Relational Contracts and Job Design*

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Abstract

This paper analyzes optimal job design in a repeated principal-agent relationship when there is only one contractible and imperfect performance measure for three tasks whose contribution to firm value is non-verifiable. The tasks can be assigned to either one or two agents. Assigning an additional task to an agent strengthens his relational contract. Therefore, broad task assignments are optimal when the performance measure strongly distorts incentives for the two-task job. This is more likely to be the case if these two tasks are substitutes.

Keywords: job design, multi-tasking, relational contracts

JEL classification: M51, M54

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1 Introduction

Measuring employee performance is often difficult because objective performance measures only imperfectly reflect an employee’s contribution to the firm. Thus, if rewards depend only on imperfect measures, employees’ incentives are not perfectly aligned with the firm’s objectives.\(^1\) The additional use of subjective performance measures, i.e., measures that are observed only by the contracting parties, may mitigate this problem. Subjective performance evaluation plays an important role in incentive contracting. Lincoln Electric, for example, motivates its workforce by using piece rates in combination with bonuses based on supervisors’ subjective assessments (Fast and Berg 1975). Thereby, workers are not only rewarded for high output but also for more complex and subtle achievements such as cooperation, innovation, or dependability. Furthermore, Hayes and Schaefer (2000) and Murphy and Oyer (2003) find empirical evidence for subjective assessment in executive compensation, and Gibbs et al. (2004) show that there is subjectivity in the determination of bonuses for auto dealership managers.

Informal agreements based on subjective performance evaluation cannot be part of an enforceable (or explicit) employment contract but have to be self-enforcing. This may be the case if the principal cares about its reputation in future relationships (Holmström 1981; Bull 1987). Baker et al. (1994) show that explicit and relational contracts\(^2\) can be complements as well as substitutes. While in some circumstances only a combination of explicit and relational contracts generates nonnegative profits, relational contracts are infeasible if objective performance measures are sufficiently close to perfect.\(^3\)

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1 See Kerr (1975) for an extensive number of examples.
2 The term relational contract denotes an informal agreement that is not enforceable by a court but is self-enforcing. Such contracts are also called “implicit” (Baker et al. 1994; MacLeod and Malcomson 1989), “self-enforcing” (Klein 1996) or “self-enforcing implicit” (Bull 1987).
3 For the interaction of explicit and relational contracts see also Schmidt and Schnitzer (1995), Pearce and Stacchetti (1998), Bernheim and Whinston (1998), Che and Yoo (2001), and Demougin and Fabel (2004). More generally, contributions to the theory of relational contracts include MacLeod and Malcomson (1989), Klein (1996), Baker et al. (2002), MacLeod (2003), and Levin
The aim of this paper is to analyze how the possibility to engage in relational contracts affects optimal job design. To do so, I first reformulate the model by Baker et al. (1994) for the case of multiple and interdependent tasks, which jointly affect non-verifiable firm value and a contractible but imperfect performance measure.

For a given set of tasks performed by a single agent, I find that relational contracts are feasible if the performance measure is sufficiently distortionary\(^4\) or firm value is sufficiently responsive to changes in effort. In both cases, the principal greatly benefits from better aligning incentives by promising an implicit bonus based on firm value. Employees trust the principal since they anticipate that it is in her interest not to renege on relational contracts.

In the next step, I examine when tasks should be split between agents. For simplicity, I consider an environment in which three tasks are to be assigned to either one or two risk-neutral agents.\(^5\) This is for example the case if some tasks are non-separable, e.g., quantity and quality in the production of a good. Furthermore, agents cannot perform the same task. Task splitting thus denotes the grouping of tasks in two different jobs where no task is part of both jobs.

If the separable task is independent of the other tasks, the principal prefers task splitting when relational contracts are not feasible. Then, she always benefits from having first-best effort in the separable task. By contrast, if she can credibly commit to pay an implicit bonus for the two-task job, she assigns all tasks to a single agent. Doing so has two effects. First, effort in the one-task job is in general no longer first-best. Second, the principal can commit to a higher implicit bonus than for the two-task job. The reason is that the performance of an agent who is responsible for three tasks is more important for the firm value. Therefore, the

\(^4\)Given that the principal’s fallback position is positive, this result is in line with Baker et al. (1994).

\(^5\)It is important to have more tasks than agents. The reason is that, besides interdependencies in the agents’ cost function, the only externality that can arise in my model is due to the misallocation of effort across tasks. Therefore, if tasks are substitutes, the first-best solution will always be implemented if the principal employs one agent for each task.
principals’ temptation to reneg on his relational contract decreases. The higher-powered implicit incentives always outweigh the loss from not having first-best effort in one task.

To illustrate this point, suppose the principal cares about the quantity and quality of a good that an agent produces and, additionally, the machine that is used for production has to be maintained. Furthermore, the only available performance measure is the quantity produced. In their seminal paper on job design, Holmström and Milgrom (1991) argue that, to counteract the misallocation of effort, the maintenance of the machine should be assigned to another agent. In my model, this is indeed optimal when only explicit contracts exist, as it is the case in Holmström and Milgrom’s static setting. However, since the performance measure disregards quality and is therefore highly distortionary, a relational contract is likely to be feasible for the production tasks in a multi-period relationship. Then, to further strengthen this contract, the production worker should also maintain the machine.6

The job design literature already provides alternative reasons for the optimality of broad task assignments. In Itoh (1994, 2001), there is also one joint performance measure for all tasks. However, in contrast to my model, agents are risk-averse and the game is one-shot. Assigning all tasks to one agent is optimal when the degree of substitutability between tasks is sufficiently low because then the effect of paying only one risk premium dominates.7 Also focussing on the aspect of performance measurement, Zhang (2003) and Hughes et al. (2005) demonstrate that, in static settings, complementarities between tasks may lead to task bundling. In my model, complementarities between the two- and one-task job also favor broad task assignments. However, complementarities within the two-task job counteract the

6In Holmström and Milgrom (1991), agents are risk-averse and there may be several performance measures. Meyer, Olsen, and Torsvik (1996) and Olsen and Torsvik (2000) extend the model by Holmström and Milgrom to a dynamic setting with limited intertemporal commitment of the principal. Focussing on the “ratchet effect”, they also show that rules for optimal job design in a static setting may no longer hold in a multi-period framework.

7Moreover, Itoh (1994, 2001) also investigates under which circumstances the principal prefers to perform a task by herself.
misallocation of effort, thereby improving explicit contracts, so that the principal may prefer task splitting.

Over the past decades, due to the introduction of new technologies and the associated higher demand for multi-skilled employees, there have been fundamental organizational changes particularly towards increased multi-tasking and employee discretion.\(^8\) Lindbeck and Snower (2001) and Dessein and Santos (2003) give possible theoretical explanations for this process, based on multitask learning and the demand for coordinating tasks, respectively. Within the empirical literature, Caroli and van Reenen (2001) show that organizational changes implying the delegation of responsibility are likely to coincide with broader task assignments. Zoghi, Levenson, and Gibbs (2005) find that multi-tasking, employee discretion, skills, and task interdependence are strongly positively correlated. Finally, the study by Dorenbosch, van Engen, and Verhagen (2005) indicates that broad task assignments support employee innovation and creativity. My analysis suggests that the observed organizational shift to more complex job assignments should be accompanied by strengthened relational contracts.

In a paper complementary to mine, Gürtler (2005) investigates optimal delegation under relational contracts. In his model, there are only two tasks, one of which the principal can perform herself. Since agents are risk-neutral and the tasks are substitutes, it is always optimal to split tasks if both of them are delegated. Therefore, the central question is whether the principal should delegate one or both tasks.

The next section introduces the model. In section 3, I examine the contracting problem with one agent. Section 4 analyzes when tasks should be split between agents. Section 5 generalizes the results to an arbitrary number of tasks and agents and discusses the model assumptions. The last section concludes.

