Bankruptcy law and firms' behavior

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Abstract

The aim of this paper is to study the impact of bankruptcy law on financing, investment, default and liquidation decisions of firms. We build a model in which a firm can finance its investment by issuing debt. The investment is risky. Because of risk, the firm may default. The firm manager takes investment and default decisions in order to maximize the value of equities. Before investment takes place, shareholders and bondholders bargain over the share of the investment that is financed through debt and the annual coupon. If it occurs, at default the firm enters an observation period after which the decision of liquidation or continuation is taken. The model is solved and calibrated in order to reproduce French firms characteristics. We then study the effect on financing, investment, default and liquidation decisions of the firms, of changes in the parameters that summarize the bankruptcy procedure.

I. Introduction

French firms seem to undertake investment projects that are less risky than that of US firms. Some people argue that this relative shyness of French firms results from bankruptcy law. A too harsh treatment of firms' financial distress would reduce risk taking. Such a relationship seems straightforward; however, in reality it is much complex since the link between the treatment of financial distress and risk taking involves several players: the firm’s shareholders, its creditors, its workers, its managers and the government. The government plays a role through the law on financial distress. Thus, risk taking depends on the whole set of

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contractual relationships between these agents that have distinct objectives. As a result, the impact of bankruptcy law on risk taking is not as intuitive as it seems at first sight.

A change in the law on firms’ financial distress has a direct impact on default and liquidation decisions. It may affect as well, all the decisions the firms takes upstream concerning investment and its financing. These decisions are taken by shareholders that seek to maximize the value of their equities. They depend on financing contracts proposed by potential creditors that seek to invest their funds at risk and return conditions similar to that prevailing on financial markets.

We consider a change of the law on financial distress that would make the French law become closer to the US one. Once we take into account the relationships between agents at stake, would such a change create incentives for the firm to engage into investments that have higher risk and returns? This paper proposes an answer to this question by concentrating on relationships between shareholders, creditors and the law. The law is summarized by its major characteristics: the existence and the length of an observation period between default and liquidation (or continuation) decisions, the change in the firm duty to its creditors during the observation period, as well as the financial distress costs during the observation period, and the degree of difficulty creditors face recover their funds in case of liquidation.

To evaluate the effects of a change in law, we construct a model of a firm behavior. The firm we consider has the opportunity finance part of a risky investment through debt. Shareholders and creditors bargain over the level and the rate of the funds financing the investment. Uncertainty on the return on investment generates a default opportunity. At default time, the firm enters the so-called observation period during which the coupon paid to the creditors is lightened but it has to pay financial distress costs. The latter encompasses very different things such as direct financial costs (lost of confidence of suppliers and of new potential creditors) as well as indirect costs (divert effort from productive activity, restructuration costs). At the end of the observation period, the firm continues its activity if market conditions improved, if not, it is liquidated. In case of liquidation, creditors recover a fraction of the net assets of the firm, which only partially covers the face value of the firm’s debt. This fraction depends on liquidation costs (cost of seizure and of the selling of the remaining assets). Of
course, the probability of default and liquidation, and the conditions under which they occur affect the bargaining between creditors and shareholders at the time the investment is undertaken. We then obtain a link between parameters that characterize the law and the firm decisions: investment, financing structure, loan rates, default, and liquidation.

Once the model is constructed and solved, it is calibrated to reproduce French firms financing characteristics. We then study the effect of changes in the parameters representing the law on financing, investment, and default choices of firms. The main results we obtain are that: (1) a better protection of creditors favors risk taking by firms, (2) the observation period is costly for all the agents and is not justified unless firms use it to restructure efficiently. Moreover, we also show that it is not possible to have both firms that choose riskier projects and a smaller default rate. Finally the structure of the firms’ financing and interest rate spreads are affected by the whole set of parameters characterizing the law. For instance, a better creditor protection translates into a higher debt at similar or smaller rates.

The model proposed here is related to the literature that considers default as a decision of the firm consistent with the objective of equity value maximization (Leland1994, Goldstein, Ju and Leland 2001, Morellec 2001, Mauer and Sakar 2004). The contribution of this literature is to consider the default decision as an irreversible choice under uncertainty that can be modeled as a real option (Dixit and Pindyck, 1994). Nevertheless, in this literature, default leads to immediate liquidation of the firm. The observation period is therefore not taken into account. Few models introduce an observation period (Franks and Torous, 1989, Longstaff, 1990) but only François and Morellec (2004) models the default decision, an observation period, the liquidation and financial decisions. We take the framework from François and Morellec (2004) and introduce upstream the investment decision, which can be dealt as a real option as well. Our model therefore mixes the approaches of François and Morellec (2004) and Mauer and Sakar (2004) from which we take the modeling of the investment decision. Moreover, in order to reproduce at best the observed survival rates of French firms, as well as the probability of liquidation at the end of the observation period, we model uncertainty by adding a jump process to the usually considered Brownian motion for the output price of the firm. Thanks to this generalization, we can take into account seldom events that have huge consequences.
The remaining of this paper is as follows. Section II deals with the decisions of a firm that is entirely financed with equity. In section III we introduce the possibility of getting into debt and a default procedure with immediate liquidation. In section IV the model is extended to the case in which there exists an observation period after the default after which the firm is liquidated or goes on with production depending on market conditions. In each section the model is solved, calibrated, and simulated. A summary of the result is finally given in section V.

II. Investment Behavior and Decision to Close of an Unlevered Firm.

In order to find the value of the firm and to characterize its investment choices, we consider in a first step the case in which the firm is only financed through equities. Of course, in this case the possibility of default is of no relevance. In a second step, we study the case in which the firm can get into debt. The possibility of default has to be taken into account.

A. Basic Assumptions of the Model

We consider a firm whose sole asset is an option to invest in a production process by paying a fixed cost $I$. Once this option is exercised, it produced one unit of good per year. The production cost is $C$; this cost is constant during time. At the time when the good is produced it is sold in a perfectly competitive market at a price $P$ per unit. The price evolution is stochastic.

Uncertainty

The price evolution is given by a geometric Brownian motion combined with a Poisson process:

$$dP = \alpha P dt + \sigma P dz + (J - 1)P dq$$  \hspace{1cm} (1)
The trend ($\alpha$) and the volatility ($\sigma$) are constant; $dz$ is the increment of a standard Wiener process and $dq$ is a Poisson process:

$$dq = \begin{cases} 
0 \text{ with probability } 1 - \lambda dt \\
1 \text{ with probability } \lambda dt 
\end{cases}$$

When $dq = 1$, the process followed by the price jumps from $P$ to $JP$. We suppose that the Poisson process $dq$ and the geometric Brownian motion $dz$ are independent.

**Gains (or Losses) of the Active Firm**

Profits of the firms are immediately taxed at a constant rate $\tau$. Moreover, we assume that the tax system is perfectly symmetrical with the possibility of losses discount.

Once option has been exercised, the firm starts producing but at any time, it has the possibility to stop the production whenever it wishes.

**Financial Environment of the Firm**

We assume that there exists a risk less financial asset which provides a constant instantaneous return $r\%$ per year. We also suppose that the price of the product which is sold by the firm can be reproduced using financial assets on the market, like for instance future contracts. This assumption allows us to appraise the investment option value as well as the value of the active firm.
B. Optimal Behavior of the Firm

The decisions of an unlevered firm only concern the price at which it should invest and that at which it should stop producing. Figure 1 provides an example of the price evolution of a firm which invests at time \( t_1 \) and stop producing at time \( t_2 \).

