Collaborative Networks in Experimental Triopolies

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Preliminary draft. Comments are welcome.

Abstract

This paper experimentally investigates the interdependence between market competition and endogenously emerging inter-firm collaboration. We restrict attention to arrangements resulting from bilateral collaboration agreements that typically characterize real world applications in which the activity concerned is a core activity of the partnering firms and risk sharing, contract enforcement and protection of proprietary knowledge are central issues. We rely on a baseline model by Goyal and Joshi (2003) which formalizes the strategic formation of collaborative networks between firms that are competing on the same product market. This model predicts strategically stable patterns of inter-firm collaboration which are empirically observed but have been ruled out in the previous theoretical literature. In a two-stage game, firms decide to form bilateral collaboration links, whose formation is costly but reduces marginal production costs, before they compete in quantity on the market. We report the results of a series of experiments. The first experiment is designed as a straightforward theory-test simulating a one-shot interaction. We manipulate the cost of link formation in different treatments. Our data almost perfectly match the predictions for both stages whenever the link formation costs are extreme and the predicted networks symmetric (empty or complete networks). In the case of intermediate link formation costs where the predicted networks are asymmetric, subjects rarely form asymmetric networks. When they do, observed and predicted quantities are less in accordance than for symmetric networks. Collusion cannot account for the observed behavior. In our second experiment we reject the conjecture that these findings are driven out by experience in a setting in which we increase the implemented number of repetitions of the two-stage game. Finally, in our third experiment we reduce the complexity of the setting by transforming the original two-stage game into a one-stage game where the formation of inter-firm networks directly

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determines firms’ payoffs. These are derived from assumed equilibrium market outputs on the
here absent competition stage. In this case, observed networks coincide with the predicted ones
indicating that experimental subjects’ limited capacity to foresee the outcomes of the market
stage may be driving the earlier discrepancies.

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*Keywords:* Endogenous formation of networks; Cournot competition; Collusion; Experiments.
1 Introduction

The globalization of markets has changed the nature of operations of the firm. Rather than from internal capabilities and market position, strategic advantages do increasingly derive from a firm’s ability to cooperate with otherwise competing firms to form business and information sharing networks. Numerous empirical studies suggest that collaborations between firms are more and more common especially in fast moving and knowledge intensive industries with short product cycles that need to reposition products and respond to changing market conditions and technological developments rapidly. Concretely, the number of strategic alliances has increased more than six-fold between 1989 and 1999 (Kang and Sakai, 2000; OECD, 2001), where the agreements that have been reached were also far larger in terms of scale and value compared to those of previous decades.

Besides drawing attention to the growing significance of strategic alliances, empirical studies have identified some of their prevalent structural features. When classifying collaborations by the number of members per alliance, the largest fraction comprises two firms (Faulkner, 1995; Vonortas, 1997), suggesting that many collaborations are bilateral.\(^1\) On the other hand, there is a small set of highly active firms involved in numerous collaborations, suggesting that individual collaborations rather than resembling isolated dyads are instead embedded within a broader network of collaborations. Here the collaborative activity is asymmetrically distributed between some few firms that are involved in most of the collaborative projects and others that get involved in just one (Delapierre and Mytelka, 1998).

These stylized features are captured by Goyal and Joshi (2003)’s oligopoly setting. Here firms strategically form bilateral collaboration links with otherwise competing firms. Concretely, the endogenously emerging horizontal collaboration of firms is modeled as a two-stage non-cooperative game: first, firms have the opportunity to form pairwise collaboration agreements according to an explicitly specified, non-cooperative formation protocol; second, firms compete on the same product market. The pairwise links involve a commitment of resources, i.e., forming a link is costly, and lead to a reduction in production costs.\(^2\) Taken together all pairwise links form a collaborative network which induces a distribution of costs across the firms in the industry. In such an environment the formation of collaboration links has two direct effects: collaboration lowers a firm’s costs but it also lowers the costs of the respective other firm which is a competitor. The impact of a collaboration link on firms not involved in it is adverse given the strengthened competitive position of collaborators.

\(^1\)Vonortas (1997) data set exclusively focuses on research alliances. Going beyond those, the claim can be qualified further: typically, bilateral relations seem to be predominant, where the incentive for collaboration originates from within the set of collaborating firms rather than being induced from the outside by state-subsidized programmes (see Delapierre and Mytelka, 1998; Hagedoorn, 1995). Furthermore, bilateral relations are considered most appropriate if the activity concerned is a core activity of the partners (Faulkner, 1995).

\(^2\)In Goyal and Joshi (2003) links are interpreted as contractual joint R&D agreements. Hence, this model captures the subset of strategic alliances which are non-equity based and in this sense relatively loose.
Consequently, the described environment features negative externalities across collaboration links.

In the theoretical literature there is a number of different approaches that model strategic alliances and try to explain their emergence. Among these the recent models focusing on the incentives of firms to collaborate and consequently on the stability of different endogenously emerging collaboration structures take the mainstream economic perspective on appraising voluntary firm collaboration according to which cooperation - quasi by definition - increases profits. This rather heroic postulate has however not been subjected to systematic testing, even though the empirical evidence on the impact of inter-firm collaboration on profitability is ambiguous (see Caloghirou, Ioannides, and Vonortas, 2003 for an overview). Experiments offer the opportunity to cleanly resolve this issue, i.e., to evaluate the general behavioral and consequently also the empirical adequacy of the suggested strategic models.

Our primary purpose in this paper is to see whether experimental data are consistent with the predictions of Goyal and Joshi (2003)’s model. We do so restricting attention to triopolies. The latter are a natural starting point as they are the least complex market structure which still allows for non-trivial network architectures. We test the considered model’s behavioral relevance concerning firstly the equilibrium network architectures it predicts. In order to provide a best-shot environment, we replace the simultaneous link-based network formation protocol assumed by Goyal and Joshi by a sequential link-based network formation protocol. Even though this complicates the derivation of theoretical predictions, it ensures their uniqueness. Besides, the sequentiality of the protocol prevents coordination failures otherwise encountered by subjects in the laboratory. The protocol we implement is a slightly modified version of the classical sequential link-based network formation protocol suggested by Aumann and Myerson (1988) and adapted by Watts (2002) featuring a recall mechanism which allows subjects in the laboratory to correct mistakes.

Next to evaluating the behavioral relevance of the model with respect to the networks it predicts, our second aim is to evaluate the behavioral relevance of its underlying assumption according to which collaboration on a pre-competitive stage does not induce players to collude on the product market. As previous experimental evidence proves that triopolies tend to produce outputs at the Nash-Cournot level we are able to clearly disentangle the impact of collaborative networks on collusive behavior. Obtained results may draw political implications. Although policy steps were taken to accommodate and actively promote strategic alliances, \(^3\) collaboration between competitors has been suspect of potentially serious effects on competition. Observing behavior in a controlled laboratory environment is a natural way to analyze the actual influence of collaborative agreements on the behavior of competitors in a market.

\(^3\)The United States and the European Union mobilized in the early 1980s to establish policies that both provided the necessary legal environment and actively promoted R&D cooperation (Vonortas, 1997).
This paper reports the results of three experiments. The first experiment consists of three different link formation cost treatments, where in each treatment subjects interact through six repetitions (rounds) of the two-stage game. From round to round, subjects are rematched according to a perfect stranger matching rule which guarantees the absence of repeated game effects. The rationale is to give best chances to the theory which models a one-shot interaction. Our data almost perfectly match the theoretical predictions in both stages of the game whenever predicted networks are symmetric. On the contrary, if asymmetric networks are predicted, experimental data are rarely in line with the predictions in both stages of the game. To further qualify and explain these findings we evaluate the impact of experience in a second experiment. In comparison with the first experiment, subjects’ time horizon is doubled but only one link formation cost treatment is considered, according to which predicted networks are systematically asymmetric. The findings of the second experiment lead us to conclude that a lack of experience does not account for the discrepancy between actual and predicted networks when the latter are asymmetric. Finally, in the third experiment, the complexity of the interactive environment is reduced as the two-stage game is boiled down to a single stage game. For a given network, subjects’ payoffs correspond to those achieved by producing equilibrium quantities on the (here absent) market stage. The third experiment consists of only one link formation cost treatment, according to which predicted networks are asymmetric. The match between actual networks and predicted networks is substantially increased in the third experiment which allows us to conclude that subjects’ limited capability to foresee the market outcomes is the appropriate explanation for the poor predictive success of strategic network formation whenever predicted networks are asymmetric.

In a general sense, our paper is a contribution to the study of group formation in oligopolies. The traditional approach to these issues has been in terms of coalitions, i.e., groups based on multilateral agreements. Models looking at the strategic formation of alliances in oligopolies recurring to coalitions are Bloch (1995), and Yi (1997). Opposed to this, the model we rely on stands in the tradition of theoretical work on strategic network formation (Aumann and Myerson, 1988; Jackson and Wolinsky, 1996; Bala and Goyal, 2000).4 Experimental research on the behavioral relevance of strategic network formation models is still young.5 To the best of our knowledge, the present paper is the first reporting experimental evidence concerning the endogenous formation of networks in an oligopoly setting.