\(^8\) See Lindbeck and Snower (2000) for an overview on the empirical evidence.
2 The model

I consider a relationship between a risk-neutral principal and one or two agents. The agents are also risk-neutral and have unlimited liability. The principal is the owner of the firm in which the agents can be employed. In each period, the probability that the principal-agent(s) relationship will be repeated in the following period is exogenously given by $\rho \in (0,1)$.

There are three tasks that jointly affect firm value $Y$. $Y$ is either high or low, $Y \in \{0,1\}$, and is realized at the end of each period. I define $N := \{1, 2, 3\}$ as the set of tasks and $e_i \geq 0$ as the non-observable effort exerted in task $i \in N$. Furthermore, $e$ denotes the vector of all efforts, $e = (e_1, e_2, e_3)^T$.\(^9\) Given $e$, the probability of high firm value is

$$\text{prob}[Y = 1|e] = \min\{f^Te, 1\} = \min\{f_1e_1 + f_2e_2 + f_3e_3, 1\},$$

(1)

where $f \in \mathbb{R}^3$ and $f_i > 0$ for all $i \in N$, i.e., all tasks are productive. The realization of $Y$ is observed by the principal and all employed agents but is non-verifiable. However, there is a verifiable performance measure $P \in \{0,1\}$ that is also realized at the end of each period, where

$$\text{prob}[P = 1|e] = \min\{g^Te, 1\},$$

(2)

$g \in \mathbb{R}^3$, $g_i > 0$ for all $i \in N$.\(^10\) Given $f$, $g$, and $e$, the realizations of $Y$ and $P$ are independent.\(^11\)

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\(^9\) All vectors are column vectors. Superscript $T$ denotes transpose.

\(^10\) Similar multi-tasking approaches are widely used in the literature, see, e.g., Feltham and Xie (1994), Datar et al. (2001), and Baker (2002).

\(^11\) Alternatively, we could interpret $Y$ as the value of a division or department in which the agents are employed. Then, the realization of $Y$ would not only depend on the effort in the three tasks under consideration. For example, none of the following results would change if we had $\text{prob}[Y = 1|e] = \min\{y + f^Te, 1\}$, where $y$ is determined by the contribution of other employees and is independent of $e$. Similarly, the realization of the performance measure $P$ could also depend on the performance of other employees.
The principal cannot perform any task from \( N \) herself. She can hire either one or two homogeneous agents to perform the tasks. If she employs two agents, agent 1 performs task 1 and agent 2 performs tasks 2 and 3, which are non-separable. It is not possible to assign the same task to both agents. Furthermore, a task assignment has to be maintained in all future periods. This also implies that the initially chosen number of agents is invariant over time.\(^{12}\)

An agent’s non-observable cost of exerting effort \( e \) is

\[
c(e) = \frac{1}{2} e^T C e = \sum_{i=1}^{3} \frac{c_{ii}}{2} e_i^2 + c_{12} e_1 e_2 + c_{13} e_1 e_3 + c_{23} e_2 e_3, \tag{3}
\]

where \( C = [c_{ij}] \) is a real, symmetric,\(^{13}\) and positive definite \((3 \times 3)\) matrix, and \( c_{ii} > 0 \) for all \( i \). If \( c_{ij} = 0, i \neq j \), tasks \( i \) and \( j \) are independent since the choice of \( e_i \) does not affect the marginal costs of \( e_j \) when tasks \( i \) and \( j \) are assigned to the same agent. If \( c_{ij} > 0 \), the marginal costs of task \( i \) increase in \( e_j \) and vice versa, so that the tasks are substitutes.\(^{14}\) If \( c_{ij} < 0 \), tasks \( i \) and \( j \) are complements.

Agents’ opportunity costs of working for the principal are zero in each period so that there are no a priori costs of employing two rather than one agent. For simplicity, I assume that \( f, g, \) and \( C \) are such that the probabilities \( f^T e \) and \( g^T e \) are always smaller than one at the optimal (first- and second-best) solution.\(^{15}\)

Timing is as follows in each period: At the beginning of the period, the principal individually offers each agent an explicit wage contract specifying some guaranteed fixed component and an explicit bonus that will be paid at the end of the period if \( P = 1 \). Additionally, the principal may offer an implicit bonus that she promises to

\(^{12}\)This can be justified if, e.g., agents have to learn how to perform a task before production can take place. Then, changing the number of agents in future periods would lead to additional learning costs for at least one task. Such costs are often at least partially borne by the firm and might be prohibitively high. I discuss the impact of this assumption in section 5.

\(^{13}\)Assuming symmetry of \( C \) is without loss of generality since each quadratic function \( c(e) \) can be represented by a symmetric matrix.

\(^{14}\)Since \( C \) is positive definite, the case of perfect substitutes, \( c(e) = 0.5(e_1 + e_2 + e_3)^2 \), which generally produces corner solutions, is excluded.

\(^{15}\)It can be shown that this is the case if \( \max\{f^T C^{-1} f, g^T C^{-1} g\} < 1 \).
pay at the end of the period if \( Y = 1 \). However, since \( Y \) is non-verifiable, an agent will rely on such a promise only if he believes that it is in the principal’s interest not to renege on it. Given the explicit and the relational contract, each agent chooses his effort level(s). Afterwards, \( Y \) and \( P \) are realized and each agent is rewarded according to his explicit contract. If \( Y = 1 \), the principal decides whether to pay the implicit bonuses to one or both agents.

The following lemma is helpful for the formal analysis of the model. It determines a square root \( S \) of the inverse of the cost matrix \( C \), denoted by \( C^{-1} \).

**Lemma 1** Let \( Q \) be the matrix whose \( i \)-th column is the orthonormal eigenvector of \( C \) associated with the eigenvalue \( \lambda_i \), \( i = 1, 2, 3 \). Define \( S := \tilde{\Lambda}Q^T \), where \( \tilde{\Lambda} := \text{diag}(\lambda_1^{-1/2}, \lambda_2^{-1/2}, \lambda_3^{-1/2}) \). Then, \( S^T S = C^{-1} \).

All proofs are in the appendix.

For example, in the simple case of independent tasks, \( C = \text{diag}(c_{11}, c_{22}, c_{33}) \), we have \( S = \text{diag}(c_{11}^{-1/2}, c_{22}^{-1/2}, c_{33}^{-1/2}) \).

In the next section, I analyze the case where the principal employs only one agent who performs all three tasks. Unless otherwise stated, the results of section 3 easily extend to the case of one agent performing \( n \geq 2 \) tasks by assuming \( f, g \in \mathbb{R}_+^n \) and defining \( C \) as a symmetric and positive definite \((n \times n)\) matrix.

### 3 Optimal contracting with one agent

#### 3.1 Pure explicit contracts

Let \( e^{FB} \) denote the vector of first-best efforts given that one agent performs all tasks, i.e.,

\[
e^{FB} = \arg\max_e f^Te - \frac{1}{2}e^TCe.
\] (4)

Assuming that first-best effort is strictly positive in each task (i.e., cost substitutabilities are not too high), \( e^{FB} = C^{-1} f \) leading to an expected profit of \( \pi^{FB} = \frac{1}{2}f^TC^{-1}f. \)
Suppose the agent does not trust the principal to pay any bonus based on the realization of $Y$, so that only a pure explicit contract is feasible. Let $\beta$ denote the explicit bonus and $||\cdot||$ the length of a vector, i.e.,

$$||f|| := \sqrt{f_1^2 + f_2^2 + f_3^2}. \quad (5)$$

Furthermore, I assume that the agent exerts strictly positive effort in each task under a pure explicit contract, i.e., $C^{-1}g > 0$ (compare the proof of proposition 1).