![Figure 1: Value of the Firm and Threshold to Stop Production](image)

Value of the Firm and Threshold to Stop Production

We first consider the value of the firm once the investment decision has been exercised. The standard argument of risk-neutral valuation imposes that \( V^U(P) \) which is the value of the firm entirely financed with equity satisfies the following differential equation:

\[
\frac{1}{2} \sigma^2 P^2 V_{PP}^U(P) + \left[ r - \delta - \lambda(1 - 1) \right] PV_{P}^U(P) - (r + \lambda) V(P) + \lambda V(JP) + (P - C)(1 - \tau) = 0, \quad P > P_A
\]

with \( P_A \) the price at which the firm decides to stop production. The general solution of equation (2) is given by:

\[
V^U(P) = \left( \frac{P}{\delta} - \frac{C}{r} \right)(1 - \tau) + A_1 P^{\beta_1} + A_2 P^{\beta_2}, \quad P > P_A
\]

Where \( A_1 \) and \( A_2 \) are constant to be determined and \( \beta_1 \) and \( \beta_2 \) are solutions of the equation:
\[
\frac{1}{2} \sigma^2 \beta (\beta - 1) + \left[ r - \delta - \lambda (J - 1) \right] \beta - (r + \lambda) + \lambda J^\beta = 0. \text{ With } \beta_1 > 1 \text{ and } \beta_2 < 0.
\]

Moreover, the value of the firm entirely financed with equity (equation 3) has to satisfy:

\[\lim_{P \to 0} V^U(P) = \left( \frac{P}{\delta - \frac{C}{r}} \right)(1 - \tau) \quad (4.a)\]

\[V^U(P_A) = 0, \quad (4.b)\]

\[\left. \frac{\partial V^U}{\partial P} \right|_{P=P_A} = 0 \quad (4.c)\]

Condition (4.a) expresses the fact that the value of the opportunity to stop production becomes zero if the price of the product becomes large. Such a condition is satisfied only if \(A_1 = 0\).

Condition (4.b) expresses the fact that at the time when production is stopped, the value of the firm is zero. Condition (4.c) ensures that the price which triggers production stopping is optimal. Substituting (3) in (4.a)-(4.c) provides the value of the unlevered firm:

\[V^U(P) = (1 - \tau) \left( \frac{P}{\delta} - \frac{C}{r} - \left( \frac{P_A}{\delta} - \frac{C}{r} \right) \left( \frac{P}{P_A} \right)^{\beta_2} \right) \quad (5)\]

With \(P_A = -\frac{\beta_2}{1 - \beta_2} \frac{\delta C}{r}\)

**Value of the Option to Invest and Investment Decision**

We not \(F(P)\) the value of the option to invest and we easily show that \(F(P)\) must satisfy the following differential equation:

\[\frac{1}{2} \sigma^2 P^2 F''_P(P) + \left[ r - \delta - \lambda (J - 1) \right] PF_P(P) - (r + \lambda)F(P) + \lambda F(JP) = 0, \quad P < P_I\]

Where \(P_I\) is the price at which the firm exercises the option to invest. The general solution has the form:

\[F(P) = \Omega_1 P^{\beta_1} + \Omega_2 P^{\beta_2}, \quad P < P_I\]
Moreover, the value of the option to invest must satisfy three boundary conditions. The first condition imposes that the value of the option is close to zero if the price of the product tends to zero. It implies \( \Omega_z = 0 \). We then note \( P^U_I \) the trigger price of the option to invest of the firm entirely financed with equity. At the price which triggers investment, shareholders are exactly indifferent between holding the option to invest and exercising it. The third condition is a "smooth pasting" condition which ensures that the investment trigger price is optimal.

\[
\lim_{P \to 0} F(P) = 0 \\
F(P^U_I) = V^U(P^U_I) - I, \\
\left. \frac{\partial F}{\partial P} \right|_{P = P^U_I} = \left. \frac{\partial V^U}{\partial P} \right|_{P = P^U_I}
\]

Finally, the value of the option to invest can be written:

\[
F^U(P) = \left[ V^U(P^U_I) - I \right] \left( \frac{P}{P^U_I} \right)^{\beta_1}
\]

And the price which triggers investment can be determined with the smooth pasting condition:

\[
\left( 1 - \frac{1}{\beta_1} \right) \left( \frac{P^U_I}{\delta} \right) - \frac{C}{r} - \beta_1 \frac{I}{1-\tau} + \left( \frac{\beta_2}{\beta_1} - 1 \right) \left( \frac{P_A}{\delta} \right) \left( \frac{P^U_I}{P_A} \right)^{\beta_2} = 0
\]

This equation can be solved numerically but not analytically.

**Numerical Resolution and Analysis of the Results**

Table 1 below gives the numerical results concerning the prices which trigger investment and production stopping in the case of an unlevered firm. Parameters of the model have been chosen in order to reproduce at best available observations on financing behavior and on default and liquidation behavior of firms (cf. table in appendix 3). Of course, as in reality firms get into debt and may face an observation period, calibration has been made using the completed model presented later in this paper (cf. part IV).
Trigger prices in the basic scenario and variants proposed depend on the chosen values and cannot be interpreted economically. Nevertheless, it is interesting to observe the way trigger prices vary depending on the institutional and economic environment of the firm.

**Table 1: Investment Behavior and production stopping for an unlevered firm.**

<table>
<thead>
<tr>
<th>Institutional and Economic environment of the firm</th>
<th>Basic scenario</th>
<th>Less uncertainty</th>
<th>Cut in the tax rate</th>
<th>Increase in the interest rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R$</td>
<td>4%</td>
<td>4%</td>
<td>4%</td>
<td>4%</td>
</tr>
<tr>
<td>$\tau$</td>
<td>0.30</td>
<td>0.30</td>
<td>0.30</td>
<td>0.30</td>
</tr>
<tr>
<td>$\alpha = -\lambda (J-1)$</td>
<td>0.02</td>
<td>0.02</td>
<td>0.01</td>
<td>0.02</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>0.8</td>
<td>0.6</td>
<td>0.8</td>
<td>0.8</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>0.1</td>
<td>0.1</td>
<td>0.1</td>
<td>0.1</td>
</tr>
<tr>
<td>$J$</td>
<td>0.8</td>
<td>0.8</td>
<td>0.9</td>
<td>0.8</td>
</tr>
<tr>
<td>$p_{I}^U$</td>
<td>30.0</td>
<td>14.7</td>
<td>29.9</td>
<td>30.0</td>
</tr>
<tr>
<td>$p_{A}^U$</td>
<td>0.10</td>
<td>0.26</td>
<td>0.10</td>
<td>0.10</td>
</tr>
</tbody>
</table>

The trend in the Brownian motion ($\alpha$) has been fixed such that the expected value of the price at any horizon must always be equal to the current price level. It is similar to impose $\alpha = -\lambda (J-1)$, since $E_{\cdot}(P(t+s)) = P(t)e^{(r+\lambda(J-1))t}$. Therefore, in the basic scenario, in the absence of shocks, the price of the good produced by the firm increases by 2% per year, but the firm has one chance over 10 to bear a shock that would cut the price of its good by $(J-1) = 20\%$. Other parameters of the model have been arbitrarily fixed to: $I = 40$, $C=1$, $\delta=0.04$.