4Both networks and coalitions can generally be used to explain the formation of strategic alliances. What needs to be delineated is the area of application of these models: the network approach (bilateral agreements) seems to be more appropriate where agreements are contractual and concern core activities whereas coalitions (multilateral agreements) seem appropriate where agreements are equity-based (as in joint ventures) or induced by policy programmes.

5Pantz and Ziegelmeyer (2005) provided results for the stylized symmetric connections model suggested by Jackson and Wolinsky (1996). Other experimental studies testing the behavioral relevance of strategic network formation models such as Callander and Plott (2005), Falk and Kosfeld (2003), Berninghaus, Ehrhart, Ott, and Vogt (2004) and Goeree, Riedl, and Ule (2005) rely on a different network conceptualization. The latter permits the unilateral formation of links which is not appropriate in the applied context of strategic alliances.
The rest of the paper is organized as follows. Section 2 introduces the theoretical background and derives some general results. Sections 3-5 respectively discuss the three different experiments. We conclude in Section 6. Proofs are provided in appendices.

2 Theoretical background

We consider a specific version of the two-stage game introduced by Goyal and Joshi (2003). A set of three firms, which are initially unconnected with zero fixed costs and identical constant returns to scale cost functions, form collaboration links on the first stage. A collaboration link is interpreted as a bilateral agreement to jointly invest in a cost-reducing technology, whose establishment and maintenance requires a positive fixed investment \( f > 0 \) referred to as link formation cost. Firms then compete in the product market on the second stage. Firms are assumed to produce homogeneous products and to compete in quantities. Opposed to Goyal and Joshi (2003) who suggest a simultaneous link formation protocol for the first stage of the game, we introduce a sequential link formation protocol.

2.1 The collaborative networks game

We first develop the required terminology and provide some definitions. We then provide a detailed but informal description of the sequential link formation protocol which corresponds to the first stage of the game, and finally we outline the features of the product market competition which corresponds to the second stage of the game.

2.1.1 Networks

Let \( N = \{1, 2, 3\} \) denote the set of ex-ante symmetric firms. A collaborative network is a list of which pairs of firms are linked to each other and is modeled as a non-directed graph. A network is thus a list of unordered pairs of firms \( ij \), where \( \{ij\} \in g \) indicates that firm \( i \) and firm \( j \) are linked under the collaborative network \( g \), \( i, j \in N \). By \( g^N = \{\{ij\} \mid i, j \in N, i \neq j\} \) we denote the set of potential links, i.e., the set of all subsets of \( N \) of size 2. \( G = \{g \subseteq g^N\} \) denotes the set of all possible collaborative networks on \( N \), and Table 1 displays the eight networks composing \( G \). Six of these eight networks reduce to the following two architectures, where architecture refers to a set of isomorphic networks: the single-link network, \( g^1 \), in which one of the three possible links within the group is formed, and the star network, \( g^* \), in which one firm (referred to as the center) is connected to any other (peripheral) firm whereas those in turn only maintain one link to the

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6Two networks are isomorphic if one can be attained from the other by relabelling firms.
center. The two remaining networks are the empty network, $g^0$, in which all firms remain isolated, and the complete network, $g^N$, in which all possible links are established.

Table 1: The eight possible networks.

<table>
<thead>
<tr>
<th>Network</th>
<th>g0</th>
<th>g1</th>
<th>g1</th>
<th>g*</th>
<th>g*</th>
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<td>3</td>
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<td>3</td>
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</table>

2.1.2 Stage 1: Link formation protocol

Initially, the three firms are unconnected. Starting from here, they can sequentially form bilateral collaboration links according to an exogenously fixed and commonly known order of play. The order of play is such that each possible pair is called exactly once. As there are three possible pairs, the order of play is given by a vector of three coordinates. We denote a complete move through the order of play as a sequence. Overall, there are six different orders of play, each consisting of the three pairs called in three consecutive periods. Within a pair, decisions are stated sequentially, where the firm deciding second is informed about the first firm’s decision before stating his own. The link between a called pair is formed if and only if both firms agree to form it. At the end of a period, all firms are informed about the then prevailing network but the firm which has not been called in the last period is not informed about the two other firms’ decisions. Afterwards, a new period starts and the next pair determined by the order of play takes a decision.

At the end of a sequence, all three firms are asked to state whether they want to modify the then prevailing network or not. If all three firms state that they do not want to make any modifications, the first stage is terminated, meaning that the network is definitive and the firms enter the production stage. This possibility of ending the first stage of the game corresponds to termination rule 1. Otherwise, an additional sequence is initiated in which all possible pairs are recalled according to the prevailing order of play. Firms then have the possibility to reconsider their respective collaboration links. Concretely, it is possible to form links that have not been formed yet, to delete links previously formed, or to keep the respective collaboration links unchanged.

After all possible pairs have been recalled, the following additional termination rule applies: if a collaborative network has not actually been modified in the course of a recall-sequence, i.e., the network at the end of the sequence is the same as at the end of the previous sequence, the first stage is terminated, the prevailing network is definitive and firms enter the production stage. This second possibility of ending the first stage of the game corresponds to termination rule 2. If the

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7In case the collaboration link is formed, the firm which has not been called in the last period infers that both called firms agreed on forming it.
network has been modified, every firm is again asked to state whether he wants to change the then prevailing network or not. In this case, the rules specified above apply.

Overall, the maximal number of sequences is five. Consequence of which is that after the fifth recall of all possible pairs, the first stage is terminated and firms enter the production stage. This last possibility of ending the first stage of the game corresponds to termination rule 3.

2.1.3 Stage 2: Market competition

Collaboration links lower marginal production costs. It is assumed that firm \(i\)'s marginal cost is linearly decreasing in its number of collaboration links. Formally, for a given collaborative network \(g\), firm \(i\)'s marginal production costs are given by

\[
c_i(g) = \lambda_0 - \lambda n_i(g),
\]

where \(\lambda_0 \geq 2\lambda\) denotes firm \(i\)'s marginal production costs when singleton, \(\lambda > 0\) denotes the cost reduction per link, and \(n_i(g)\) denotes the number of collaboration links that firm \(i\) maintains in network \(g\).\(^8\)

We presume quantity competition in a homogeneous product triopoly where the inverse demand function is given by

\[
p = \alpha - \sum_{i \in N} q_i,
\]

where \(q_i \geq 0\) denotes the quantity produced by firm \(i\) and \(\alpha > 0\) measures the size of the market.\(^9\) For a given collaborative network \(g\), firm \(i\)'s profit net of link formation costs when producing the quantity \(q_i\) is thus given by

\[
\pi_i(g) = (\alpha - q_i - q_j - q_k)q_i - (\lambda_0 - \lambda n_i(g))q_i - fn_i(g),
\]

where \(j, k \in N\), and \(i \neq j \neq k\).

2.2 Predictions

In this section, we first determine, for any given collaborative network, the quantities produced by the firms on the market stage. We do so by assuming, on the one hand, that firms maximize individual profit, and, on the other hand, that connected firms maximize their joint profit while competing against other firms. That is, we determine both the (Nash-Cournot) equilibrium quantities and the quantities produced under collusion of connected firms. Then, we characterize the

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\(^8\lambda_0 \geq 2\lambda\) ensures that \(c_i(g) \geq 0, \forall i \in N, \forall g \in G\).

\(^9\)We assume that, whatever the sum of the produced quantities, the market size is large enough so that prices are always positive.
networks supported by the subgame perfect equilibria of the collaborative networks game, i.e., the networks resulting from subgame perfect equilibrium play under the assumption that firms maximize individual profit. Collusive networks, i.e., those networks formed by firms that anticipate collusion on the production stage of the game, will be provided later only for the parametrization implemented in the laboratory (see Section 3). Finally, we provide a complete characterization of the networks which are efficient from an overall societal perspective, i.e., those networks that maximize total welfare.

2.2.1 Predicted market outputs

The equilibrium output can be written as

\[ q_i^E(g) = (\alpha - \lambda_0 + 3\lambda n_i(g) - \lambda \sum_{i \neq j} n_j(g))/4, \quad (1) \]

\( i \in N \) and \( g \in G \), where \( \alpha - \lambda_0 > 2\lambda \), so that, for any collaborative network, each firm produces a strictly positive quantity in equilibrium. Firm \( i \)'s equilibrium profits are given by

\[ \pi_i^E(g) = (q_i^E(g))^2 - f n_i(g), \forall i \in N, \forall g \in G. \quad (2) \]

We now derive the predicted quantities for the case in which the collaboration links formed on the first stage induce connected firms to collude when setting their market quantities, i.e., connected firms maximize their joint profit. We refine collusion by assuming that two firms with the same number of collaboration links produce the same quantity and that, for a given collaborative network, each firm’s collusive profit has to be strictly greater than its respective equilibrium profit.