**Proposition 1** Suppose that $C^{-1}f, C^{-1}g > 0$. In the optimal pure explicit contract with one agent, the bonus $\tilde{\beta}$ and the principal’s resulting profit $\tilde{\pi}$ are

$$\tilde{\beta} = \frac{||f_c||}{||g_c||} \cos \theta_c > 0, \quad \tilde{\pi} = \frac{||f_c||^2}{2} \cos^2 \theta_c, \quad (6)$$

where $f_c := Sf$, $g_c := Sg$, and $\theta_c$ denotes the angle between $f_c$ and $g_c$. The first-best solution $e^{FB}$ is implemented if and only if there is a $\mu \in \mathbb{R}$ such that $f = \mu g$.

Proposition 1 is a special case of Theorem 1 and 3 in Schnedler (2004), who examines the optimality of congruent performance measures with a risk-averse agent. However, in contrast to Schnedler, I use the concept by Baker (2002) and Gibbons (2005) to measure the quality of the performance measure. Baker and Gibbons analyze the case of independent and equally costly tasks in which

$$\tilde{\beta} = \frac{||f||}{||g||} \cos \theta, \quad \tilde{\pi} = \frac{||f||^2}{2c_{11}} \cos^2 \theta, \quad (7)$$

where $\theta$ denotes the angle between $f$ and $g$. Thus, there are two features that determine the optimal explicit bonus $\tilde{\beta}$: scaling, as given by $||f||/||g||$, and alignment,

\footnote{The circumstances under which this happens are discussed in section 3.2.}

\footnote{All three authors model the congruence problem slightly differently. They define firm value as $f^T e$ plus a noise term, and the performance measure is $g^T e$ plus another noise term. In Baker (2002), the agent is also risk-averse.}

\footnote{That is, $C = c_{11} I$, where $I$ is the identity matrix. Then, $f_c = c_{11}^{-1/2} f$ and $g_c = c_{11}^{-1/2} g$.}
as given by $\cos \theta$. The higher $\cos \theta$ the better aligned are $f$ and $g$ and, therefore, the more useful is the performance measure for efficiently directing effort to the different tasks. As a result, the optimal explicit bonus and expected profit increase in $\cos \theta$.

Proposition 1 shows that this intuition carries over to the case of differently costly and possibly interdependent tasks if we characterize the quality of the performance measure by the angle between $f_c$ and $g_c$. That is, the marginal effects of effort on firm value and the performance measure $f$ and $g$, respectively, must be appropriately weighted by the agent’s costs. By considering $g_c$ instead of $g$, we account for the fact that the agent’s effort decision is not only determined by the tasks’ relative importance for the performance measure but also by their relative costs and possible interdependencies. Similarly, the efficient allocation of effort is characterized by $f_c$ rather than $f$ since the former vector also takes into account the agent’s costs.

To better capture the intuition, consider the following example of independent but differently costly tasks, where

$$
f^T = \frac{1}{2}(1, 1, 1), \quad g^T = \frac{1}{2}(1, 1, 2), \quad C = \text{diag}(1, 1, \sigma), \quad \sigma > 1,
$$

so that\(^{19}\)

$$
f_c^T = \frac{1}{2}(1, 1, \sigma^{-1/2}), \quad g_c^T = \frac{1}{2}(1, 1, 2\sigma^{-1/2}).
$$

Here, the performance measure puts too much weight on task 3. However, $f_c$ and $g_c$ indicate that this problem is the less severe the more costly is task 3, since then cost considerations make the agent direct relatively more effort towards tasks 1 and 2, while the efficient effort in task 3 also decreases. Indeed, it is easily verified that $\cos \theta_c$ increases in $\sigma$ and equals one when $\sigma$ approaches infinity. By proposition 1,

$$
\tilde{\beta} = \frac{1 + \sigma}{2 + \sigma} \quad \text{and} \quad \tilde{\pi} = \frac{(1 + \sigma)^2}{4\sigma(2 + \sigma)},
$$

\(^{19}\)The assumption $\sigma > 1$ ensures that all probabilities are smaller than 1.
which means that the optimal explicit contract employs higher-powered incentives when task 3 becomes more expensive. However, the principal’s profit decreases in $\sigma$ because the negative cost effect dominates the positive effect from having a better aligned performance measure.

Although the relationship between $f$ and $g$ is in general no longer relevant to characterize the quality of the performance measure when tasks are interdependent, first-best effort is still implemented if and only if $f$ and $g$ are perfectly aligned.\(^{20}\) I henceforth assume that the principal cannot implement first-best effort by a pure explicit contract, i.e., $\cos \theta < 1$. Furthermore, I will say that a performance measure $g'$ is superior to $g$ if $\cos \theta'_c > \cos \theta_c$, where $\theta'_c$ denotes the angle between $f_c$ and $g'_c$. The next section analyzes the optimal combination of explicit and relational contracts.

### 3.2 Combining explicit and relational contracts

This section extends part of the analysis in Baker et al. (1994) to the case of multiple and possibly interdependent tasks. Assume the principal offers the agent an explicit contract as described in the foregoing section. Additionally, suppose that she can credibly promise to pay an implicit bonus $\gamma$ if $Y = 1$. Then the agent chooses $e(\beta, \gamma)$ to solve the problem

$$\max_e \alpha + \beta g^T e + \gamma f^T e - \frac{1}{2} e^T C e,$$

i.e.,

$$e(\beta, \gamma) = C^{-1}(\beta g + \gamma f).$$

When determining the optimal combination of the explicit and implicit bonus, the principal must take into account that her promise to pay $\gamma$ if firm value is high

\(^{20}\)This generalizes a result from Datar, Kulp, and Lambert (2001) who show that the principal prefers a perfectly aligned performance measure when tasks are independent and the agent is risk-neutral (or the performance measure is noiseless). However, Schnieder (2004) shows that this result does no longer hold if the agent is risk-averse.
must be trustworthy to the agent. To model the role of trust, I assume that if the principal once reneges on the relational contract, the agent will never trust her again to pay an implicit bonus. Thus, if the principal breaks the relational contract, her fallback position is a pure explicit contract leading to profit $\tilde{\pi}$ in all future periods.

For any $\beta$ and $\gamma$, the fixed component of the explicit contract will be such that the agent receives just his reservation utility. Therefore, the principal chooses $\beta$ and $\gamma$ to solve the problem

$$\max_{\beta, \gamma} f^T e(\beta, \gamma) - \frac{1}{2} e(\beta, \gamma)^T C e(\beta, \gamma)$$

s.t. $\gamma \leq \sum_{t=1}^{\infty} \rho^t \left[ f^T e(\beta, \gamma) - \frac{1}{2} e(\beta, \gamma)^T C e(\beta, \gamma) - \tilde{\pi} \right]$.  

Inequality (14) is the principal’s commitment constraint. It says that her short-term profit from reneging on the relational contract, $\gamma$, must not exceed the associated expected long-term loss, which is given by the term on the right-hand side. Otherwise, the agent would anticipate that the principal will not stick to the informal agreement. Solving the principal’s problem yields the following proposition.