Investment timing is sensitive to the institutional and economic environment of the firm, while production stopping is nearly independent from this environment. Therefore, a decrease in uncertainty and a fall in the tax rate always reduce the threshold at which the firm invests while an increase in the risk less interest rate increases this threshold. The price at which the firm stops producing is nearly insensitive to uncertainty parameters (except that of the geometric Brownian motion).

In the case in which the firm is entirely financed with equity, production stopping is a seldom event. Indeed, table 1 shows that the discrepancy between the price which trigger investment
and that which stops production is such that the firm decides to stop producing only if the
price becomes 90% less than the price at which is has invested and started producing.

III. Optimal Behavior of the levered firm in the absence of observation period

Consider now the case in which the cost of exercising the option is at least partially financed
with debt. Before exercising the option to invest we suppose that the firm finalizes an
agreement with the bondholders under which bondholders will pay the firm $K$ when the firm
exercises the option to invest, in return for a continuous coupon payment of $R$ per year (the
interest rate at which the firm gets into debt is then $i = R/K$). Debt has an infinite maturity
barring default. We assume that the timing of the investment decision is at the discretion of
the shareholders and cannot be contracted in advance by bondholders. Nevertheless,
bondholders anticipate that equity holders choose an exercise strategy that maximizes equity
value. As such, bondholders will require that their future commitment of $K$ to finance the
exercise of the investment option be fair relative to the agreed coupon and the closing
strategy.

Finally, if the firm can get into debt, its decisions concerning prices at which it invests and
liquidates and the contract with bondholders which expresses the amount of debt and the
coupon.

A. Security values

Consider first $R$ and $K$ as given; our objective here is to compute the market values of equity,
of debt and of the firm, once the option to invest has been exercised. The differential equation
which describes the levered firm value is the same as equation (2), except that income
accruing to shareholders is now $(P\cdot C\cdot R) (1 - \tau)$ per unit time. The analogous general solution
for the levered firm is:

$$E(P) = \left( P \frac{C + R}{\delta - \frac{C + R}{r}} \right) (1 - \tau) + B_1 P^{\beta_1} + B_2 P^{\beta_2}, \quad P > P_B$$

(6)
Where $B_1$ and $B_2$ are constant to be determined, $\beta_1$ and $\beta_2$ are the same as those presented just below equation (3). $P_B$ is the price at which shareholders decide to default.

Moreover, the general solution (6) must satisfy the following boundary conditions:

\[
\lim_{P \to \infty} E(P) = \left( \frac{P}{\delta} - \frac{C + R}{r} \right) (1 - \tau) \quad (7.a)
\]
\[
E(P_B) = 0, \quad (7.b)
\]
\[
\frac{\partial E}{\partial P}\bigg|_{P = P_B} = 0 \quad (7.c)
\]

Condition (7.a) expresses that the value of the option to default is zero when the good price becomes large. This condition is met only if $B_1 = 0$. Condition (7.b) expresses that at the time of default, the firm value is zero. Condition (7.c) ensures that the price triggering default is optimally chosen by shareholders. Substituting (6) in (7.a)-(7.c) we get the value of the levered firm:

\[
E(P) = (1 - \tau) \left( \frac{P}{\delta} - \frac{C + R}{r} - \left( \frac{P_B}{\delta} - \frac{C + R}{r} \right) \left( \frac{P}{P_B} \right)^{\beta_2} \right) \quad (8)
\]

With $P_B = \frac{-\beta_2 \delta(C + R)}{1 - \beta_2} \frac{r}{r}$

Note that $P_B \geq P_A$ since $R \geq 0$.

**B. Market value of the debt of the firm**

Since debt has an infinite maturity, the general solution for the debt value is:

\[
D(P) = \frac{R}{r} + C_1 P^{\beta_1} + C_2 P^{\beta_2}, \quad P > P_B \quad (9)
\]

Where $C_1$ and $C_2$ are constant to be determined and boundary conditions are:

\[
\lim_{P \to \infty} D(P) = \frac{R}{r} \quad (10.a)
\]
\[
D(P_B) = (1 - b)V''(P_B) \quad (10.b)
\]
Condition (10.a) implies \( C_1 = 0 \), which expresses that if the default probability decreases, the market value of debt equals that of a riskless asset infinitely paying a coupon \( R \). Condition (10.b) expresses that at the time of default, bondholders receive the unlevered value of the firm less the liquidation costs. The latter are assumed to be proportional to the firm value \( 0 \leq b \leq 1 \). Substituting (9) into (10.a) and (10.b), we get the market value of the (risky) debt of the firm:

\[
D(P) = \frac{R}{r} + \left( 1 - b \right) V^U(P_b) - \frac{R}{r} \left( \frac{P}{P_b} \right)^{\beta^2}
\]

(11)

Finally, the total value of the levered firm is the sum of the equity value (equation 8) and of the market value of the debt (equation 11), that is:

\[
V^L(P) = V^U(P) + \frac{\pi R}{r} \left[ 1 - \left( \frac{P}{P_b} \right)^{\beta^2} \right] - b V^U(P_b) \left( \frac{P}{P_b} \right)^{\beta^2}
\]

(12)

The market value of the levered firm is equal to the value of the unlevered firm plus the expected tax shield of debt minus expected liquidation costs.

C. Investment Decision, Interest Rate and Optimal Financing Structure of the Firm

This firm has an option to invest and a contract with bondholders in which the latter commit to provide \( K \) dollars to help the firm to finance its investment in exchange for a coupon \( R \). Since the option to invest is the sole asset of the firm, the current value of the firm is equal to the value of this option. Our objective here is to determine the value of the option to invest and the price which trigger the exercise of this option.

Value of the option to invest

We note \( F(P) \) the value of the option to invest. We easily show that \( F(P) \) must satisfy the following differential equation:
\[
\frac{1}{2} \sigma^2 P^2 F_{pp}(P) + \left[r - \delta - \lambda(J - 1)\right] PF_{p}(P) - \left(r + \lambda\right) F(P) + \lambda F(JP) = 0, \quad P < P_I \quad (13)
\]

Where \( P_I \) is the price at which the firm exercises the option to invest. The general solution is of the form:

\[
F(P) = \Omega_1 P^{\beta_1} + \Omega_2 P^{\beta_2}, \quad P < P_I
\]

Besides, the value of the option to invest must check three boundary conditions. Condition (14.a) imposes that the option value becomes close to zero if the good price tends to zero. It implies \( \Omega_2 = 0 \). To find the two other conditions, recall that the exercise timing is chosen by the shareholders. To do so, they maximize the equity value as opposed to the value of the firm. We then note \( P_I \) the price level that triggers investment. Condition (14.b) expresses that at \( P_I \), shareholders are exactly indifferent between holding the option to invest or exercise it. Condition (14.c) is a smooth pasting condition ensuring that the trigger price is optimal.

\[
\lim_{P \to 0} F(P) = 0 \quad (14.a)
\]
\[
F(P_I) = E(P_I) - (I - K), \quad (14.b)
\]
\[
\frac{\partial F}{\partial P} \bigg|_{P=P_I} = \frac{\partial E}{\partial P} \bigg|_{P=P_I} \quad (14.c)
\]

Finally, the option to invest may be written:

\[
F(P) = \left[E(P_I) - (I - K)\right] \left(\frac{P}{P_I}\right)^{\beta_1} \quad (15)
\]