If no collaboration link is established, the collusive outcome is, by definition, identical to the equilibrium one.

If only one collaboration link is established, quantities are set collusively by the two connected firms which compete against the isolated firm. Consequently, the market output is given by \((\alpha - \lambda_0 + 2\lambda)/3, (\alpha - \lambda_0 - \lambda)/3\) where the first element corresponds to the total quantity produced by the two connected firms. However, the collusive profit of any of the two connected firms which equals \(1/2 \) \( ((\alpha - \lambda_0 + 2\lambda)/3)^2 - f \), is strictly smaller than its equilibrium profit. Therefore, the two connected firms will not collude and the market output as well as the firms’ profits will be identical to the equilibrium ones.

If two collaboration links are established, collusion leads to the center of the star network being the only firm producing a strictly positive quantity, and this monopoly quantity equals \((\alpha - \lambda_0 + 2\lambda)/2\). As side payments are ruled out in our framework, each firm’s produced quantity has to be
large enough to render its collusive profit strictly larger than its equilibrium profit. This constraint does not determine how firms share the monopoly profit through allocating production. Specifying a collusive quantity for each firm can only be done in an ad-hoc way. However, there is a unique triple of collusive quantities which fulfills this constraint for the parametrization implemented in the laboratory. We will provide it after having introduced our laboratory setting.

Finally, if the complete network is formed, quantities are set collusively by the three connected firms, and the total quantity produced on the market is given by \( (\alpha - \lambda_0 + 2\lambda)/2 \). Each firm’s collusive profit, which equals \((1/12)(\alpha - \lambda_0 + 2\lambda)^2 - 2f\), is obviously strictly greater than its equilibrium profit.

### 2.2.2 Predicted networks

#### Subgame perfect equilibrium networks

Proposition 1 provides a complete characterization of the subgame perfect equilibrium networks, i.e., the networks supported by the subgame perfect equilibria of the collaborative networks game when firms maximize individual profit.

**Proposition 1**

(i) For \( f < \lambda (\alpha - \lambda - \lambda_0)/4 \), \( g^N \) is the unique subgame perfect equilibrium network. (ii) For \( \lambda (\alpha - \lambda - \lambda_0)/4 < f < \lambda (\alpha + \lambda - \lambda_0)/4 \), \( g^1 \) is the unique subgame perfect equilibrium network. (iii) For \( f > \lambda (\alpha + \lambda - \lambda_0)/4 \), \( g^\emptyset \) is the unique subgame perfect equilibrium network.

Under the questionable assumption that firms foresee the equilibrium market outputs but are **myopic** when forming collaboration links, the outcome of the collaborative networks game is a pairwise stable network in the sense of Jackson and Wolinsky (1996). For \( f < \lambda (\alpha - \lambda - \lambda_0)/4 \), \( g^N \) is the unique pairwise stable network. For \( f > \lambda (\alpha + \lambda - \lambda_0)/4 \), \( g^\emptyset \) is the unique pairwise stable network. Finally, for \( \lambda (\alpha - \lambda - \lambda_0)/4 < f < \lambda (\alpha + \lambda - \lambda_0)/4 \), \( g^1 \), where the first pair according to the order of play is connected, is the unique pairwise stable network (see Appendix 1). Thus, for a given link formation cost, subgame perfection and pairwise stability predict the same network (architecture), which we refer to as an incentive-compatible network.

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10When a myopic firm considers forming a collaboration link with another firm, he simply asks himself whether he may expect to be better off with this link than without it, given the previously existing structure.

11Goyal and Joshi (2003) extend Jackson and Wolinsky (1996)’s definition of pairwise stability by adding an individual rationality condition for participation in the network, namely that each firm should find it profitable to maintain its collaboration links in the network rather than not having any links. Relying on this extended definition does not alter the characterization of the pairwise stable networks (see Appendix 1).
Efficient networks

Taking the social planner’s perspective, we now assume that the set of collaborative relations among firms is a subject of design. However, produced quantities are still at the discretion of the firms, meaning that, for a given set of collaboration links, firms produce equilibrium quantities. For a given network $g \in G$, total welfare is defined as the sum of consumer surplus and aggregate profits of the firms:

$$W(g) = \frac{1}{2} \left( \sum_{i \in N} q_i^E(g) \right)^2 + \sum_{i \in N} \pi_i^E(g).$$

A network $g^{eff}$ is efficient if $W(g^{eff}) \geq W(g)$, for all $g \in G$. Proposition 2 provides a complete characterization of the socially efficient networks.

**Proposition 2**

(i) For $f < \left( \lambda(5(\alpha - \lambda_0) + 3\lambda) \right)/16$, $g^N$ is the unique efficient network. (ii) For $f = \left( \lambda(5(\alpha - \lambda_0) + 3\lambda) \right)/16$ and $g^*$ are the unique efficient networks. (iii) For $\left( \lambda(5(\alpha - \lambda_0) + 3\lambda) \right)/16 < f < \left( 5\lambda(\alpha + \lambda - \lambda_0) \right)/16$, $g^*$ is the unique efficient network. (iv) For $f = \left( 5\lambda(\alpha + \lambda - \lambda_0) \right)/16$, $g^*$ and $g^1$ are the unique efficient networks. (v) For $\left( 5\lambda(\alpha + \lambda - \lambda_0) \right)/16 < f < \left( \lambda(5(\alpha - \lambda_0) + 7\lambda) \right)/16$, $g^1$ is the unique efficient network. (vi) For $f = \left( \lambda(5(\alpha - \lambda_0) + 7\lambda) \right)/16$, $g^1$ and $g^0$ are the unique efficient networks. (vii) For $f > \left( \lambda(5(\alpha - \lambda_0) + 7\lambda) \right)/16$, $g^0$ is the unique efficient network.

For some link formation cost ranges, there is a tension between incentive-compatible and efficient networks. More precisely, if $f < \left( \lambda(\alpha - \lambda - \lambda_0) \right)/4$, both incentive compatibility and efficiency considerations lead to the complete network. If $\left( \lambda(\alpha - \lambda - \lambda_0) \right)/4 < f < \left( \lambda(\alpha + \lambda - \lambda_0) \right)/4$, there is a tension as the incentive-compatible architecture is the single-link network whereas the efficient network is the complete network. A tension also arises if $\left( \lambda(\alpha + \lambda - \lambda_0) \right)/4 < f < \left( \lambda(5(\alpha - \lambda_0) + 7\lambda) \right)/16$, as the incentive-compatible network is the empty network whereas the efficient network is either the star network or the single-link network. If $f > \left( \lambda(5(\alpha - \lambda_0) + 7\lambda) \right)/16$, both incentive compatibility and efficiency considerations lead to the empty network.

3 First experiment

In the course of a session, subjects go through six repetitions of the collaborative networks game in a perfect stranger condition which ensures that nobody meets any of the other subjects more than once. This design enables subjects to get accustomed to the structure of the game while ruling out repeated game effects. Furthermore, as pairwise stable networks are identical (isomorphic) to subgame perfect equilibrium networks for a given link formation cost, the design leads to the same prediction irrespective of subjects’ assumed degree of rationality in the formation process (however, subjects have to anticipate that equilibrium quantities will be produced on the market stage). For this reason we consider our first experiment a natural starting point to test the behavioral adequacy of Goyal and Joshi (2003)’s model in a controlled way.
3.1 Parametrization and specific predictions

In the laboratory we implement the above specified triopoly setting in the following parametrization: we assume a market size of $\alpha = 30$, a marginal production cost if singleton of $\lambda_0 = 14$, and a marginal cost reduction per collaboration link of $\lambda = 6$. Hence, for a given collaborative network $g \in G$, firm $i$’s profit is given by

$$\pi_i(g) = (30 - q_i - q_j - q_k)q_i - (14 - 6n_i(g))q_i - f n_i(g),$$

where $i, j, k \in N$, and $i \neq j \neq k$. We restrict the quantity space to the set of integers from 0 to 10, i.e., $q_i \in \{0, 1, \ldots, 10\} \forall i \in N$, and we consider three different values for the link formation cost: a ‘negligible’ one ($f = 5$), a medium one ($f = 20$), and a high one ($f = 35$). As will be shown below, the chosen parametrization guarantees that the equilibrium quantities are - with the exception of the one produced by the center in $g^*$ - in the interior of the action space which allows to clearly identify both collusion and competitive behavior on the market stage.\(^{12}\) The chosen parametrization also leads to interesting predictions with respect to the networks as the efficient network remains unchanged whatever the considered value of the link formation cost while the subgame perfect equilibrium (pairwise stable) network and the collusive network changes conditional on whether the link formation cost is negligible, medium or high. Consequently, there is no tension between incentive-compatibility and efficiency in the negligible cost treatment, while a tension is present in the other two treatments and becomes stronger when the formation cost increases from medium to high.