**Proposition 2** Under the optimal combination of explicit and relational contracts with one agent, the implicit bonus is

$$\gamma^* = \begin{cases} 
1 & \text{if } 2\phi \leq \eta \\
2 \left( 1 - \frac{2}{\eta} \right) & \text{if } \phi < \eta < 2\phi \\
0 & \text{if } \eta \leq \phi 
\end{cases}$$

(15)

and the explicit bonus is $\beta^* = (1 - \gamma^*)\tilde{\beta}$. The principal receives the expected profit

$$\pi = \begin{cases} 
\pi^{FB} & \text{if } 2\phi \leq \eta \\
\tilde{\pi} + 2\phi \left( 1 - \frac{2}{\eta} \right) & \text{if } \phi < \eta < 2\phi \\
\tilde{\pi} & \text{if } \eta \leq \phi 
\end{cases}$$

(16)
where
\[ \phi := (1 - \rho)/\rho, \quad \eta := 2(\pi^{FB} - \tilde{\pi}) = ||f_c||^2(1 - \cos^2 \theta_c). \] (17)

The higher the implicit bonus the principal can credibly commit to, \( \gamma^* \), the better aligned are the agent’s incentives and thus the higher is the principal’s expected profit. If \( \gamma^* = 1 \), the agent is the residual claimant and therefore chooses the first-best effort allocation. Since the explicit bonus \( \beta^* \) decreases in \( \gamma^* \), explicit and relational contracts are substitutes as in Baker et al. (1994) when the principal’s fallback position is positive.\(^{21}\)

The principal can commit to an implicit bonus if her expected loss from reneging on the relational contract is large. This is the case if the probability that the principal-agent relationship will continue is high (i.e., \( \phi \) is low), and if the profit under a pure explicit contract is significantly lower than the first-best profit, i.e., \( \eta \) is large. Therefore, relational contracts exist in environments where pure explicit contracts perform poorly due to bad aligned performance measures.\(^{22}\)

\(^{21}\)In Baker et al., the agent’s opportunity costs may be positive so that the principal’s profit under a pure explicit contract may be negative. Then the principal’s fallback position is to shut down earning zero profit, a case that is not examined in this paper.

\(^{22}\)Given that the principal’s fallback position is positive, Baker et al. (1994) derive the same result.
Depicting $\gamma^*$ and $\pi$ for fixed $||f_c||$ and varying $\cos^2 \theta_c$, figure 1 shows how the optimal implicit bonus and the principal’s profit vary with the quality of the performance measure while holding the first-best profit constant.\textsuperscript{23} The principal benefits from a superior performance measure if and only if relational contracts are infeasible. Otherwise, the benefit from having a higher powered explicit contract is always offset by the loss from having weaker implicit incentives.

On the other hand, holding $\cos \theta_c$ constant while increasing $||f_c||$ fixes the quality of the performance measure but increases each tasks’ expected marginal productivity (after accounting for the agent’s cost) by the same factor.\textsuperscript{24} Since $\pi^{FB}$ increases more strongly in $||f_c||$ than $\tilde{\pi}$, the overall effect on $\eta$ is positive and, therefore, both $\gamma^*$ and $\pi$ increase in $||f_c||$. I summarize the comparative statics in the following corollary.

**Corollary 1** \(i\) For fixed $||f_c||$, the optimal implicit bonus decreases in $\cos \theta_c$. The principal’s expected profit strictly increases in $\cos \theta_c$ if and only if she cannot commit to an implicit bonus, i.e., if $\cos^2 \theta_c \geq 1 - \phi / ||f_c||^2$. \(ii\) For fixed $\cos \theta_c$, the optimal implicit bonus and expected profit increase in $||f_c||$.

Since the quality of the performance measure depends on the agent’s costs, it is interesting to ask how, given $f$ and $g$, the properties of the effort cost function affect the existence of relational contracts. Formally, we have to analyze how $\eta$ varies with $C$. It turns out that the results are unambiguous only if the agent performs two tasks.

**Proposition 3** If the agent performs only two tasks, $\eta$ decreases in $c_{11}$ and $c_{22}$ but increases in $c_{12}$. Thus, relational contracts are more likely to exist if tasks are substitutes rather than complements.

As $c_{12}$ increases, every arbitrary effort allocation becomes more costly so that first-best profit $\pi^{FB}$ or, equivalently, $||f_c||$ falls. However, in the case of two tasks,\textsuperscript{23} Note that $\pi^{FB} = \frac{\gamma^2}{2} C^{-1} = \frac{||f_c||^2}{2}$.\textsuperscript{24} Formally, $||f_c||$ increases while $\cos \theta_c$ remains constant if, e.g., $f$ is multiplied by $\lambda > 1$ or $C$ is multiplied by $\mu < 1$.
cos θc also decreases in \( c_{12} \). To understand the intuition, first suppose the tasks are independent and the performance measure places too much weight on task 2 so that, under a pure explicit contract, the agent allocates too much effort to this task. For example, consider a professor whose tasks are teaching and doing research, where the latter is relatively more easily to measure. If \( c_{12} \) increases so that teaching and research become substitutes, the professor will devote even less time to teaching. Therefore, optimal explicit incentives and the performance of a pure explicit contract decrease. By contrast, complementarities among teaching and research counteract the misallocation of effort.\(^{25}\) Although both \( ||f_c|| \) and \( \cos \theta_c \) decrease, the second effect dominates so that \( \eta \) increases.

Proposition 3 also shows that the existence of relational contracts is less likely if \( c_{ii}, i = 1, 2, \) is high. The higher these ”direct” costs of task \( i \), the lower is the loss from using a distortionary performance since the agent’s first- and second-best effort levels are relatively small.

If the agent performs more than two tasks, their interaction becomes more complex so that the results of proposition 3 do not extend.\(^{26}\) For example, consider the case

\[
f^T = \frac{1}{5}(1, 1, 1), \quad g^T = \frac{1}{5}(1, 1, 2), \quad C = \begin{pmatrix}
1 & c_{12} & 0 \\
c_{12} & 1 & 0 \\
0 & 0 & 1
\end{pmatrix}, \quad 0 \leq c_{12} < 1. \quad (18)
\]

Here, if \( c_{12} = 0 \), first-best effort is identical in all tasks while, under a pure explicit contract, the agent neglects tasks 1 and 2. However, as \( c_{12} \) increases, it becomes efficient to allocate relatively more effort to task 3, so that the agent’s effort allocation under an explicit contract becomes closer to first-best, i.e., \( \eta \) decreases in \( c_{12} \). To illustrate, suppose a professor is supposed to teach, do administrative work (tasks

\(^{25}\)This trade-off was already explained by Holmström and Milgrom (1991).

\(^{26}\)However, one can show that in the case of three independent tasks \( \eta \) still decreases in \( c_{ii} \) for all \( i \).
1 and 2) and research (task 3). Neglecting tasks 1 and 2 is, at least to some extent, desirable if these tasks are substitutes.

4 When should tasks be split?

In the previous section, I focussed on the case of one agent. In this section, I analyze under which circumstances the principal benefits from splitting tasks between two agents. Obviously, there is no use of task splitting if the principal can set first-best incentives when employing one agent, i.e., if $\gamma^* = 1$. Therefore, I henceforth consider the case $\gamma^* < 1$ which is, by proposition 2, equivalent to $\eta < 2\phi$.

If there is task splitting, agent 1 performs task 1 and agent 2 performs the non-separable tasks 2 and 3. As in the introductory example, the non-separable tasks could be quantity and quality in the production of a good. Alternative examples are monitoring subordinates and generating reports; selling and advisory service; or doing research and publishing papers.\(^{27}\)

I assume that, if the principal breaks a relational contract with one agent, both agents will not rely on relational contracts in all future periods. This implies that an agent can observe whether or not the principal kept an implicit agreement with his colleague.\(^{28}\) Furthermore, agent 1 observes the explicit and relational contract offered to agent 2 and vice versa.

To characterize the optimization problem of agent 2, I define

$$\hat{e} := (e_2, e_3)^T, \quad \hat{f} := (f_2, f_3)^T, \quad \hat{g} := (g_2, g_3)^T, \quad \hat{C} := \begin{pmatrix} c_{22} & c_{23} \\ c_{23} & c_{33} \end{pmatrix}. \quad (19)$$

\(^{27}\)The non-separable tasks could also be interpreted as two dimensions of only one task, e.g., producing, supervising etc.