The debt level
The appropriate value of $K$ is easy to determine. Bondholders will not agree to give $K$ dollars to the firm when equity holders choose to invest, unless $K$ is a fair price for the debt. Since they know that once the contract is finalized they cannot force equity holders to choose a firm-value-maximizing exercise strategy, they value the debt (and thereby determine $K$) under the assumption that shareholders will seek to maximize equity value and not firm value. Therefore, the incentive-compatible $K$ must be equal to the market value of debt at the investment trigger.

$$K = D(P_I) = \frac{R}{r} + \left( (1-b)V^U(P_B) - \frac{R}{r} \left( \frac{P_I}{P_B} \right) \right)^{\beta^2}$$  \hspace{1cm} (16)$$

Substituting this $K$ value in equation (15) and using the fact that $E(P_I) + D(P_I) = V(P_I)$, we get the option value to invest.

$$F(P) = \left[ V^L(P_I) - I \right] \left( \frac{P}{P_I} \right)^{\beta^1}$$  \hspace{1cm} (17)$$

And the price which triggers investment can be determined using condition (14.c):

$$\left(1 - \frac{1}{\beta^1} \right) \frac{P_I}{\delta} - \frac{C + R}{1 - \tau} - \frac{I - K}{1 - \tau} + \left( \frac{\beta^2}{\beta^1} - 1 \right) \left( \frac{P_B}{\delta} - \frac{C + R}{r} \right) \left( \frac{P_I}{P_B} \right)^{\beta^2} = 0$$  \hspace{1cm} (18)$$

Where $K$ is given by equation (16). Equation (18) can be solved numerically but not analytically.

**Interest rate at which the firm get into debt**

So far, we have considered a given $R$. Equation (18) provides the optimal financing structure for a given $R$. How to determine the level or the interest rate $i = R/K$ that are optimal for the firm?

Since $R$ and $K$ are decided in advance in the contract between shareholders and bondholders, we consider that the firm selects the interest rate which maximizes the value of the option to
invest among the contracts proposed by bondholders (cf. figure 2.b below). The curve of the contracts proposed by the bank is such that at any point the market value of the debt at the time of investment equals the face value of debt.

D. Numerical Resolution and results analysis

Results of the numerical resolutions of the equations describing the optimal behavior of the firm are given in Table 2. We focus on the effect of the institutional and economic environment of the firm on the investment, liquidation, and financing (interest rate and leverage) decisions. Moreover, by comparing the behavior of the levered firm (Table 1) with that of the unlevered firm (Table 2) one can appraise what changes if the firm can get into debt.

The effect of Debt on the Investment and Default Decisions of the firm

Comparing prices that trigger investment in Table 1 and 2, one can note that firms that have an access to debt invest earlier than unlevered firms (cf. figure 2.a). It comes from the fact that since interest payments can be deduced from taxable benefits, production becomes more profitable from the point of view of the firm. Indeed, we easily show that if the tax rate is zero, firms to not get into debt.

The counterpart to the possibility of debt is that firm can default on their debt. The possibility to have debt then makes investment easier but also increases the probability of default. In the basic scenario, the survival rate at 5 years of levered firms is 75% against 99% for unlevered firms. The increase in default risk comes on the one hand from the fact that levered firms have an incentive to default for a price higher than the one at which unlevered firms stop producing. On the other hand, levered firms invest for a smaller price level compared to what unlevered firms do. Numerical simulations show that the increase in default risk essentially comes from the higher price at which levered firms default.
Optimal Financing Structure and Interest Rate

A levered firm has a higher default risk than an unlevered firm. The contract between the firm and the bank concerns both the financing structure and the interest rate (as shown in bold in figure 2.b). The bank has to make sure that at the time of investment the face value of the debt equals its market value; the latter being a decreasing function of the debt rate and an increasing function of the interest rate. Among all the contracts (financing structure/interest rate) which makes the bank indifferent, the firm chooses the one which maximizes its equity value. In figure 2.b, it is the contract which reaches the higher "iso-option value" \( F \). In the basic scenario, firms choose a debt rate around 28% for a 13.4% interest rate.

Such financing structure/interest rate combinations results from a trade-off for the firm: at the time of investment, it could choose a higher level of debt but would then increase its default probability that would lead the bank to require a higher interest rate. One finally computes that at the time of default the market value of debt is only 15% of the face value.

Effect of the Institutional and Economic Environment on the Decision of the Firm
More uncertainty\(^3\) increases the investment trigger and decreases the default trigger. These two effects, joined with higher price volatility generate finally a higher default probability. As a consequence, the firm has less debt and a higher interest rate. This results on the one hand from a change in the slope of the curve of the contracts proposed the bank and on the other hand from a change in the shape of the "iso-option value" curves.

An increase in the tax rate has a direct effect on the optimal level of debt and on the interest rate, which both become higher. Effects on trigger prices are similar to those exhibited when comparing unlevered and levered firms: the investment trigger is smaller while the default trigger is higher. Survival rates are slightly smaller. Bankruptcy occurs when debt has lost a larger part of its value.

An increase in the risk less interest rate increases both the investment trigger and the default one. The firm has a little less debt and a higher interest rate. But default occurs when the debt has lost less of its initial value. Survival rates are nearly insensitive to an increase in the risk less rate.

For higher liquidation costs, liquidation occurs for a smaller price. Survival rates are slightly higher. Default occurs when debt has lost a larger fraction of its initial value. The option value of investment gets smaller and investment is made for a higher level of the price. Optimal debt is a little smaller and the interest rate is higher.

---

\(^3\) We choose here the case in which uncertainty increases following an increase in the uncertainty parameter of the geometric Brownian motion while other parameters remain unchanged.
Tableau 2: Investment and default/liquidation behavior of a levered firm in the absence of bankruptcy procedure.

<table>
<thead>
<tr>
<th></th>
<th>Basic scenario</th>
<th>More uncertainty</th>
<th>Increase in the tax rate</th>
<th>Increase in the risk less interest rate</th>
<th>Increase in the liquidation costs</th>
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<td>$R$</td>
<td>0.04</td>
<td>0.04</td>
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<td>$\tau$</td>
<td>0.3</td>
<td>0.3</td>
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<td>0.3</td>
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<tr>
<td>$\lambda$</td>
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<td>0.1</td>
<td>0.1</td>
<td>0.1</td>
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<tr>
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<td>0.8</td>
<td>0.8</td>
<td>0.8</td>
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</tr>
<tr>
<td>$\alpha = -\lambda(J-1)$</td>
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<td>0.02</td>
<td>0.02</td>
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<td>0.02</td>
</tr>
<tr>
<td>$\sigma$</td>
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<td>0.9</td>
<td>0.8</td>
<td>0.8</td>
<td>0.8</td>
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<tr>
<td>$B$</td>
<td>0.1</td>
<td>0.1</td>
<td>0.1</td>
<td>0.1</td>
<td><strong>0.3</strong></td>
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<tr>
<td><strong>Behavior of the firm</strong></td>
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<td></td>
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<td></td>
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<tr>
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<td>16.3</td>
<td>13.0</td>
<td>14.6</td>
<td>15.0</td>
</tr>
<tr>
<td>$P_B$</td>
<td>0.83</td>
<td>0.78</td>
<td>0.85</td>
<td>0.86</td>
<td>0.82</td>
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<td>Debt/equity</td>
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<td>24%</td>
<td>36%</td>
<td>27%</td>
<td>25%</td>
</tr>
<tr>
<td>Market value of the debt at the time of default / face value of the debt</td>
<td>15.5%</td>
<td>14.7%</td>
<td>13.9%</td>
<td>16.5%</td>
<td>12%</td>
</tr>
<tr>
<td>Interest rate (coupon / face value of debt)</td>
<td>13.1%</td>
<td>14.8%</td>
<td>13.7%</td>
<td>14.2%</td>
<td>13.2%</td>
</tr>
<tr>
<td>Survival rate at 1 year</td>
<td>100%</td>
<td>100%</td>
<td>100%</td>
<td>100%</td>
<td>100%</td>
</tr>
<tr>
<td>Survival rate at 3 years</td>
<td>91%</td>
<td>88%</td>
<td>90%</td>
<td>91%</td>
<td>92%</td>
</tr>
<tr>
<td>Survival rate at 5 years</td>
<td>75%</td>
<td>69%</td>
<td>63%</td>
<td>75%</td>
<td>77%</td>
</tr>
</tbody>
</table>