3.1.1 Predicted market outputs

For a given network $g \in G$, part of the equilibrium quantities are given by Equation (1) but as outputs are integer valued in the experiment, asymmetric equilibrium configurations also arise. In an asymmetric equilibrium, two firms which have the same number of links produce different quantities but the industry output has to be identical to the one obtained in the symmetric equilibrium. Table 2 summarizes the equilibrium quantities and profits to any observable network assuming that in any given network each firm produces a strictly positive quantity.

As an illustration, if $g^*$ is the observed network then, in the symmetric equilibrium, the center produces 10 units whereas each of the two peripherals produces 4 units which leads to an industry output of 18 units. In an asymmetric equilibrium, if the center produces only 9 units then one of the peripherals has to produce 4 units and the other peripheral has to produce 5 units so that the

\(^{12}\) The parametrization has also been chosen to enable a future extension to quadropolies where the most important constraint has been to get low absolute equilibrium quantities also for $n = 4$ in order to be able to provide payoff tables to the subjects.
Table 2: Equilibrium quantities and profits (symmetric quantities and profits are bold).

<table>
<thead>
<tr>
<th>Network</th>
<th>Number of links</th>
<th>Equilibrium quantities</th>
<th>Industry output</th>
<th>Equilibrium profits</th>
</tr>
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<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>( f = 5 )</td>
</tr>
<tr>
<td>( g^N )</td>
<td>2</td>
<td>6.78</td>
<td>21</td>
<td>32.39, 46</td>
</tr>
<tr>
<td>( g^* )</td>
<td>2</td>
<td>9.10</td>
<td>18</td>
<td>80.90, 50.60</td>
</tr>
<tr>
<td>( g^1 )</td>
<td>1</td>
<td>3.45</td>
<td>18</td>
<td>7.11, 15</td>
</tr>
<tr>
<td>( g^2 )</td>
<td>1</td>
<td>6.78</td>
<td>15</td>
<td>37.44, 51</td>
</tr>
<tr>
<td>( g^3 )</td>
<td>0</td>
<td>1.2</td>
<td>15</td>
<td>1.2, 1.2</td>
</tr>
<tr>
<td>( g^4 )</td>
<td>0</td>
<td>3.45</td>
<td>12</td>
<td>16</td>
</tr>
</tbody>
</table>

industry output also equals 18 units. Table 2 clearly shows that firms which form collaboration links have a strong advantage over unconnected firms. In case of symmetric outcomes, the center in a star network has a 25% cost advantage compared to the two peripheral players which translates into a 56% market share for the center while the two peripheral firms equally divide the remaining market among themselves. In the single-link network there are two low cost (connected) firms, facing one high-cost (unconnected) firm. Here, the low cost firms have a relative cost advantage of 57% which gets reflected in these two firms dividing 93% of the market between themselves.

Table 3 summarizes the collusive quantities and profits for any observable network under the assumption that two firms with the same number of links produce the same quantity.\(^{13}\)

<table>
<thead>
<tr>
<th>Network</th>
<th>Number of links</th>
<th>Collusive quantities</th>
<th>Industry output</th>
<th>Collusive profits</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>( f = 5 )</td>
</tr>
<tr>
<td>( g^N )</td>
<td>2</td>
<td>5</td>
<td>15</td>
<td>55</td>
</tr>
<tr>
<td>( g^* )</td>
<td>2</td>
<td>9</td>
<td>13</td>
<td>125</td>
</tr>
<tr>
<td>( g^1 )</td>
<td>1</td>
<td>2</td>
<td>13</td>
<td>13</td>
</tr>
<tr>
<td>( g^2 )</td>
<td>1</td>
<td>7</td>
<td>15</td>
<td>44</td>
</tr>
<tr>
<td>( g^3 )</td>
<td>0</td>
<td>1</td>
<td>15</td>
<td>1</td>
</tr>
<tr>
<td>( g^4 )</td>
<td>0</td>
<td>4</td>
<td>12</td>
<td>16</td>
</tr>
</tbody>
</table>

Table 3: Collusive quantities and profits.

### 3.1.2 Predicted networks

We restrict the set of orders of play to the following six ones: 12, 23, 31; 23, 12, 31; 31, 12, 23; 12, 31, 23; 23, 31, 12; and 31, 23, 12. Within pairs, the firm quoted first in a pair is the one deciding first. Consequently, whatever the considered link formation cost, the subgame perfect equilibrium network is identical to the collusive network. The equilibrium outcome of the collaborative networks\(^{13}\)In the previous section, we determined that, when firms collude after having formed the complete network, the total quantity produced on the market is given by \((\alpha - \lambda_0 + 2\lambda)/2\). Given our parametrization, one third of this total quantity is not an integer. The collusive quantity provided in Table 3 for the complete network corresponds to the integer which maximizes firms’ joint profit.
game both under collusion and individual profit maximization on the market stage is the complete network if \( f = 5 \), it is the empty network if \( f = 35 \), and it is the single-link network if \( f = 20 \). In the latter case, which pair is connected depends on the assumed tie-breaking rule for firms which are indifferent between forming (keeping) and not forming (deleting) a collaboration link. In the following, we assume that an indifferent firm forms (keeps) with a strictly positive probability. Therefore, if \( f = 20 \), the single collaboration link is formed in period 1, which entails that myopia leads to the same predicted network as farsightedness (see Appendix 2 for a complete characterization of the subgame perfect equilibrium networks and the collusive networks for the chosen parametrization). Moreover, given that the aggregate production level is unaffected, it is easy to check that the equilibrium network structures remain unchanged irrespective of whether symmetric or asymmetric equilibria are considered. Table 4 summarizes the predicted networks under the assumption of individual profit maximization, collusion, and total welfare maximization.

<table>
<thead>
<tr>
<th>Link formation cost</th>
<th>Pairwise stable or subgame perfect equilibrium network</th>
<th>Collusive network</th>
<th>Efficient network</th>
</tr>
</thead>
<tbody>
<tr>
<td>Negligible</td>
<td>( g^N )</td>
<td>( g^N )</td>
<td>( g^N )</td>
</tr>
<tr>
<td>Medium</td>
<td>( g^1 )</td>
<td>( g^1 )</td>
<td>( g^N )</td>
</tr>
<tr>
<td>(first pair is connected)</td>
<td>(first pair is connected)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>High</td>
<td>( g^0 )</td>
<td>( g^0 )</td>
<td>( g^N )</td>
</tr>
</tbody>
</table>

Table 4: Predicted networks.

Our parametrization entails that the incentive-compatible/collusive network in the negligible as well as in the high cost treatment is symmetric, whereas the incentive-compatible/collusive network in the medium cost treatment is asymmetric. The medium cost treatment allows us to analyze the endogenous emergence of ex-post asymmetries, whereas the other two extreme treatments serve as a baseline, testing whether subjects respond to changes in the link formation cost and whether behavior complies to the predictions in cases in which the structural predictions are rather straightforward.

### 3.2 Experimental design

In the course of a session, subjects interact in groups of three for six repetitions of the collaborative networks game, where the latter are referred to as *rounds*. From one round to another, groups are rematched according to a *perfect stranger* matching rule which ensures that no two subjects interact more than once with each other. In a given round, each group is composed of a subject labelled ‘person 1’, a subject labelled ‘person 2’, and a subject labelled ‘person 3’. Labels are
randomly assigned to subjects at the beginning of a session and are kept for its entire duration. The order of play is varied between any two rounds such that subjects go through all six possible orders of play in a session. The succession and specification of orders is the same for all groups in all sessions and is provided together with the instructions, i.e., it is public knowledge. Subjects are confronted with the six orders of play in the following succession in all sessions: 12, 23, 31 (round 1); 23, 12, 31 (round 2); 31, 12, 23 (round 3); 12, 31, 23 (round 4); 23, 31, 12 (round 5); and 31, 23, 12 (round 6). Within pairs, we deviated from the natural order as the subject quoted first in a pair is the one deciding first. This ensures that all subjects are equally often the first to decide in a pair. In a given session, subjects face only one link formation cost.

Subjects are paid the sum of their payoffs from all rounds, where equilibrium payoffs (viz. payoffs according to Table 2 where the predicted networks are pairwise stable/subgame perfect equilibrium/collusive) are identical across treatments as the conversion rate of experimental points into euros has been adapted accordingly. More precisely, in the negligible cost treatment, 100 experimental points exchanged for 5 euros. In the medium cost treatment, 100 experimental points exchanged for 10 euros, and, in the high cost treatment, 100 experimental points exchanged for 13 euros.

During a round, subjects receive the following feedback on their screens: within a called pair, the subject deciding second is informed about the decision taken by the first subject and vice versa. The subject which is not part of the pair deciding in the current period is updated only about the outcome (and not about individual decisions) at the end of the period, i.e., he gets informed whether the link under consideration has been formed (kept) or not.