\(^{28}\)I make this assumption because it is prevalent in the literature (see, e.g., Bull (1987)). However, in the case of three tasks and two agents, it can be shown that the results do not change when only the agent who was cheated does no longer rely on relational contracts. I will discuss the impact of this assumption in more general settings in section 5.
Furthermore, let $\beta_1, \gamma_1$ and $\beta_2, \gamma_2$ denote the explicit and implicit bonus for agent 1 and agent 2, respectively. Analogously, $\alpha_1$ and $\alpha_2$ denote the fixed components. Suppose that the implicit bonuses $\gamma_1$ and $\gamma_2$ are credible. Then, given the effort $\hat{e}$ of agent 2, agent 1 chooses $e_1$ to solve the problem
\[
\max_{e_1} \alpha_1 + \beta_1 g^T e + \gamma_1 f^T e - \frac{c_{11}}{2} e_1^2.
\] (20)

Analogously, given effort $e_1$ of agent 1, agent 2 chooses $\hat{e}$ to solve
\[
\max_{\hat{e}} \alpha_2 + \beta_2 g^T \hat{e} + \gamma_2 f^T \hat{e} - \frac{1}{2} \hat{e}^T \hat{C} \hat{e}.
\] (21)

It follows that\(^{29}\)
\[
e_1(\beta_1, \gamma_1) = e_{11}^{-1}(\beta_1 g_1 + \gamma_1 f_1), \quad \hat{e}(\beta_2, \gamma_2) = \hat{C}^{-1}(\beta_2 \hat{g} + \gamma_2 \hat{f}),\] (22)
i.e., an agent’s effort choice does not depend on the effort choice of his colleague. However, the joint effort of both agents determines the probabilities of high firm value and a favorable performance measure and, therefore, also the expected payment to each agent. Thus, when deciding whether to accept the contract offered by the principal, each agent must anticipate the effort choice of his colleague. Moreover, each agent can trust the principal to pay his individual bonus only if the principal finds it beneficial to pay both implicit bonuses simultaneously. Thus, if an agent does not know the implicit bonus offered to his colleague, he cannot judge the credibility of the promise that the principal made to him. Therefore, the assumption that each agent observes the contract offered to his colleague simplifies the analysis.\(^{30}\)

Let $\gamma_1^*$ and $\gamma_2^*$ denote the optimal implicit bonus for agent 1 and agent 2, respec-

\(^{29}\)It can be shown that $\hat{C}$ is positive definite. I also assume that $\hat{C}^{-1} \hat{f}, \hat{C}^{-1} \hat{g} > 0$. If $c_{12} = c_{13} = 0$, this already follows from $C^{-1} f, C^{-1} g > 0$.

\(^{30}\)In the case of three tasks this assumption can be dropped. As we will see, agent 1 exerts first-best effort in his task under a pure explicit contract. Thus, by knowing $f$ and $g$ and the tasks assigned to himself, each agent can anticipate which contract will be offered to his colleague.
tively. As in the previous section, $\gamma^*$ denotes the optimal implicit bonus with one agent performing all tasks alone. Furthermore, define $\hat{f}_c := \hat{S}\hat{f}$, $\hat{g}_c := \hat{S}\hat{g}$, where $\hat{S}$ is derived according to Lemma 1 so that $\hat{S}^T\hat{S} = \hat{C}^{-1}$, and $\hat{\theta}_c$ denotes the angle between $\hat{f}_c$ and $\hat{g}_c$.

Given that tasks are split, first-best effort is $f_{11}^2$ in task 1 and $\hat{C}^{-1}\hat{f}$ in tasks 2 and 3, and the resulting profit is $\pi^B_S = \frac{f_{11}^2}{2} + \|\hat{f}_c\|^2/2$. Let $\hat{\pi}_S$ denote the principal’s profit with pure explicit contracts under task splitting.

**Proposition 4** Assume that tasks are split. Then $\gamma^*_1 = 0$ and the principal’s expected profit is

$$\pi_S = \begin{cases} 
\frac{f_{11}^2}{2} + \frac{||\hat{f}_c||^2}{2} & \text{if } 2\phi \leq \hat{\eta} \\
\frac{f_{11}^2}{2} + \frac{||\hat{f}_c||^2}{2} \cos^2 \hat{\theta}_c + 2\phi \left(1 - \frac{\phi}{\hat{\eta}}\right) & \text{if } \phi < \hat{\eta} < 2\phi \\
\frac{f_{11}^2}{2} + \frac{||\hat{f}_c||^2}{2} \cos^2 \hat{\theta}_c & \text{if } \hat{\eta} \leq \phi
\end{cases},$$

where $\hat{\eta} := 2(\pi^B_S - \hat{\pi}_S) = ||\hat{f}_c||^2(1 - \cos^2 \hat{\theta}_c)$.

Under task splitting, agent 1 cannot misallocate effort across tasks. Thus, he chooses the efficient effort in task 1 under a pure explicit contract, and his contribution to the principal’s expected profit is always $\frac{f_{11}^2}{2}$. Due to the assumption of additively separable “production functions” $f^T e$ and $g^T e$, the optimal contracting problem with agent 2 is equivalent to the one considered in section 3.2 so that his contribution to $\pi_S$ is given by (16), after taking into account that now the agent performs only tasks 2 and 3. Agent 2 exerts the efficient effort if either the congruency problem disappears under task splitting because the distortion was only caused by task 1 (i.e., $\cos \hat{\theta}_c = 1$), or if an implicit bonus $\gamma^*_2 = 1$ is feasible (i.e., $\hat{\eta} \geq 2\phi$).

The next proposition is the main result of this paper. It characterizes optimal job design when task 1 is independent of the other tasks.

**Proposition 5** Suppose $\gamma^* < 1$ and task 1 is independent of tasks 2 and 3, i.e., $c_{12} = c_{13} = 0$. Then, $\hat{\eta} \leq \eta$ and $\gamma^*_2 \leq \gamma^*$. Furthermore,
(i) if $\hat{\eta} \leq \eta \leq \phi$, i.e., $\gamma^*_2, \gamma^* = 0$, the principal prefers task splitting,

(ii) if $\phi < \hat{\eta} \leq \eta$, i.e., $\gamma^*_2, \gamma^* > 0$, the principal prefers to assign all tasks to one agent,

(iii) if $\hat{\eta} \leq \phi < \eta$, i.e., $\gamma^*_2 = 0$ and $\gamma^* > 0$, the principal prefers task splitting if and only if $\hat{\eta} \leq \eta - 4\phi(1 - \phi\eta^{-1})$.

To understand the intuition for these results, first assume relational contracts are not feasible ($\gamma^*_2, \gamma^* = 0$). Assigning all tasks to one agent does not affect the effort allocation in tasks 2 and 3, since performing task 1 does not influence the costs of the other tasks. However, it introduces another tension between the non-separable tasks and task 1, and thus requires further compromising on the strength of explicit incentives. In the introductory example, assigning the maintenance of the machine to the production worker will probably require to further decrease piece rates, which were already relatively low under task splitting due to quality considerations. Therefore, the principal always benefits from assigning task 1 to another agent, i.e., $\tilde{\pi} \leq \tilde{\pi}_S$.31

Furthermore, when task 1 is independent, the first-best effort allocation is the same whether tasks are split or not, i.e., $\pi^{FB} = \pi^{FB}_S$. Thus, $\tilde{\pi} \leq \tilde{\pi}_S$ immediately implies $\hat{\eta} \leq \eta$ which is equivalent to $\gamma^*_2 \leq \gamma^*$. Reneging on an implicit agreement with an agent who is responsible for a broad range of tasks leads to larger losses for the principal. Consequently, she can strengthen an agent’s relational contract if she assigns another task to him. Moreover, the higher-powered relational contract always outweighs the loss from not having first-best incentives in the separable task. Therefore, the principal always benefits from assigning all tasks to one agent if a relational contract is already feasible for the two-task job (i.e., $\gamma^*_2 > 0$). This is the case if performance in the two-task job is difficult to measure (i.e., $\cos\hat{\theta}_c$ is small) and/or this job makes an important contribution to firm value (i.e., $||\hat{f}_c||$ is large).