IV. Default, Observation Period, Decision on Continuation or Liquidation

We consider that the firm can get into debt but also that there exists a bankruptcy procedure in two stages. In a first stage, the firm states to be in default and enters an observation period during which the coupon it has to pay is reduced but financial distress costs have to be paid. In a second stage, which starts at the end of the first one, the firm has to liquidate or not.

A. Default, Observation Period and bankruptcy trigger

The firm starts to be in default when the price reaches a level $P_D$ (which will be determined later on). The firm is then under the protection of the law during an observation period $T$.

Firm's Cash-flow during the observation period
During the observation period $T$, the firm pays financial distress costs $\varphi$ and goes on with production (it thus pays the production costs) but it only pays a fraction $\theta$ of the coupon $R$.

$$\int [P - (C + \theta R + \varphi)](1 - \tau) dt$$

**Bankruptcy trigger**

At the end of the period $T$, if the price is still under $P_D$ the firm is liquidated. Creditors can get back $(1-b)V_u$, and shareholders get nothing. The probability that the firm is not liquidated at the end of the observation period is equal to $\Phi^-$, with:

$$\Phi^- = \sum_{k=0}^{\infty} p_k(\lambda T)(1 - \phi(v_k))$$

Where $\phi(v_k)$ is the cumulative function of a normal $(0.1)$ distribution with

$$v_k = -\frac{\alpha}{\sigma} \sqrt{T} - \frac{k \ln J}{\sigma \sqrt{T}} + \frac{1}{2} \sigma \sqrt{T}$$

Figure 3 provides an example of the price evolution for a firm that is liquidated after the observation period (3.a) for a firm that goes on with production after the observation period (3.b).

**Firm value at the time of default**
The firm value at the time of default, \( V^D(P) \), is the discounted sum of cash-flows during the observation period plus the expected value of the levered firm at the end of the observation period (it may then be liquidated):

\[
V^D(P) = \int_0^T (E(P(t+s)) - (C + \theta R + \varphi))(1-\tau)e^{-rt} ds + \int_0^T (\theta R)e^{-rt} ds \\
+ e^{-rT} \left[ \int_P^\infty (E(P) + D(P))df(P(t+T)) + (1-b)\int_0^P V^U(P(t+T))df(P(t+T)) \right]
\]

For creditors to accept the observation period rather than an immediate liquidation, the observation period must at least as profitable as immediate bankruptcy.

B. Solving the model

Default trigger and equity value before default

Outside the observation period, the equity value is still given by:

\[
E(P) = \left[ \frac{P}{\delta} - \frac{C+R}{r} \right](1-\tau) + B_2P^{\beta_2} \quad P > P_D
\]

Where \( B_2P^{\beta_2} \) is the default option.

At the time of default, the equity value is such that:

\[
E^D(P) = \int_0^T (E(P(t+s)) - (C + \theta R + \varphi))(1-\tau)e^{-rt} dt + e^{-rT} \int_P^\infty E(P(t+T))df(P(t+T))
\]

That can be rewritten (see appendix):

\[
E^D(P) = \left[ \frac{1-e^{-\delta t}}{\delta} P - \frac{1-e^{-r_t}}{r}(C + \theta R + \varphi) \right](1-\tau) \\
+ e^{-rT} \left[ (1-\tau)\left( \frac{(\Phi^{-1})P}{\delta} - \frac{\Phi(C+R)}{r} \right) + \Phi^{-2}B_2P^{\beta_2} \right]
\]
Where
\[
\Phi_1 = e^{\alpha \sum_{k=0}^{\infty} p_k (\lambda T) e^{k\ln J} (1 - \phi(v_1))} \quad \text{With} \quad v_1 = -\frac{\alpha}{\sigma} \sqrt{T} - \frac{k \ln J}{\sigma \sqrt{T}} - \frac{1}{2} \sigma \sqrt{T}
\]

And
\[
\Phi_2 = e^{\beta_1 (\lambda T) e^{\alpha \sum_{k=0}^{\infty} p_k (\lambda T) e^{k\ln J} (1 - \phi(v_1))}} \quad \text{With} \quad v_2 = -\frac{\alpha}{\sigma} \sqrt{T} - \frac{k \ln J}{\sigma \sqrt{T}} - \left(\beta_2 - \frac{1}{2}\right) \sigma \sqrt{T}
\]

The default trigger is determined by shareholders who maximize equity value. Optimality conditions are given by:

\[
E(P_D) = E^D(P_D)
\]
\[
\frac{\partial E}{\partial P} \bigg|_{P=P_0} = \frac{\partial E^D}{\partial P} \bigg|_{P=P_0}
\]

These conditions determine \( P_D \) and \( B_2 \) (see appendix).

\[
P_D = \frac{\beta_2}{1 - \beta_2} \frac{\delta}{r} \frac{[1 - e^{\gamma T}] R}{e^{\gamma T} - e^{\gamma T} \Phi_1}
\]

We show easily that if \( \delta \) gets close to \( r \) in the case in which \( \theta = 1 \) and \( \varphi = 0 \) the default trigger is higher than the liquidation trigger in the absence of observation period. Introducing a default procedure that would only be a delay before bankruptcy increases the bankruptcy trigger. This result is qualitatively similar to that of François and Morellec (2004). Nevertheless, the mechanism generating this result differs from the one invoked by François and Morellec (2004) : it is not the possibility of renegotiation between creditors and shareholders which generates a higher default trigger but rather the fact that an observation period of a given size makes it compulsory for the firm to anticipate the possibility of bankruptcy and provides therefore an incentive to default before equity value is zero (as it would be the case without observation period) ; the default trigger is thus higher.
Equity value of the firm may then be written:

\[
E(P) = (1 - \tau) \left[ \frac{P}{\delta} - \frac{C + R}{r} + \frac{\Phi^(-) e^{-\beta T} - e^{-\beta T}}{\beta \delta [1 - \Phi^(-) e^{-\beta T}]} P_D \left( \frac{P}{P_D} \right)^{\beta_2} \right]
\]

Debt value with a bankruptcy procedure

Outside the observation period, the debt value is still given by:

\[
D(P) = \frac{R}{r} + C_2 P^{\beta_2}, \quad P > P_D,
\]

where \( C_2 P^{\beta_2} \) is the default option value. At the time of default, it is such that:

\[
D^D(P) = \int_0^T (\theta R) e^{-sT} ds + e^{-sT} \left[ \int_{P_D}^P D(P) df(P(T)) + (1 - b) \int_0^P V^U(P(T)) df(P(T)) \right]
\]

Whose expression after having computed the integrals is given in appendix.