On the decision screen for the market stage, the network that has been finalized on the first stage, i.e., the network that is consequently payoff-relevant, is displayed. Subjects then enter their production decisions. Afterwards, a ‘result screen’ is displayed to every subject on which he is again reminded of the network in place, of his own chosen quantity, and which additionally informs him about the two others’ aggregate production level and his resulting payoff for the given round.

3.2.1 Practical procedures

Overall, we ran one session in both the negligible and high cost treatments, and three sessions in the medium cost treatment. Therefore, we have collected one independent observation in the extreme cost treatments (negligible and high), based on a total of 36 (18 × 2) subjects and three independent observations in the medium cost treatment, based on a total of 54 (18 × 3) subjects each. In the considered case of six rounds, the implemented matching rule required exactly 18 subjects per session, where each subject took part in only one session facing just one of the three link formation cost conditions.
The experiment has been conducted using the computerized network of the Max Planck Research Laboratory at Jena, Germany.\footnote{Based on an application developed by Boun My (2003) for Visual Basic.} In total 24 subjects were invited for each session, 18 of which took part, and all participants were students of Jena University. Additional subjects were invited to ensure that only subjects that fully understood the procedures (tested by a pre-experimental questionnaire) participated. More generally, only subjects reading for mathematical/physical (mathematics, statistics, computing, chemistry, engineering, physics), some life/environmental (biochemistry, biology), medical (including psychology) as well as social science (only economics, business administration, law) degrees were invited guaranteeing subjects’ acquaintance with abstract problems. Approximately 60% of the participants were male.

Subjects interacted via computer terminals, which were visually isolated from each other. Communication other than through taken decisions was not allowed. Subjects were fully informed about the rules of the game by written instructions which were turned into public knowledge by reading them out loudly. The instructions were neutrally formulated (subjects were referred to as ‘persons’ rather than as firms, and they were ‘forming connections’ and ‘choosing numbers’ rather than forming collaboration links and producing quantities). In addition to the written instructions in which benefits and costs were provided in functional form, each subject received three payoff tables, each specifying individual payoffs from the various output combinations subject to the own number of links (zero, one or two).\footnote{Each payoff table was printed in poster size (A3). The three tables were cella-taped onto the partition screens of each cubicle so that they were comfortably readable by subjects.} Consequently, each subject had complete information about the relationship between decisions and profits applying to himself and all others. Additionally, each subject was provided with a ‘history sheet’ for voluntary use, making it easy to keep track of own and others’ decisions and outcomes during the experiment if a subject wished to do so.

Already in their invitation subjects were informed that participation in the experiment was conditional on questionnaire performance and that there was no show-up fee.\footnote{Subjects were recruited and invited using ORSEE (Greiner, 2003).} Subjects, whose questionnaire results indicated that they had not sufficiently understood the game (those that made two or more mistakes), were replaced and paid the minimal compensation of 2 euros for answering the questionnaire. The questionnaire comprised ten questions, four checking the understanding of the payoff calculation and six more general questions concerning the link formation protocol, etc. The questionnaire was identical in all treatments and sessions. Overall, 77% of all invited subjects made no mistakes in the questionnaire. Due to no-show ups, we could not replace all subjects that made mistakes in all sessions. 91% of the subjects finally participating had made no mistakes in the questionnaire, the rest made one mistake. The latter we briefed individually about what their mistake has been before starting the experiment. Consequently, we are confident that all
participating subjects fully understood the rules of the game and the experimental procedures and were able to read the payoff tables. Participating subjects received 4 euros if they answered the questionnaire without any mistakes and 2 euros if they made one mistake. The amount earned for the questionnaire served as an initial endowment to cover potential later losses. As these could not generally be excluded, subjects were necessarily required to sign a letter of agreement before the start of the experiment informing them about the possibility of losses and asking them to cover losses from the money earned for the questionnaire.\textsuperscript{17}

After the questionnaire and the replacement of subjects, we took subjects through three practice rounds to familiarize them with the software. Here they were required to make entrances publicly announced by the experimenter. Own decisions were excluded by the software. This was done in order to disable subjects to gain experience about optimal strategies or others’ behavior. In the course of the practice rounds, subjects were taken through three of the possible networks (starting with the complete network, then moving to the star and the empty network). The production levels to be chosen on the market stage were set to 0 for all subjects in all practice rounds. After the practice rounds, the actual experiment was started. The sessions took approximately 1 hour and 30 minutes irrespective of the level of the link formation cost.

3.3 Results

Table 5 summarizes our findings at the aggregate level by providing, for each session, the relative frequencies of each network (architecture), and the associated average quantities depending on the number of links.

In the negligible link formation cost treatment all observed networks comply with the prediction (complete network), and there is a high adequacy of the observed average quantity compared to the equilibrium quantity. Hence, on average, after having formed all possible collaboration links, subjects do not collude on the market stage. This is a clear indication against the conjecture that the formation of collaboration links induces collusion, especially as the complete network is the most favorable set of collaborative relationships for collusion. Also in the high link formation cost treatment, there is a high fit between predicted and observed networks (success rate of 86%), and when the empty network is observed, the observed average quantity almost perfectly matches the predicted quantity.

In the medium cost treatment the data of all three sessions look very similar. We will therefore

\textsuperscript{17}By signing they also agreed to cover losses exceeding this amount by incomes from future experiments (which in fact never applied). One subject did not agree to sign and consequently could not take part to the experiment. We are confident that most of our subjects valued future participation more than having to cover a loss. The Jena participant database currently includes 1650 registered subjects, 13\% of which have just taken part to one experiment at the moment (which includes subjects that just registered, which are on average 70 subjects per month). Moreover, about 70\% of all registered subjects have taken part to 5 or more experiments. This shows that the fraction of subjects that potentially did not value future participation highly is for sure clearly below 10\%.}
<table>
<thead>
<tr>
<th>Link formation cost</th>
<th>$g^N$</th>
<th>$g^*$</th>
<th>Peripheral</th>
<th>One link</th>
<th>No link</th>
<th>$g^0$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Center</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Negligible</td>
<td>1.00: 6.83</td>
<td>0.00: —</td>
<td>—</td>
<td>0.00: —</td>
<td>0.00: —</td>
<td>—</td>
</tr>
<tr>
<td></td>
<td>(1.06; 108)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Medium - Session A</td>
<td>0.64: 6.68</td>
<td>0.31: 7.09</td>
<td>0.31: 5.23</td>
<td>0.05: 5.50</td>
<td>0.05: 3.00</td>
<td>0.00: —</td>
</tr>
<tr>
<td></td>
<td>(1.14; 69)</td>
<td>(1.97; 11)</td>
<td>(0.69; 22)</td>
<td>(0.58; 4)</td>
<td>(0.00; 2)</td>
<td>—</td>
</tr>
<tr>
<td>Medium - Session B</td>
<td>0.69: 6.48</td>
<td>0.20: 7.14</td>
<td>0.20: 4.64</td>
<td>0.11: 5.50</td>
<td>0.11: 2.00</td>
<td>0.00: —</td>
</tr>
<tr>
<td></td>
<td>(1.42; 75)</td>
<td>(1.77; 7)</td>
<td>(1.50; 14)</td>
<td>(1.19; 8)</td>
<td>(0.82; 4)</td>
<td>—</td>
</tr>
<tr>
<td>Medium - Session C</td>
<td>0.64: 7.16</td>
<td>0.28: 8.20</td>
<td>0.28: 5.05</td>
<td>0.08: 5.50</td>
<td>0.08: 2.67</td>
<td>0.00: —</td>
</tr>
<tr>
<td></td>
<td>(1.57; 69)</td>
<td>(1.87; 10)</td>
<td>(1.32; 20)</td>
<td>(0.84; 6)</td>
<td>(0.58; 3)</td>
<td>—</td>
</tr>
<tr>
<td>Medium - Overall</td>
<td>0.66: 6.77</td>
<td>0.26: 7.50</td>
<td>0.26: 5.02</td>
<td>0.08: 5.50</td>
<td>0.08: 2.44</td>
<td>0.00: —</td>
</tr>
<tr>
<td></td>
<td>(1.41; 213)</td>
<td>(1.89; 28)</td>
<td>(1.17; 56)</td>
<td>(0.92; 28)</td>
<td>(0.73; 9)</td>
<td>—</td>
</tr>
<tr>
<td>High</td>
<td>0.00: —</td>
<td>0.00: —</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>0.86: 3.98</td>
</tr>
<tr>
<td></td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>(1.05; 10)</td>
<td>(0.71; 5)</td>
</tr>
</tbody>
</table>

Table 5: Relative frequencies of observed networks and associated average quantities.

discuss the overall results. First, most of the observed networks (about 2/3 of the cases) comply with the efficient prediction as they are complete networks. The observed average quantity is again very well in line with the equilibrium quantity whenever a complete network has been formed. Second, about one quarter of the observed networks are star networks which do not meet any of the predictions. When a star network has been formed, the observed average quantities do not comply with the predicted quantities, neither for the center nor for the peripheral subjects, as their chosen quantities are much closer to each other than predicted. The same result is observed whenever a single-link network is formed which happened in about 8% of the cases.