31 The inequality is binding if $\frac{f_1}{g_1} = \frac{\tilde{T}^c - \hat{C}}{g^c - \hat{g}}$ (compare the proof of proposition 5).
In the example of the production worker, a relational contract is likely to be feasible under task splitting if poor quality is not easily detected or traced back to the responsible agent. In this case, the worker should be made responsible for an additional task so that an appropriate allocation of effort across tasks becomes even more important for the firm.

Finally, if $\gamma^* > 0$ but $\gamma^*_2 = 0$, the principal prefers to split tasks if both $\eta$ and $\hat{\eta}$ are relatively small. Then, if tasks are not split, the relational contract is rather weak, while the pure explicit contracts under task splitting perform relatively well. Figure 2 illustrates these findings. It also shows that, for all possible combinations of $\eta$ and $\hat{\eta}$, tasks are more often assigned to one agent than they are split.

Regarding the nature of the non-separable tasks with respect to the agent’s costs, we obtain the following result.

**Proposition 6** Suppose $c_{12} = c_{13} = 0$. Since $\hat{\eta}$ increases in $c_{23}$, tasks are more likely to be assigned to one agent if the non-separable tasks are substitutes.

As we already know from proposition 3, substitutabilities within the two-task job make the existence of a relational contract for this job more likely since they decrease the quality of any given imperfect performance measure. By the above
argumentation, the principal then benefits from assigning an additional task to the agent performing the two-task job since thereby his implicit incentives can be improved.

If task 1 is not independent of the other tasks, the first-best effort allocation is no longer identical under both scenarios, which adds an additional trade-off to the problem of optimal job design. If task 1 is complementary to the other tasks, assigning all tasks to one agent has the further advantage of decreasing overall effort costs, so that one would expect it to be more often optimal. Indeed, it is easy to construct examples where, although only explicit contracts are feasible under both job design schemes, it is optimal not to split tasks if $c_{12}$ and $c_{13}$ are negative and sufficiently small. Similarly, if $c_{12}$ and $c_{13}$ are positive, the principal may prefer task splitting even if it weakens relational contracts. Thus, to strengthen implicit contracts without causing cost disadvantages, it may be better to encourage a worker to suggest improvements in the production process rather than to assign the maintenance task to him.

To summarize, the principal benefits from assigning additional responsibilities to agents who already perform tasks that make an important but difficult to measure contribution to the firm. In general, this will be more likely to be the case for management than for production tasks. For instance, Lazear (2000) shows that piece rates combined with an effective form of quality control can provide workers with incentives to produce high output without neglecting quality. If production workers can be closely monitored, even the number of hours worked may serve as a good proxy for performance. Usually, the output of supervisors and managers is less concrete and therefore more difficult to measure. By contrast, financial measures like divisional profit or stock price may serve well for higher-level executives. Thus,

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32I could not construct an example where $\hat{\eta} > \eta$, i.e., where assigning an additional (possibly complementary) task to an agent brings pure explicit contracts closer to the first-best solution (conditioned on job design). Therefore, I conjecture $\gamma_2 \leq \gamma$ still holds when all tasks are interdependent, but have not been able to prove it.

33Murphy and Oyer (2003) show that bonuses are less discretionary for CEOs than for lower-level
my analysis suggests that jobs tend to consist of more tasks on middle hierarchy levels.

Furthermore, suppose firm value \( Y \) depends on production as well as management tasks as described in footnote 11. Then, management tasks are also more likely to be assigned to one agent than production tasks, because management tasks generally affect firm value more strongly.

Finally, since implicit bonuses also increase in the probability that the principal-agent relationship continues, employees that are more likely to stay with the firm should also perform more tasks.

5 Discussion

In this section, I discuss the generalization of the analysis to more tasks and agents as well as some of the model assumptions.

If the number of tasks increases, the analysis in section 4 becomes more complicated. If there are, for instance, four tasks and two agents, it may be optimal to pay both agents an implicit bonus. Under the assumption that both agents lose trust if the principal reneges on a relational contract with one of them, the principal’s only commitment constraint is

\[
\phi(\gamma_1 + \gamma_2) \leq f^T e - \sum_{l=1,2} \left( \alpha_l + \beta_l g^T e + \gamma_l f^T e \right) - \tilde{\pi}_S. \tag{23}
\]

Naturally, the optimal implicit bonuses cannot be determined independently of each other. This means, in particular, that the derivation of the optimal contract for one agent cannot be boiled down to the problem analyzed in section 3.2 by just dropping the tasks performed by the other agent. This feature greatly simplified the analysis in the case of three tasks. However, it can be reestablished by changing the modelling

\footnote{Compare constraint (45) in the appendix. Of course, all vectors are now four-dimensional.}
of trust.

Assume that if the principal reneges on one relational contract only the agent who was cheated loses trust. This assumption leads to additional commitment constraints for the principal and, therefore, limits the set of implementable implicit bonuses relative to (23). Then, assuming that each set of non-separable tasks is independent of all other tasks, it can be shown that agents’ optimal contracts are independent of each other, and proposition 5 can be extended to the case of splitting $n$ tasks between $l < n$ agents. In particular, assigning all tasks to one agent will be optimal if at least one agent receives an implicit bonus under each arbitrary task splitting. If, on the other hand, it is not credible to promise an implicit bonus to an agent who performs all tasks, task splitting always increases profits.

While I was not able to derive clear-cut results for the general optimal task assignment under the initial modelling of trust, it is clear how results will change relative to the case just described. If, under task splitting, all agents lose trust when the principal reneges on only one relational contract, the temptation to renege is smaller. As explained above, this results in a larger set of implementable relational contracts. Thus, task splitting will more frequently be preferred to assigning all tasks to one agent.

I made the assumption that the principal cannot change the task assignment in future periods. This affects her fallback position after breaking implicit agreements and, therefore, may be critical for the results derived in proposition 5. If the principal initially employs two agents and agrees on a positive implicit bonus with agent 2, she would not want to dismiss one agent after reneging on the relational contract because task splitting is superior under pure explicit contracts.

However, if there is initially a single agent performing all tasks, the principal would benefit from splitting tasks after breaking the relational contract. Thus, in this case the assumption of inflexible job design matters. It worsens the fallback position of the principal and, therefore, leads to a higher feasible implicit bonus for
the single agent. Hence, ex ante it is in the principal’s interest to commit to not splitting tasks in the future. I assumed that such a commitment is possible because the costs of hiring another agent (e.g., learning costs) are higher than the benefits.\textsuperscript{35} If such a commitment is not possible, assigning all tasks to a single agent will be less often preferred. However, as long as changing the organization of work within the firm is costly, there still are circumstances in which broad task assignments are optimal.

Furthermore, I assumed that agents’ reservation utility is zero. Now suppose that an agent’s alternative wage per period is $\bar{w} > 0$, where $\bar{\pi} - 2\bar{w} > 0$, i.e., the expected profit under pure explicit contracts is still positive. Then it is easily verified that the principal’s expected profit is $\pi - \bar{w}$ if all tasks are performed by one agent, and $\pi_S - 2\bar{w}$ under task splitting.\textsuperscript{36} Thus, task splitting becomes less attractive. At the end of section 4, I argued that jobs should be more complex on higher hierarchy levels. This conclusion is strengthened by positive but not too high opportunity costs since alternative wages will, in general, increase in the hierarchy level.

\section{Conclusion}

This paper provided some insights into the optimal interplay between job design and relational contracts. I showed that the existence of relational contracts favors broader task assignments because thereby implicit agreements can be further strengthened. The principal may benefit from a high-powered relational contract even if this contract prevents first-best effort in one task. Relational contracts exist if contractible performance measures are highly distortionary or firm value strongly responds to effort changes in the given tasks. Thus the principal benefits from as-

\textsuperscript{35}Explicitly, I assume that $K \geq \pi_S - \hat{\pi}$, where $K$ denotes the costs of learning how to perform a task which have to be borne by the firm, e.g., trainee programs or opportunity costs from having senior staff to teach the new colleague.