At the time of default, we have: \( D(P_D) = D^D(P_D) \). This equation provides the parameter value \( C_2 \) (see appendix).

**Investment Decision and Debt Level**

To determine the debt level and the price which trigger investment, we use the method already used to compute (i) the investment trigger, (ii) the debt level and (iii) the interest rate that has already been presented in the case of a levered firm in absence of default procedure.

**C. Numerical Resolution, Calibration of the Model and Analysis of the Results**

Results of the numerical resolutions of the model for several sets of the institutional and economic parameters are given in Table 3. We analyze first the ability of the calibrated model to reproduce the observed behavior of firms (see appendix). We then study the effect of
parameters which characterize the bankruptcy procedure: length of the observation period, financial distress costs, fraction of the coupon that is paid during the observation period and liquidation costs.

**Calibration of the Model (basic scenario)**

The liquidation cost $b$ has been fixed to 10%, which is the upper boundary of the evaluation provided by Malécot (1995) for French firms. Note nevertheless that higher liquidation costs have been recently observed in the case of the bankruptcies of Moulinex and Alstom. Recall that $(1-b)$ may be interpreted as the fraction of the value of the firm which non-prioritary creditors can get back in case of liquidation. Indeed, in our model all the production costs (wages and money due to suppliers) as well as taxes are paid at each time and in particular during the observation period. Parameter $b$ represents then the liquidation cost (cost of seizure and of selling of the remaining assets).

We have few observations on the financial distress costs during the observation period. Recall that in reality they encompasses very different things such as direct financial costs, loss of confidence of suppliers and of new potential creditors, as well as indirect costs due to the diversion of effort form productive activity towards restructuring. This broad definition is reflected in the broad range of the evaluations existing in the literature: from 3% of the firm value to 57% of the claims. In our scenario, the costs of financial distress vary from 0.5% (basic scenario) to 15% of the market value of the debt at default time.

We find again the same problem of lack of information for the parameter $\theta$ which is the fraction of the coupon which is paid during the observation period. In arbitrary way, we make it vary between 50% (basic scenario) to 30%.

Uncertainty parameters of the geometric Brownian motion ($\sigma$) and those of the Poisson process ($\lambda, J$) have been calibrated in order to reproduce at best the survival rate of firms after investment, as well as the fraction of bankruptcy at the end of the observation period (fixed at 1 year as it is observed). We obtain a survival rate at 5 years (62%) which correspond to the survival rate observed at 3 years. Note however that the observed rate is the one prevailing
for all firms that is for both levered and unlevered firms, while the model refers to levered firms. It is quite intuitive that if restricting to levered firms, the survival rate should be lower. The fraction of firms that are liquidated at the end of the observation period (66%) is in the range of the observed values (62 – 72%).

The debt/equity ratio expressed in market values is 27% at the time of investment. We do not have observations of this ratio. Indeed the sole available data concern the debt/equity ratio at any time of the firm life after investment. This latter ratio is between 60% and 140% for French firms and 25% and 55% for US firms.

**Introducing an observation period.**

To get the effects of the existence of an observation period, one can study the firm behavior when introducing an observation period without changing the financial burden of the firm (no cut in the coupon paid to creditors nor financial distress costs). We compare the behavior of this firm (first column in Table 3) with that of a firm for which there is no observation period (first column of Table 2).

The existence of an observation period provides an incentive for firms to get into default earlier than they do with no observation period, which is consistent with analytical results. The default trigger is significantly affected by the existence of an observation period while the other parameters of the model have only a marginal effect. Moreover, the observation period provides incentives for firms to invest earlier.

A smaller investment trigger and a higher default and liquidation trigger have the immediate effect to reduce the survival rate from 75% to 61%. Since the firm goes bankrupt for a rather high price, the ratio between the market value of debt and its face value is high at the time of default which means that creditors can get back a larger part of their claims. Firms can then have a larger debt (32%) at smaller rates (12.7%). By fixing the financing structure (28%), we show that in the absence of observation period, the interest rate the firm faces would be even smaller (12.3%) ; nevertheless, this combination debt-interest rate would not be optimal for shareholders (see figure 2.b).
The default procedure

During the observation period firms benefit from a cut in the coupon and must pay a cost of financial distress. The introduction of the cut in the coupon and of the cost of financial distress provides an incentive for firms to invest later without affecting the default trigger. The survival rate at 5 years is higher. Since the coupon is no longer fully paid during the observation period, banks require a higher interest rate for the same level of debt. In Table 4, the whole set of variants has been redone by fixing the financing structure. We observe that a fall in the part of the coupon that is paid (from 50% to 30%) for a given debt (27%) translates into interest rates that are significantly higher (13.9% against 13.4%). Finally, the optimal behavior of the firm, when the part of the coupon it has to pay is cut, consists into reducing debt while the interest rate becomes higher.
Table 3: Investment behavior, financing structure, default and liquidation decisions with a bankruptcy procedure.

<table>
<thead>
<tr>
<th></th>
<th>Observation period with no financial consequences</th>
<th>Basic scenario</th>
<th>More uncertainty</th>
<th>Increase in tax</th>
<th>Cut in the part paid of the coupon during default</th>
<th>Increase in the costs of financial distress</th>
<th>Increase in the liquidation costs</th>
<th>Lengthening of the observation period</th>
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</thead>
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<tr>
<td><strong>Institutional and economic environment of the firm</strong></td>
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<td></td>
</tr>
<tr>
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</tr>
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<td>0.8 0.8 0.8 0.8 0.8 0.8 0.8 0.8</td>
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<td></td>
<td></td>
</tr>
<tr>
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<td>0.1 0.1 0.1 0.1 0.1 0.1 0.1 0.1</td>
<td>0.1 0.1 0.1 0.1 0.1 0.1 0.1 0.1</td>
<td>0.1 0.1 0.1 0.1 0.1 0.1 0.1 0.1</td>
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<td></td>
</tr>
<tr>
<td>$J$</td>
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<td>0.1 0.1 0.1 0.1 0.1 0.1 0.1 0.1</td>
<td>0.1 0.1 0.1 0.1 0.1 0.1 0.1 0.1</td>
<td>0.1 0.1 0.1 0.1 0.1 0.1 0.1 0.1</td>
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</tr>
<tr>
<td>$\alpha=\lambda(J-1)$</td>
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<td>0.02 0.02 0.02 0.02 0.02 0.02 0.02 0.02</td>
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<td>0.5 0.5 0.5 0.5 0.5 0.5 0.5 0.5</td>
<td>0.5 0.5 0.5 0.5 0.5 0.5 0.5 0.5</td>
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<td>0.1 0.1 0.1 0.1 0.1 0.1 0.1 0.1</td>
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<td></td>
<td></td>
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</tr>
<tr>
<td>$B$</td>
<td>0.1 0.1 0.1 0.1 0.1 0.1 0.1 0.1</td>
<td>1 1 1 1 1 1 1 1</td>
<td>1 1 1 1 1 1 1 1</td>
<td>1 1 1 1 1 1 1 1</td>
<td>1 1 1 1 1 1 1 1</td>
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<td>1.63 1.65 1.67 1.63 1.66 1.59 2.19 2.39</td>
<td>1.63 1.65 1.67 1.63 1.66 1.59 2.19 2.39</td>
<td>1.63 1.65 1.67 1.63 1.66 1.59 2.19 2.39</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Behavior of the firm**