Our findings in both the negligible and the high cost treatments speak in favor of equilibrium behavior. In the medium cost treatment, where the equilibrium network is asymmetric and in this sense non-trivial, subjects deviate from the equilibrium prediction in a way that is in fact considerably welfare-enhancing in the sense that the deviation induces a higher total welfare than the equilibrium total welfare. Table 6 provides, for each treatment, the average observed total welfare as well as the efficiency rate in case there exists a tension between the incentive-compatible network and the efficient network. In the latter case, the efficiency rate is defined as the difference between the average observed total welfare and the equilibrium total welfare divided by the difference between the maximal total welfare and the equilibrium total welfare. But clearly, the positive welfare implication can only be considered as an unintended side-effect as total welfare incorporates the surplus of consumers who are actually not present in the laboratory. In fact, by

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18 Under the negligible link formation cost, the average observed total welfare equals 98.55% of the equilibrium/maximal total welfare.
deviating from the equilibrium prediction, subjects achieve considerably lower average payoffs than the equilibrium ones (see below).

<table>
<thead>
<tr>
<th>Link formation cost</th>
<th>Average observed total welfare</th>
<th>Efficiency rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>Negligible</td>
<td>332.06</td>
<td>—</td>
</tr>
<tr>
<td>Medium</td>
<td>220.24</td>
<td>64.13%</td>
</tr>
<tr>
<td>High</td>
<td>120.42</td>
<td>1.12%</td>
</tr>
</tbody>
</table>

Table 6: Average observed total welfare and efficiency rates.

We resort to a linear mixed-effects model\(^{19}\) to estimate the produced quantities in the different observed networks, and their evolution over time.\(^{20}\) The fixed effects consist of an intercept, the continuous variables *Round*, *Total_Number_of_Links*, and *Own_Number_of_Links* as well as the interaction effects *Round* : *Total_Number_of_Links* and *Round* : *Own_Number_of_Links*. The random effects only appear at the individual level, and they consist of random intercepts and random slopes (random effects for *Own_Number_of_Links*). Assuming a within-subject first-order autoregressive form of the error term provides a substantially better fit of the data than the independent errors model (the estimated autocorrelation is 0.162).\(^{21}\) We additionally model non-constant variance by considering an exponential error model for the inter-individual variability. The results of the statistical analysis are displayed in Table 7.\(^{22}\)

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient estimate</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Intercept</td>
<td>3.641</td>
<td>&lt; 0.01</td>
</tr>
<tr>
<td><em>Round</em></td>
<td>0.072</td>
<td>0.082</td>
</tr>
<tr>
<td><em>Total_Number_of_Links</em></td>
<td>-0.063</td>
<td>0.809</td>
</tr>
<tr>
<td><em>Own_Number_of_Links</em></td>
<td>1.551</td>
<td>&lt; 0.01</td>
</tr>
<tr>
<td><em>Round</em> : <em>Total_Number_of_Links</em></td>
<td>-0.137</td>
<td>0.049</td>
</tr>
<tr>
<td><em>Round</em> : <em>Own_Number_of_Links</em></td>
<td>0.202</td>
<td>0.043</td>
</tr>
</tbody>
</table>

Random Effects: Intercept = 0.433; *Own_Number_of_Links* = 0.464

Number of observations: 540
Number of subjects: 90

Table 7: Results of the estimation of the produced quantities for the different observed networks.

According to the regression results, the estimated quantity for the empty network equals

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\(^{19}\)In each session, subjects interact for several rounds. We have therefore collected repeated measurement data, i.e., data where subjects have multiple measurements over time. Analyzing these data requires recognizing and estimating variability both between and within subjects. Mixed-effects models serve this purpose.

\(^{20}\)Acceptance or rejection of the null hypothesis is always based on a 5 percent level of significance.

\(^{21}\)Since our observations are taken longitudinally on the same subjects, within-subjects errors are likely to be auto-correlated.

\(^{22}\)A normal plot of the within-subjects standardized residuals indicates that the assumption of normality for the within-subjects errors is plausible. The considered model is the outcome of a selection process which started with a full model including the variable *Link_Formation_Cost* as well as all possible interaction effects, and then sequentially dropped insignificant variables on the basis of likelihood ratio tests. Even though it is more appropriate as the dependent variable belongs to the set \(\{0,\ldots,10\}\), we conjecture that a 2-sided censored Tobit regression model would not alter qualitatively our results and that even quantitative differences would be minor. Indeed, only 3.89% of our observations are censored.
3.641 units, and this estimated value is not significantly different from the symmetric equilibrium/collusive quantity of 4 units as the 95% confidence interval for the intercept is given by [3.268, 4.014]. Over the course of a session, this quantity does not evolve. If the single-link network is formed, the estimated quantity of the unconnected subjects slightly decreases from 3.504 units in the first round to 2.819 units in the last round whereas the estimated quantity of the connected subjects slightly increases from 5.257 units in the first round to 5.582 units in the last round. Over time, estimated quantities come closer to the equilibrium/collusive quantities. In the last round, according to their respective 95% confidence intervals, the estimated quantity of the unconnected subjects differs significantly from the symmetric equilibrium/collusive quantity contrary to the estimated quantity of the connected subjects. As the value 2 belongs to the 95% confidence interval of the estimation for unconnected subjects, the asymmetric equilibrium could be, in this case, a better predictor of the observations. If the star network is formed, the estimated quantity of the centers slightly increases from 6.873 units in the first round to 7.523 units in the last round whereas the estimated quantity of the peripherals slightly decreases from 5.120 units in the first round to 4.760 units in the last round. Again, even though estimated quantities come closer to the equilibrium/collusive quantities over time, the estimation for the centers is clearly far away from any of the predicted quantities. If the complete network is formed, the estimated quantity almost perfectly matches the symmetric equilibrium quantity as it slightly decreases from 6.736 units in the first round to 6.701 units in the last round.23 In summary, observed quantities in symmetric networks (empty or complete network) are very much in line with the equilibrium quantities which speaks against collusion. In contrast, observed quantities differ from the equilibrium quantities when an asymmetric network is formed (single-link or star network).

In the medium cost treatment, given the observed quantities, connected subjects in the single-link network achieve an average payoff of about 24 units whereas subjects in the complete network only achieve an average payoff 9 units. Consequently, even though there is a discrepancy between observed and predicted quantities, there remains a strong actual payoff improvement from forming the single-link network.24

Our discussion up to this point has not looked at whether or not the observed quantities do in fact meet the predicted equilibrium/collusive quantity triples, in the sense that each of the three quantities matches the equilibrium/collusive quantity. In the negligible, medium, and high cost treatment, respectively 5.56% (5.56%, 0%), 0.93% (7.41%, 0%) and 11.11% (19.44%, 0%) of subjects with two collaboration links are very large (see the estimated random slope).25 Average payoffs in both the negligible and the high cost treatment are almost identical to the symmetric equilibrium payoffs whereas in the medium cost treatment average payoffs amount to less than 60% of the equilibrium payoffs.
11.11%) of the observed outcomes comply with the symmetric equilibrium outcome (asymmetric equilibrium outcome, collusive outcome). Even though the asymmetric equilibrium does a better job in organizing the data, which is obviously due to its larger set of predicted outcomes, the overall level of compliance with any of the predicted outcomes is rather low in all treatments.

3.4 Discussion

We observe a very high predictive success in the extreme cost conditions, which is where the predicted networks are symmetric. There is a perfect fit of observed and predicted (complete) networks in the low formation cost treatment. In the high formation cost treatment we observe some single-link networks in the early rounds, but there is a clear convergence to the predicted empty network over time with a perfect fit of observed and predicted (empty) networks reached in the last two rounds of the session.

Opposed to that, the predictive success of the equilibrium network is very low in all sessions of the medium cost treatment. One possible explanation could have been that subjects deviate from the predicted production levels in asymmetric but not in symmetric networks. This is in fact the case. Whereas the average production levels almost perfectly match the predictions in both the complete and the empty network, we observe a systematic deviation from the predicted quantities whenever the formed network is asymmetric. Here the low-cost subjects (the centers in the star and the connected subjects in the single link network) do not fully exploit their comparative cost advantage by underproducing, while the high-cost subjects (the peripherals in the star and the singleton in the single link network) systematically overproduce. Consequently, the observed asymmetry in production levels (and payoffs) is lower than predicted. Irrespective of this systematic deviation in production levels, the actual average payoff in the single link network is still higher than in the complete network. Concluding, the actually observed discrepancy between actual predicted quantities in asymmetric as opposed to symmetric networks does not explain why we do not observe single-link networks.