\textsuperscript{36}The problem becomes more complex if $\bar{\pi} - 2\bar{w} < 0$ since then optimal explicit and implicit bonuses depend on $\bar{w}$. 

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signing additional tasks to agents who already perform tasks that are particularly important for the firm but whose contribution is difficult to measure.

I assumed that there is only one exogenously given contractible performance measure. In many situations, the principal can invest in generating additional performance measures, thereby improving the performance of explicit contracts. However, doing so increases the costs of performance measurement. The analysis in this paper also suggests that appropriate job design can be a substitute to generating performance measures.

Appendix

Proof of lemma 1. Since $C$ is positive definite, $C^{-1}$ exists. Furthermore, because $C$ is also real and symmetric, $\lambda_1, \lambda_2, \lambda_3$ are real and strictly positive and $C = QAQ^T$, where $\Lambda := \text{diag}(\lambda_1, \lambda_2, \lambda_3)$ (see, e.g., Lütkepohl (1996)). Since $Q^T = Q^{-1}$, $C^{-1} = QA^{-1}Q^T$, so that by definition of $S$, $S^T S = C^{-1}$.

Proof of proposition 1. Let $\alpha$ (fixed component) and $\beta$ (bonus) denote the components of the explicit wage contract. The principal’s optimization problem is

$$\max_{\alpha, \beta, e} f^T e - (\alpha + \beta g^T e) , \tag{24}$$

s.t. $e = \arg\max_{\tilde{e}} \alpha + \beta g^T \tilde{e} - \frac{1}{2} \tilde{e}^T C \tilde{e} , \tag{25}$

$$0 \leq \alpha + \beta g^T e - \frac{1}{2} e^T C e . \tag{26}$$

From the incentive compatibility constraint (25), the agent exerts $e(\beta) = \beta C^{-1} g$. For any $\beta$, the principal optimally chooses $\alpha$ such that the agent’s participation constraint (26) is binding. Thus, the optimal bonus $\tilde{\beta}$ maximizes $f^T e(\beta) - \frac{1}{2} (e(\beta))^T C e(\beta)$ and we obtain

$$\tilde{\beta} = \frac{f^T C^{-1} g}{g^T C^{-1} g} \quad \text{and} \quad \tilde{\pi} = \frac{1}{2} \frac{(f^T C^{-1} g)^2}{g^T C^{-1} g} . \tag{27}$$
Applying $C^{-1} = S^T S$ and $f_c^T g_c = \|f_c\| \|g_c\| \cos \theta_c$ yields (6). Furthermore, $\tilde{\pi} = \pi^{FB}$ if and only if $\cos^2 \theta_c = 1$, i.e., if there is a $\mu \in \mathbb{R}$ such that $f_c = \mu g_c \iff Sf = \mu Sg \iff S(f - \mu g) = 0$. Since $\det(S) = \det(\tilde{\Lambda}) \det(Q^T) > 0$, the last equation holds if and only if $f - \mu g = 0$.

**Proof of proposition 2.** Let $\zeta$ denote the Lagrange multiplier of (14). The first-order condition for the optimal $\beta$ is

$$(1 + \zeta)[f^T C^{-1} g - g^T C^{-1}(\beta g + \gamma f)] = 0. \quad (28)$$

Thus, given $\gamma$, the optimal explicit bonus is$^{37}$

$$\beta(\gamma) = (1 - \gamma) \frac{f^T C^{-1} g}{g^T C^{-1} g} = (1 - \gamma) \tilde{\beta}. \quad (29)$$

Substitution of $e(\beta, \gamma)$ and $\beta$ in (13) and (14) and some transformations yield the simplified problem

$$\max_{\gamma} \frac{1}{2} \left( f^T C^{-1} g \right)^2 + \frac{\gamma(2 - \gamma)}{2} \left( f^T C^{-1} f - \frac{(f^T C^{-1} g)^2}{g^T C^{-1} g} \right) \quad (30)$$

subject to

$$\phi \gamma \leq \frac{\gamma(2 - \gamma)}{2} \left( f^T C^{-1} f - \frac{(f^T C^{-1} g)^2}{g^T C^{-1} g} \right), \quad (31)$$

which is equivalent to

$$\max_{\gamma} \frac{\|f_c\|^2}{2} \cos^2 \theta_c + \frac{\gamma(2 - \gamma)}{2} \|f_c\|^2 (1 - \cos^2 \theta_c) \quad (32)$$

subject to

$$\phi \gamma \leq \frac{\gamma(2 - \gamma)}{2} \|f_c\|^2 (1 - \cos^2 \theta_c), \quad (33)$$

The objective function (32) increases in $\gamma$ for $\gamma \leq 1$ and attains its maximum at $\gamma = 1$. Thus, the principal chooses the highest $\gamma \leq 1$ that satisfies (33), which yields (15). Equation (16) then follows from substituting $\gamma^*$ in (32).

$^{37}$Note that $f^T C^{-1} g = g^T C^{-1} f$ since $C^{-1}$ is symmetric, which follows from symmetry of $C$. 

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Proof of proposition 3. For the time being, suppose \( f, g \in \mathbb{R}^2_+ \) and \( C \) is a symmetric, positive definite \((2 \times 2)\) matrix. Using a program like Mathematica to simplify the expression for \( \eta \), it can be shown that

\[
\eta = f^T C^{-1} f - \frac{(f^T C^{-1} g)^2}{g^T C^{-1} g} = \frac{(f_2 g_1 - f_1 g_2)^2}{c_{22} g_1^2 + c_{11} g_2^2 - 2 c_{12} g_1 g_2}.
\]  

(34)

Since \( \gamma^* \) increases in \( \eta \), proposition 3 follows.

Proof of proposition 4. First consider the principal’s fallback position when she reneges on one or both relational contracts. In this case, she will offer in each following period the explicit contracts that solve the problem

\[
\max_{\alpha_l, \beta_l, e_l} f^T e - \sum_{l=1,2} (\alpha_l + \beta_l g^T e),
\]

\( \text{s.t.} \quad e_1 = c_{11}^{-1} \beta_1 g_1, \quad \hat{e} = \hat{C}^{-1} \beta_2 \hat{g}, \)

(35)

\[0 \leq \alpha_1 + \beta_1 g^T e - \frac{c_{11}}{2} e_1^2,\]

(37)

\[0 \leq \alpha_2 + \beta_2 g^T e - \frac{1}{2} e^T \hat{C} \hat{e}.\]

(38)

The solution to this problem is straightforward and leads to optimal explicit bonuses of \( \beta_1 = f_1/g_1, \beta_2 = (\hat{f}^T \hat{C}^{-1} \hat{g})/(\hat{g}^T \hat{C}^{-1} \hat{g}) \) and an expected profit of

\[
\tilde{\pi}_S = \frac{f_1^2}{2 c_{11}} + \frac{||\hat{f}||^2}{2} \cos^2 \hat{\theta}_c.
\]

(39)

Thus, the optimal combination of explicit and relational contracts is determined by
solving

\[
\max_{\alpha_l, \beta_l, \gamma_l, e_l} \quad f^T e - \sum_{l=1,2} (\alpha_l + \beta_l g^T e + \gamma_l f^T e),
\tag{40}
\]

s.t. \[ e_1 = c_{11}^{-1} (\gamma_1 f_1 + \beta_1 g_1), \tag{41} \]
\[ \hat{e} = \hat{C}^{-1} (\gamma_2 \hat{f} + \beta_2 \hat{g}), \tag{42} \]
\[ 0 \leq \alpha_1 + \beta_1 g^T e + \gamma_1 f^T e - \frac{c_{11}}{2} e_1^2, \tag{43} \]
\[ 0 \leq \alpha_2 + \beta_2 g^T e + \gamma_2 f^T e - \frac{1}{2} \hat{e}^T \hat{C} \hat{e}, \tag{44} \]
\[ \phi(\gamma_1 + \gamma_2) \leq f^T e - \sum_{l=1,2} (\alpha_l + \beta_l g^T e + \gamma_l f^T e) - \tilde{\pi}_S. \tag{45} \]