- $P_I$: 13.6 14.4 16.8 13.5 14.8 15.3 15.4 14.7
- $P_D$: 1.63 1.63 1.65 1.67 1.63 1.66 1.59 2.19
- $P_D / P_I$: 12% 11.3% 9.8% 12.4% 11% 10.9% 10.3% 14.9%

- Debt/Equity at the time of investment: 32% 27% 23% 25% 27% 24% 26%
- Market value of debt at the time of default/face value of debt: 42.2% 35.5% 35.5% 34.9% 32.6% 35.0% 32.3% 43.7%
- Interest rate (coupon / face value of the debt): 12.7% 13.4% 15.1% 13.9% 13.6% 13.2% 13.4% 13.5%
- Survival rate at 1 year: 99% 99% 98% 99% 99% 99% 99% 98%
- Survival rate at 3 years: 80% 81% 76% 79% 81% 82% 83% 75%
- Survival rate at 5 years: 61% 62% 56% 60% 63% 63% 64% 56%
- % de liquidation at the end of the observation period: 66% 66% 67% 66% 66% 66% 66% 71%
Table 4: Investment behavior, financing structure, default and liquidation decisions with a bankruptcy procedure for a given level of debt.

<table>
<thead>
<tr>
<th>0</th>
<th>Basic scenario</th>
<th>More uncertainty</th>
<th>Increase in tax</th>
<th>Cut in the part paid of the coupon during default</th>
<th>Increase in the costs of financial distress</th>
<th>Increase in the liquidation costs</th>
<th>Lengthening of the observation period</th>
</tr>
</thead>
<tbody>
<tr>
<td>Institutional and economic environment of the firm</td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$R$</td>
<td>0.04</td>
<td>0.04</td>
<td>0.04</td>
<td>0.04</td>
<td>0.04</td>
<td>0.04</td>
<td>0.04</td>
</tr>
<tr>
<td>$\tau$</td>
<td>0.3</td>
<td>0.3</td>
<td>0.4</td>
<td>0.3</td>
<td>0.3</td>
<td>0.3</td>
<td>0.3</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>0.1</td>
<td>0.1</td>
<td>0.1</td>
<td>0.1</td>
<td>0.1</td>
<td>0.1</td>
<td>0.1</td>
</tr>
<tr>
<td>$J$</td>
<td>0.8</td>
<td>0.8</td>
<td>0.8</td>
<td>0.8</td>
<td>0.8</td>
<td>0.8</td>
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<td>0.3</td>
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<tr>
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<td>0.1</td>
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</tr>
<tr>
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<tr>
<td>$T$</td>
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<td>1</td>
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<td>1</td>
<td>1</td>
<td>1</td>
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</table>

Behavior of the firm

$P_I$ | 14.4 | 15.6 | 15.0 | 14.3 | 15.3 | 14.7 | 14.5 |

$P_D$ | 1.63 | 1.80 | 1.48 | 1.70 | 1.66 | 1.70 | 2.24 |

$P_D / P_I$ | 11.3% | 11.6% | 9.9% | 11.8% | 10.9% | 11.5% | 15.5% |

Debt/Equity at the time of investment | 27% | 27% | 27% | 27% | 27% | 27% | 27% |

Market value of debt at the time of default/face value of debt | 35.5% | 37.3% | 32.6% | 33.4% | 35.0% | 33.4% | 44.1% |

Interest rate (coupon / face value of the debt) | 13.4% | 15.7% | 13.1% | 13.9% | 13.2% | 13.8% | 13.6% |

Survival rate at 1 year | 99% | 93% | 99% | 99% | 99% | 99% | 99% |

Survival rate at 3 years | 81% | 72% | 83% | 80% | 82% | 80% | 76% |

Survival rate at 5 years | 62% | 52% | 65% | 61% | 63% | 62% | 56% |
The costs of financial distress do not affect interest creditors get during the observation period; therefore, they have nearly no effect on the financing structure and on the interest rate. Nevertheless, they affect the investment trigger: firms decide to invest later.

If the observation period gets longer, the firm invests later and default earlier which reduces survival rates. It marginally affects the optimal financing of the firm which reduces slightly its debt at a little higher interest rate.

**Liquidation**

At the end of the observation period, the firm is liquidated if the price is less than the default trigger. Shareholders do not get anything and creditors get the value of the firm less the liquidation costs. The higher these costs, the later both investment and default. They have the same level of interest rate but for smaller debt levels. For a same financing structure (see Table 4), they would face a higher interest rate. Such a mechanism is not perfectly comparable with that exhibited in the case of a reduction of the part paid of the coupon. In both cases, creditors' protection is smaller but here nobody gets any benefits from an increase in the liquidation costs while a cut in the part paid of the coupon benefit to shareholders.

**V. Synthesis of the results and conclusion**

**Creditors protection and risk taking by firms**

This paper seeks to appraise whether a change in the bankruptcy law could provide incentive for firms to engage into riskier (but more profitable) activities. Here, a better creditor's protection translates into either a cut in liquidation costs or by an increase in part paid of the coupon during the observation period. In both cases indeed, for a given price level, firms accept to invest for a higher level of uncertainty. We have therefore shown that an increase in creditor's protection would encourage firms to invest in riskier
investments. The better creditor's protection gives them the possibility to have a higher debt at similar or smaller interest rates.

Practically, it means that any policy that makes it easier for the creditor to attract the net assets of the firm in case of liquidation would improve the relationship between creditors and shareholders. Note however that is also translates into a smaller survival rate: on the one hand firms getting involved into riskier projects are defaulting more; on the other hand, default and liquidation are decided for higher price levels. The efficiency of such a policy cannot only be measured through the average observed default rate but also through the initial risk taking.

Studying the link between part paid of the coupon during the observation period and risk taking we obtain that nay fall of this part paid results must come with an increase in the part of the net assets of the firm the creditors get in case of liquidation to prevent firms to engage into even less risky projects.

**The role of the observation period**

An important result of our model is that the existence of a delay between default and liquidation or continuation provides the firms with an incentive to inter into default earlier. This translates into a significant reduction of the survival rates. The observation period is therefore costly and can only be justified if firms use it to reorganize efficiently. We reach a boundary of the model developed here which does not take into account the active behavior of firms which reorganize during the observation period.

**Transitory effects versus long run effects**

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4 To make it easier for the creditor to attract the net assets of the firm can be an harmonization or a simplification of the collateral laws. If we only care on claims hold by banks, it can also be the renunciation by the state to its prioritary claims, but we then ignore the social cost of such a policy.
So far, we have analyzed the long run effects of a change of the law on firms' financial distress. Our model describes the effects of the law on the whole set of decisions the firms takes from the birth of the firm. What would be the effects of a change in the law on firms that already exist?