25One possible conjecture is that subjects that have been comparatively disadvantaged by the formed network, e.g., being the singleton in the single link network, try to punish their interaction partners by overproducing on the production stage. But in fact our results qualitatively replicate results obtained by Rassenti, Reynolds, Smith, and Szidarovsky (2000) who implement Cournot competition with exogenously imposed individual cost asymmetries. In none of their 15 experiments do their average observations meet the qualitative prediction that produced quantities should decrease with increasing production costs. Instead, in one experiment they even observe that the highest cost subjects produce the highest average quantities. Concluding, the discrepancy between predicted and actual quantities that we observe seems to be a more general phenomenon in asymmetric settings [see also Mason and Phillips (1997) and Mason, Phillips, and Nowell (1992)] that does not result from punishment in response to endogenously emerging asymmetries.

26Most of the actual single-link networks are not formed strategically. We observe an (almost) immediate finalization of the predicted networks in the extreme formation cost treatments. The average number of sequences in the low formation cost treatment is 1 and 1.19 in the high formation cost treatment. Basically, the same holds for the medium cost treatment where groups on average (across the three sessions) go through 1.24 sequences before finalizing the network. This indicates that subjects make use of the recall possibility implied in the implemented formation protocol to correct errors but do not strategically exploit it to induce the predicted single link network in the medium cost
At least three explanations can be put forward to account for our data. The first is that six rounds are not enough for subjects to gain the necessary experience and understanding to be able to handle the non-trivial case they are confronted with in the medium cost treatment. The second is that subjects are unable to foresee the market output while forming collaboration links. The third is that subjects dislike asymmetric networks as they lead to asymmetric payoffs. Even after reducing the theoretically predicted payoff asymmetry between the connected and the singleton firms in the actually observed single-link networks, actual average payoffs remain considerably different between the two positions. However, this line of argumentation is not completely convincing. Indeed, we chose the six orders of play, which were public knowledge, so that, in the medium cost treatment, each subject would have been connected four times in case of equilibrium play. Payoffs would then have been identical for each subject, and, as already mentioned, much higher than the actual payoffs.

Next, we will test whether or not a higher level of experience is sufficient to dissolve the discrepancy between the observed and predicted networks in the medium cost treatment.

4 Second experiment

We subject the results attained in the previously described first experiment to a robustness check: as the first design has been set up to resemble a one-shot interaction, subjects have basically no possibility to get accustomed to the strategic environment. This said, it is an obvious conjecture that consistence between theoretical predictions and observed behavior can be augmented by increasing the number of rounds. The impact of more experience may be considerable given a decision environment whose strategic complexity is unquestioned. The second experiment complements the first by doubling the number of rounds (12 rounds). Due to operational constraints, we here move to a stranger matching rule, according to which it is guaranteed that no subject faces the same two other subjects more than once.\(^{27}\) By also here relying on random rematching we intend to minimize the impact of the reputation and signalling effects which allows us to still compare our data to the previously specified one-shot game predictions.

Apart from the change in the number of rounds (and the matching rule) the experimental procedures and design have been kept unchanged. Subjects just went through the first design’s succession of the six different orders of play twice. The total payoff was given by the payoffs earned in the twelve rounds (where we adjusted the conversion rate such that the incentives per session were kept unchanged across the first and second experiment).

In the second experiment we exclusively implemented the non-trivial medium cost treatment. In fact, in 2/3 of the observed single-link networks the singleton actively excluded himself.\(^{27}\)The matching in the first six rounds has been identical to the one in the first experiment.
(f = 20), where we ran three sessions. Consequently, we collected three independent observations based on a total of 54 (3 × 18) participants.28

4.1 Results

Table 8 summarizes our findings at the aggregate level by providing, for each session, the relative frequencies of each network (architecture), and the associated average quantities depending on the number of links.

<table>
<thead>
<tr>
<th>Relative frequency: Average quantity</th>
<th>$g^N$</th>
<th>Center</th>
<th>$g^*$</th>
<th>Peripheral</th>
<th>One link</th>
<th>$g^1$</th>
<th>No link</th>
<th>$g^0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>(quantity’s std. deviation; # obs.)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Session A: First six rounds</strong></td>
<td>0.69: 6.89</td>
<td>0.17: 6.67</td>
<td>0.17: 4.75</td>
<td>0.14: 5.50</td>
<td>0.14: 3.00</td>
<td>0.00: —</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(1.38; 75)</td>
<td>(1.03; 6)</td>
<td>(1.29; 12)</td>
<td>(1.18; 10)</td>
<td>(0.71; 5)</td>
<td>—</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Session A: Last six rounds</strong></td>
<td>0.75: 6.94</td>
<td>0.08: 6.33</td>
<td>0.08: 5.00</td>
<td>0.17: 5.83</td>
<td>0.17: 2.67</td>
<td>0.00: —</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.89; 81)</td>
<td>(1.53; 3)</td>
<td>(0.63; 6)</td>
<td>(0.83; 12)</td>
<td>(1.03; 6)</td>
<td>—</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Session B: First six rounds</strong></td>
<td>0.75: 6.68</td>
<td>0.22: 6.75</td>
<td>0.22: 5.56</td>
<td>0.03: 4.00</td>
<td>0.03: 3.00</td>
<td>0.00: —</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(1.42; 81)</td>
<td>(1.83; 8)</td>
<td>(1.55; 16)</td>
<td>(0.00; 2)</td>
<td>(—; 1)</td>
<td>—</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Session B: Last six rounds</strong></td>
<td>0.86: 6.72</td>
<td>0.03: 9.00</td>
<td>0.03: 4.50</td>
<td>0.11: 5.50</td>
<td>0.11: 2.75</td>
<td>0.00: —</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.80; 93)</td>
<td>(—; 1)</td>
<td>(0.71; 2)</td>
<td>(1.31; 8)</td>
<td>(0.50; 4)</td>
<td>—</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Session C: First six rounds</strong></td>
<td>0.75: 6.58</td>
<td>0.19: 7.43</td>
<td>0.19: 5.21</td>
<td>0.06: 4.75</td>
<td>0.06: 3.00</td>
<td>0.00: —</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(1.19; 81)</td>
<td>(1.62; 7)</td>
<td>(1.12; 14)</td>
<td>(0.50; 4)</td>
<td>(1.41; 2)</td>
<td>—</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Session C: Last six rounds</strong></td>
<td>0.83: 7.10</td>
<td>0.11: 7.25</td>
<td>0.11: 5.25</td>
<td>0.03: 8.00</td>
<td>0.03: 2.00</td>
<td>0.03: 4.33</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.79; 90)</td>
<td>(0.50; 4)</td>
<td>(0.89; 8)</td>
<td>(2.83; 2)</td>
<td>(—; 1)</td>
<td>(1.15; 3)</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Overall: First six rounds</strong></td>
<td>0.73: 6.71</td>
<td>0.20: 6.95</td>
<td>0.20: 5.21</td>
<td>0.07: 5.12</td>
<td>0.07: 3.00</td>
<td>0.00: —</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(1.33; 237)</td>
<td>(1.53; 21)</td>
<td>(1.35; 42)</td>
<td>(1.09; 16)</td>
<td>(0.76; 8)</td>
<td>—</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Overall: Last six rounds</strong></td>
<td>0.82: 6.92</td>
<td>0.07: 7.13</td>
<td>0.07: 5.06</td>
<td>0.10: 5.91</td>
<td>0.10: 2.64</td>
<td>0.01: 4.33</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.84; 264)</td>
<td>(1.25; 8)</td>
<td>(0.77; 16)</td>
<td>(1.34; 22)</td>
<td>(0.81; 11)</td>
<td>(1.15; 3)</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Overall: All rounds</strong></td>
<td>0.77: 6.82</td>
<td>0.13: 7.00</td>
<td>0.13: 5.17</td>
<td>0.09: 5.58</td>
<td>0.09: 2.79</td>
<td>0.01: 4.33</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(1.10; 501)</td>
<td>(1.44; 29)</td>
<td>(1.22; 58)</td>
<td>(1.29; 38)</td>
<td>(0.79; 19)</td>
<td>(1.15; 3)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 8: Relative frequencies of observed networks and associated average quantities.

The data of the first six rounds confirm the picture that has been drawn from the medium cost treatment in the first experiment. The observed frequencies of the different networks are on average across all sessions almost identical to those of the first experiment. The same holds with respect to the observed average quantities. Again, in symmetric networks these match the equilibrium quantities, while there is a systematic deviation in asymmetric networks (stars and single-link networks) confirming the result obtained in the first experiment according to which the observed difference in quantities between the different positions in the network is much lower than predicted. The second half of the second experiment allows us to investigate whether the compliance with the predictions increases when subjects accumulate more experience. A first finding is that star

2850% of the participants were male. Overall, 87% of all invited subjects made no mistakes in the questionnaire. We could replace all subjects that made mistakes in all sessions, i.e., all participating subjects made zero mistake in the questionnaire.
networks, which do not match any prediction, are driven out over time. But instead of enforcing the predicted single-link networks, we observe an increase in the relative frequency of complete networks over time.\textsuperscript{29} The observed frequency of single-link networks is unaffected by greater experience as their fraction is stable (at a very low level) across the first and second half of the second experiment. Concluding, a lack of experience does not seem to be a reasonable explanation for the discrepancy between predicted and observed networks when the formation cost is medium.