Note that (45) also implies that the principal will not break the relational contract with only one of the two agents. It is easy to verify that the agents’ participation constraints (43) and (44) bind at the optimal solution. Thus, by substituting \( e_1 \) and \( \hat{e} \) and defining

\[
\pi(\beta_1, \gamma_1, \beta_2, \gamma_2) := \frac{f_1}{c_{11}} (\beta_1 g_1 + \gamma_1 f_1) + \hat{f}^T \hat{C}^{-1} (\beta_2 \hat{g} + \gamma_2 \hat{f}) - \frac{1}{2c_{11}} (\beta_1 g_1 + \gamma_1 f_1)^2 - (\beta_2 \hat{g} + \gamma_2 \hat{f})^T \hat{C}^{-1} (\beta_2 \hat{g} + \gamma_2 \hat{f}), \tag{46} \]

the problem can be simplified to

\[
\max_{\beta_1, \gamma_1, \beta_2, \gamma_2} \pi(\beta_1, \gamma_1, \beta_2, \gamma_2) \tag{47}
\]

s.t. \[ \phi(\gamma_1 + \gamma_2) \leq \pi(\beta_1, \gamma_1, \beta_2, \gamma_2) - \frac{f_1^2}{2c_{11}} - \frac{||\hat{f}_c||^2}{2} \cos^2 \hat{\theta}_c. \tag{48} \]

From the first-order condition for \( \beta_1 \),

\[
\beta_1(\gamma_1) = (1 - \gamma_1) \frac{f_1}{g_1} \text{ for } 0 \leq \gamma_1 \leq 1. \tag{49} \]
After substituting $\beta_1$, the principal’s problem becomes

$$\max_{\beta_2, \gamma_2} \frac{f_1^2}{2c_{11}} + \pi(0, 0, \beta_2, \gamma_2)$$

$$\text{s.t. } \phi(\gamma_1 + \gamma_2) \leq \pi(0, 0, \beta_2, \gamma_2) - \frac{||\hat{f}_c||^2}{2} \cos^2 \theta_c.$$  \hfill (50)

Thus, the principal cannot do better than setting $\gamma_1 = 0$. The remaining optimization problem corresponds to the one considered in section 3.2 so that $\pi_S$ follows from (16).

**Proof of proposition 5.** It is easily verified that, since $c_{12} = c_{13} = 0$,

$$C^{-1} = \xi \begin{pmatrix}
(\xi c_{11})^{-1} & 0 & 0 \\
0 & c_{33} & -c_{23} \\
0 & -c_{23} & c_{22}
\end{pmatrix} \quad \text{and} \quad \hat{C}^{-1} = \xi \begin{pmatrix}
c_{33} & -c_{23} \\
-c_{23} & c_{22}
\end{pmatrix}, \quad (52)$$

where $\xi := (c_{22}c_{33} - c_{23}^2)^{-1}$. By (15) and the proof of proposition 4, $\gamma_2^* \leq \gamma^*$ if and only if $\hat{\eta} \leq \eta$, i.e.,

$$\begin{align*}
||\hat{f}_c||^2(1 - \cos^2 \hat{\theta}_c) &\leq ||f_c||^2(1 - \cos^2 \theta_c) \quad \text{if and only if} \quad \hat{f}_c^T \hat{C}^{-1} \hat{f} - \left(\hat{f}_c^T \hat{C}^{-1} \hat{g}\right)^2 \\
&\leq f_c^T C^{-1} f - \left(f_c^T C^{-1} g\right)^2 \\
&\leq f_1^2 \left(\frac{f_1 c_{11}}{g_1} + \tau\right)^2 \leq f_1^2 \left(\frac{f_1 c_{11}}{g_1} + \frac{\tau^2}{v}\right).
\end{align*}$$

(53)  \hfill (54)  \hfill (55)  \hfill (56)

where $\tau := \hat{f}_c^T \hat{C}^{-1} \hat{g}$ and $v := \hat{g}^T \hat{C}^{-1} \hat{g}$. Further transformation of (56), using that $v > 0$ since $\hat{C}^{-1}$ is positive definite and $\hat{g} > 0$, yields that (53) is equivalent to $0 \leq (\tau g_1 - v f_1)^2$ and thus holds.

(i) From $\gamma^* = 0$ it follows that $\gamma_2^* = 0$. Thus, task splitting leads to a weakly
higher expected profit if and only if
\[
\frac{||\hat{f}_c||^2}{2} \cos^2 \theta_c \leq \frac{\hat{f}_1^2}{2c_{11}} + \frac{||\hat{f}_c||^2}{2} \cos^2 \hat{\theta}_c
\]  
(57)
which is equivalent to (55) and thus holds.

(ii) By assumption, $\gamma^* < 1$. Thus we now consider the case $0 < \gamma^*_2 \leq \gamma^* < 1$. Since
\[
\frac{\hat{f}_1^2}{c_{11}} = f^T C^{-1} f - \hat{f}^T \hat{C}^{-1} \hat{f} = ||f_c||^2 - ||\hat{f}_c||^2,
\]  
(58)
task splitting leads to a lower expected profit than no task splitting iff
\[
\frac{||f_c||^2}{2} - \frac{||\hat{f}_c||^2}{2} + \frac{||\hat{f}_c||^2}{2} \cos^2 \hat{\theta}_c + 2\phi \left( 1 - \frac{\phi}{||\hat{f}_c||^2(1 - \cos^2 \hat{\theta}_c)} \right) \leq \frac{||f_c||^2}{2} \cos^2 \theta_c + 2\phi \left( 1 - \frac{\phi}{||f_c||^2(1 - \cos^2 \theta_c)} \right)
\]  
(59)
Because $\gamma^*_2 < 1$, we have $||\hat{f}_c||^2(1 - \cos^2 \hat{\theta}_c) < 2\phi$ and, therefore, the left-hand side of (59) strictly increases in $||\hat{f}_c||^2(1 - \cos^2 \hat{\theta}_c)$. Furthermore, (59) binds iff $||\hat{f}_c||^2(1 - \cos^2 \hat{\theta}_c) = ||f_c||^2(1 - \cos^2 \theta_c)$. Thus, by (53), (59) holds.

(iii) If $\gamma^*_2 = 0$ and $\gamma^* > 0$, the principal prefers task splitting if and only if
\[
\hat{\pi} + 2\phi \left( 1 - \frac{\phi}{\hat{\eta}} \right) \leq \frac{\hat{f}_1^2}{2c_{11}} + \frac{||\hat{f}_c||^2}{2} \cos^2 \hat{\theta}_c
\]  
(60)
\[
\Leftrightarrow \frac{1}{2}(||f_c||^2 - \eta) + 2\phi \left( 1 - \frac{\phi}{\eta} \right) \leq \frac{\hat{f}_1^2}{2c_{11}} + \frac{1}{2}(||\hat{f}_c||^2 - \hat{\eta})
\]  
(61)
\[
\Leftrightarrow \hat{\eta} \leq \eta - 4\phi \left( 1 - \frac{\phi}{\eta} \right)
\]  
(62)

**Proof of proposition 6.** By proposition 5, if $\phi < \hat{\eta}$, all tasks are assigned to one agent. By the proof of proposition 3,
\[
\hat{\eta} = \frac{(f_3 g_2 - f_2 g_3)^2}{c_{33} g_2^2 + c_{22} g_3^2 - 2c_{23} g_2 g_3}.
\]  
(63)
References


