominate.
Proposition 2 The government’s expected cost of implementing a given expected output \( \bar{q} > 0 \) are always lower under the GPS than under the tournament scheme.

Proof. The proof is by contradiction. Assume that the government’s expected costs for implementing a given expected output \( \bar{q} \) are lower under the tournament, i.e.

\[
p_t(\bar{q})\bar{q} < p_a(\bar{q})\bar{q} \quad \iff \quad q_t(\bar{p}_t) > q_a(\bar{p}_t),
\]

where \( \bar{p}_t := p_t(\bar{q}) \) is the price required to implement quantity \( \bar{q} \) under the tournament. By (12) and (22), this is the case if and only if at \( \bar{p}_t \)

\[
\frac{g_a}{g_a + \nu} < \frac{g_t}{2g_t + \nu},
\]

\[\iff \quad g_a < \frac{\nu g_t}{g_t + \nu}.\]  

A necessary condition for this inequality to hold is \( g_a < g_t \) or, equivalently, \( x_a < x_t \). Applying (24) and \( \frac{\partial g_j}{\partial x_i} \geq 0\), this requires

\[
\frac{\nu \frac{\partial g_j}{\partial x_i} \big|_{x_a}}{(g_a + \nu)^2} < \frac{\frac{\partial g_i}{\partial x_i} \big|_{x_a} (g_a + \nu) - g_a \frac{\partial g_i}{\partial x_i} \big|_{x_a} \frac{\partial g_i}{\partial x_i} \big|_{x_a} (g_a + \nu)}{(2g_a + \nu)^2},
\]

from which it follows that \( g_a > \frac{\nu}{2}(1 + \sqrt{5}) \). However, this condition is in contradiction to inequality (28) which can hold only if \( g_a < \nu \).

For the tournament to be better than the GPS, there must be a price \( p \) such that \( q_t > q_a \). That is, the winner of the tournament must produce at least twice as much output as each firm under the GPS. This would require that the production cost function is not too convex (i.e. \( s \) is low), and that the tournament winner’s expected innovation is substantially higher than the average innovation under the GPS (see 28). However, whenever investment incentives are high under the tournament, they are also relatively high under the GPS. Furthermore, whenever \( s \) is low the GPS leads to the better innovation (see Proposition 1). Therefore, expected output under the GPS is always higher.

6 Concluding Remarks

We have motivated our analysis by the problem of promoting new technologies such as renewable energies. While not being competitive yet, energy production from renewables is characterized by steep learning curves. This has been captured by assuming that production cost can be reduced by non-contractible investments into innovations, which partly spill over to other firms. These spillovers, together with our assumption that marginal production cost increase in the level of output, provide a strong rationale for inducing production from both firms in our model.
However, with non-contractible innovation investments firms disregard the beneficial effect that R&D spillovers have on other firms. Therefore, we have considered the alternative instrument of a research tournament, which provides additional investment incentives since firms try to outperform each other. Furthermore, under the tournament subsidies are targeted at the most successful innovator. Nevertheless, we find that the government’s expected cost of inducing a targeted output level are always lower under the GPS. Furthermore, in many cases the GPS also induces more innovation.

For parsimony, we restricted our analysis to the case of two firms. The extension to \( n \) identical firms is straightforward. Holding investments constant, the average innovation of the tournament winner increases in \( n \). However, due to a lower probability of winning, investment incentives are reduced if more firms participate in the tournament. The overall effect is such that, for an arbitrary number of firms, the GPS always dominates a tournament scheme under which only the best innovator produces.

Therefore, the paper provides strong support for the system of guaranteeing a fixed output price for renewables, which has been applied rather successfully in several EU countries. This conclusion seems to be further strengthened if we allow for output spillovers, which occur if firms learn from each other during the production process, e.g. through reverse engineering. Accordingly, they would lower production costs only under the GPS, where both firms produce.

Another issue which has not been considered is that firms can take measures to prevent spillovers. Since spillovers tend to be more detrimental under the tournament, firms are more likely to do so under this scheme. Noting that spillovers are beneficial from a social point of view, this would further strengthen the case for the GPS.

References