If the terms of the contracts between creditors and shareholders are fixed, one shows easily that any creditor protection strengthening would favor creditors without affecting shareholders. In particular, the default trigger remains unchanged. If the terms of the contracts can be changed, firms should get smaller interest rates. We reach here another boundary of our model in which a renegotiation period after period of the contract between creditors and shareholders cannot be considered.
References


Appendix:

Equity value at the time of default

At the time the firm defaults, its equity value is such that:

\[
E^D(P) = \int_0^T \left( E_i(P(t)) - (C + \theta R + \varphi) \right)(1-\tau)e^{-rt} dt + e^{-rt} \int_0^T E(P(t + T)) df(P(t + T))
\]

With

\[
\int_0^T E(P(t + T)) df(P(t + T)) = (1-\tau) \left( \Phi^-_1 \right) \frac{P}{\delta} - \Phi^- \frac{C + R}{r} + \Phi^- B_2 P^B_1
\]

Where

\[
\Phi^-_1 = e^{\alpha \sigma T} \sum_{k=0}^{\infty} p_k(\lambda T) e^{\frac{k}{2} \sigma \nu T} (1 - \phi(v_k^1)) \quad \text{With} \quad v_k^1 = \frac{-\alpha}{\sigma \sqrt{T}} - \frac{k}{2} \ln J - \frac{1}{2} \sigma \sqrt{T}
\]

\[
\Phi^-_2 = e^{\beta \alpha (\beta_2 - 1) \sigma^2 T} \sum_{k=0}^{\infty} p_k(\lambda T) e^{\frac{k}{2} \sigma \nu T} (1 - \phi(v_k^2)) \quad \text{With} \quad v_k^2 = \frac{-\alpha}{\sigma \sqrt{T}} - \frac{k}{2} \ln J - \beta_2 \frac{1}{2} \sigma \sqrt{T}
\]

And

\[
\int_0^T \left( E_i(P(t)) - (C + \theta R + \varphi) \right)(1-\tau)e^{-rt} dt = \int_0^T \left( P e^{(r-\delta)t} - (C + \theta R + \varphi) \right)(1-\tau)e^{-rt} dt
\]

\[
= \left[ \frac{1-e^{-\delta T}}{\delta} P - \frac{1-e^{-rT}}{r} (C + \theta R + \varphi) \right] (1-\tau)
\]

We finally have:

\[
E^D(P) = \left[ \frac{1-e^{-\delta T}}{\delta} P - \frac{1-e^{-rT}}{r} (C + \theta R + \varphi) \right] (1-\tau)
\]

\[
+ e^{-rt} \left[ (1-\tau) \left( \frac{(\Phi^-_1) P}{\delta} - \frac{\Phi^- (C + R)}{r} \right) + \Phi^- B_2 P^B_1 \right]
\]

Determination of the default trigger
The default trigger is determined by shareholders that maximize equity value. Optimality conditions are:

\[ E(P_D) = E^D(P_D) \]

\[ \frac{\partial E}{\partial P} \bigg|_{P=P_0} = \frac{\partial E^D}{\partial P} \bigg|_{P=P_0} \]

\[ E(P_D) = E^D(P_D) \]

\[ \Leftrightarrow \]

\[ \left[ 1 - e^{-(\delta+\lambda(J-1))T} \right] \frac{1}{\delta} P_D - \frac{1 - e^{-\tau T}}{r} (C + \theta R + \varphi) \right] (1 - \tau) \]

\[ + e^{-\tau T} \left[ \left( \frac{P_D \Phi^{-1}}{\delta} - \Phi^{-} \frac{C + R}{r} \right) (1 - \tau) + \Phi^{-} B_2 P_D^{\beta_2} \right] \]

\[ = \left( \frac{P_D}{\delta} - \frac{C + R}{r} \right) (1 - \tau) + B_2 P_D^{\beta_2} \]

\[ \frac{\partial E}{\partial P} \bigg|_{P=P_0} = \frac{\partial E^D}{\partial P} \bigg|_{P=P_0} \]

\[ \Leftrightarrow \]

\[ (1 - \tau) \frac{1}{\delta} e^{-(\delta+\lambda(J-1))T} + e^{-\tau T} \left[ \left( \frac{\Phi^{-1}}{\delta} \right) (1 - \tau) + \Phi^{-} B_2 P_D^{\beta_2 - 1} \right] \]

\[ = \frac{(1 - \tau)}{\delta} + B_2 P_D^{\beta_2 - 1} \]

The solution of this 2 equations 2 unknowns system is given in the text.

Debt value with a bankruptcy procedure
\[ D^\beta(P) = \int_0^T (\theta R)e^{-\theta s} ds + e^{-\theta T} \bigg[ \int_{P_0}^P D(P) df(P(T)) + (1-b) \int_{P_0}^P V^U(P(T)) df(P(T)) \bigg] \]

With

\[ \int_0^T (\theta R)e^{-\theta s} ds = \frac{\theta R}{r} \left( 1 - e^{-\theta r} \right) \]

\[ \int_{P_0}^P D(P(T)) df(P(T)) = \frac{R}{r} \Phi^- + C_2 P_D^2 \Phi^- \]

\[ \int_0^{P_0} V^U(P(T)) df(P(T)) = (1-\tau) \int_{P_*}^P \frac{P}{\delta} \left( \frac{P_A}{r} - C \right) \left( \frac{P_A}{P_A} \right) \beta^2 df(P(T)) \]

\[ = (1-\tau) \left[ \Phi^+ \frac{P_D}{\delta} - \Phi^+ \frac{C}{r} \right] \left( \frac{P_A}{\delta} - C \right) \left( \frac{P_D}{P_A} \right) \beta^2 \]

\[ - (1-\tau) \left[ 2 \Phi^+ \frac{P_D}{\delta} - \Phi^+ \frac{C}{r} \right] \left( \frac{P_A}{r} - C \right) \left( \frac{P_D}{P_A} \right) \beta^2 \]

With

\[ \Phi^+ = e^{\alpha T} \sum_{k=0}^\infty p_k(\lambda T)e^{\alpha n_j} (\phi(v_k)) \]

\[ \Phi^+ = e^{(\alpha T)} \sum_{k=0}^\infty p_k(\lambda T)e^{\alpha n_j} (\phi(v_k)) \]

\[ \Phi^+ = \sum_{k=0}^\infty p_k(\lambda T)(\phi(v_k)) \]

\[ \Phi^+ = e^{\alpha T} \sum_{k=0}^\infty p_k(\lambda T)e^{\alpha n_j} (\phi(v_k)) \]

With \( v_k^A = \frac{\alpha}{\sigma} \sqrt{T} - \frac{k \ln J}{\sigma T} - \frac{1}{2} \frac{\ln P_a}{\sigma T} + \frac{\ln P_a - \ln P_a}{\sigma T} \)
\[ \Phi^{a^2} = e^{\beta_2 (\beta_1 \sigma^2) / 2} \sum_{k=0}^{\infty} p_k (\lambda T) e^{k \ln J} \left( \phi(v^A_{i^2}) \right) \] With

\[ v^A_{i^2} = -\frac{K}{\sigma \sqrt{T}} - \frac{k}{\sigma \sqrt{T}} \left( \beta_2 - \frac{1}{2} \right) - \frac{\ln P_A - \ln P_D}{\sigma \sqrt{T}} \]

\[ \sum_{k=0}^{\infty} p_k (\lambda T) (\phi(v^A_{i^2})) \] With

\[ v^A_{i^2} = -\frac{K}{\sigma \sqrt{T}} - \frac{k}{\sigma \sqrt{T}} \left( \beta_2 - \frac{1}{2} \right) - \frac{\ln P_A - \ln P_D}{\sigma \sqrt{T}} \]
Available data on firm financing, default and liquidation