The results of a similar statistical analysis to the one performed in the previous section are displayed in Table 9.\textsuperscript{30}

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient estimate</th>
<th>$p$-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Intercept</td>
<td>3.579</td>
<td>$&lt; 0.01$</td>
</tr>
<tr>
<td>Round</td>
<td>0.042</td>
<td>$&lt; 0.01$</td>
</tr>
<tr>
<td>Total Number of Links</td>
<td>-0.357</td>
<td>$&lt; 0.01$</td>
</tr>
<tr>
<td>Own Number of Links</td>
<td>2.009</td>
<td>$&lt; 0.01$</td>
</tr>
</tbody>
</table>

Random Effects: Intercept = 0.825; Own Number of Links = 0.459

Number of observations: 648
Number of subjects: 54

Table 9: Results of the estimation of the produced quantities for the different observed networks.

According to the statistical analysis, if the single-link network is formed, the estimated quantity of the unconnected subjects slightly increases from 3.264 units in the first round to 3.726 units in the last round whereas the estimated quantity of the connected subjects slightly increases from 5.273 units in the first round to 5.735 units in the last round. If the star network is formed, the estimated quantity of the centers slightly increases from 6.925 units in the first round to 7.387 units in the last round whereas the estimated quantity of the peripherals slightly increases from 4.916 units in the first round to 5.378 units in the last round. In asymmetric networks, and even though at the absolute level estimated quantities are similar to the first experiment ones, they now only come closer to the equilibrium/collusive quantities over time for the subjects with most collaboration links. If the complete network is formed, the estimated quantity almost perfectly matches the symmetric equilibrium quantity as it increases from 6.568 units in the first round to 7.03 units in the last round. Like in the first experiment, observed quantities in the complete network are very much in line with the equilibrium quantities which speaks against collusion, whereas observed quantities differ from the equilibrium quantities when an asymmetric network is formed (single-link or star network). As before, actual average payoffs in the predicted single-link

\textsuperscript{29}Deviations from the incentive-compatible network lead to an overall increase in the total welfare. Compared to the medium cost treatment of the first experiment, the efficiency rate is even slightly higher with a level of 72.42\% (average observed total welfare of 226.54).

\textsuperscript{30}The here reduced model is smaller than the previous one as interaction effects are insignificant and allowing for heteroscedastic within-subject errors does not improve the estimation procedure. Like in the first experiment, assuming a within-subject first-order autoregressive form of the error term provides a substantially better fit of the data than the independent errors model (the estimated autocorrelation is 0.322).
networks (even though lower than the predicted ones) provide a strong incentive to reinforce the single-link network.

Looking at the compliance of quantity triples with the three benchmarks, we again find that most of the observed triples are different from both the equilibrium and the collusive triples (5.56% of the observed triples are identical to the symmetric triple, 12.04% are identical to the asymmetric triples, and only 0.93% are identical to the collusive triple across all sessions). Even though at a very low level, compliance increases from the first to the second half of the sessions which accounts for an overall higher level compared to the first experiment.

4.2 Discussion

The results of the second experiment substantiate the genuineness of the previously established discrepancy between observed and predicted networks in the medium cost treatment. Even after subjects have gained experience through more interactions, the observed fraction of the predicted single-link networks remains quite stable at a very low level. Again, irrespective of strong actual payoff incentives in their favor, single-link networks are in advanced rounds not much reinforced. The overall fraction of asymmetric networks decreases over time as star networks are driven out. The move mainly goes towards an increase in the fraction of complete networks as a result of which the total welfare level increases over time, overall moving beyond the level of the first experiment.

In summary, the first potential explanation for the discrepancy between actual and predicted network in the medium cost treatment has been ruled out by the findings of the second experiment. Below, we report on an experimental test of the previously mentioned second potential explanation, according to which the observed regularities are driven by subjects’ inability to foresee the market output while forming collaboration links. The third experiment also allows us to evaluate the degree to which inequity aversion can account for the discrepancy between actual and predicted networks in the medium cost treatment.

5 Third experiment

The third experiment tests whether the previously observed deviations from the predicted single-link network in the medium cost treatment can be attributed to subjects’ inability to foresee the market output while forming collaboration links. In order to test this conjecture we radically reduce the complexity of the interactive environment. Concretely, subjects interact with each other only on the first stage of the collaborative networks game. Individual payoffs associated to a given network are derived from (assumed) symmetric equilibrium play on the market stage. In other words, for a given network and depending on the number of collaboration links he has established,
a subject’s payoff in a given round is given by Equation (2) where the link formation cost equals 20.

Except for the absence of the second stage of the collaborative networks game in each round, the experimental design and the practical procedures remained identical to those of the second experiment. Experimental instructions described the link formation protocol in an exhaustive way, no market stage was mentioned but subjects were described the payoffs associated to each position in each of the eight possible networks. Subjects went through 12 rounds, and, from round to round, they were rematched according to a stranger matching rule. We ran one session, meaning that we collected one independent observation based on a total of 18 subjects.31

5.1 Results and discussion

We define two measures of predictive success for the incentive-compatible formation of networks: the relative frequency of observed networks which are single-link networks, and the relative frequency of observed networks which are single-link networks such that the first pair called according to the order of play is connected. The first measure is clearly weak in the sense that it allows for non-strategic network formation whereas, according to the second measure, the actual network has to be identical to the predicted one. Table 10 provides the levels of those two measures for the second and third experiment, and for the medium cost treatment of the first experiment.

<table>
<thead>
<tr>
<th>Rounds</th>
<th>Weak measure of predictive success</th>
<th>Strong measure of predictive success</th>
</tr>
</thead>
<tbody>
<tr>
<td>First experiment</td>
<td>1-6</td>
<td>8.33%</td>
</tr>
<tr>
<td>Second experiment</td>
<td>1-6</td>
<td>7.41%</td>
</tr>
<tr>
<td></td>
<td>7-12</td>
<td>10.19%</td>
</tr>
<tr>
<td>Overall</td>
<td></td>
<td>8.80%</td>
</tr>
<tr>
<td>Third experiment</td>
<td>1-6</td>
<td>61.11%</td>
</tr>
<tr>
<td></td>
<td>7-12</td>
<td>86.11%</td>
</tr>
<tr>
<td>Overall</td>
<td></td>
<td>73.61%</td>
</tr>
</tbody>
</table>

Table 10: Measures of predictive success for the incentive-compatible formation of networks.

The findings in the third experiment are in sharp contrast to those of the first two experiments. In the second half of the third experiment’s unique session, almost all actual networks are single-link networks and a large majority of them correspond exactly to the predicted one. The proportion of single-link networks in the first two experiments is about 9% and the proportion of actual networks which correspond exactly to the predicted one is below 5%. Therefore, the subjects’ inability to foresee the market output while forming collaboration links seems to be a reasonable explanation.

31 In the third experiment, 56% of the participants were male. Overall, 86% of all invited subjects made no mistakes in the questionnaire. All participating subjects made zero mistake.
to account for the discrepancy between observed and predicted networks when the link formation cost is medium. Still, far from all actual networks in Experiment 3 are identical to the predicted one.

In an attempt to account for the remaining inadequacy between actual and predicted networks, we resort to a generalized linear mixed-effects models to estimate the strong measure of predictive success in each experiment, and its evolution over time. The dependent variable is a logit transformation of the strong measure of predictive success. In addition to the intercept, there are 5 fixed effects: a dummy variable, labelled Experiment, which takes the value 0 in the medium cost treatment of the first experiment and in the second experiment, and the value 1 in the third experiment;\textsuperscript{32} the continuous variables Round and Round\textsuperscript{2}; and the interaction effects Experiment : Round and Experiment : Round\textsuperscript{2}. Random effects only appear at the group level and they only consist of random intercepts.\textsuperscript{33} The results of the regression are displayed in Table 11.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient estimate</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Intercept</td>
<td>-6.765</td>
<td>&lt; 0.01</td>
</tr>
<tr>
<td>Experiment</td>
<td>6.042</td>
<td>&lt; 0.01</td>
</tr>
<tr>
<td>Round</td>
<td>1.351</td>
<td>0.019</td>
</tr>
<tr>
<td>Round\textsuperscript{2}</td>
<td>-0.104</td>
<td>0.022</td>
</tr>
<tr>
<td>Experiment : Round</td>
<td>-1.365</td>
<td>0.036</td>
</tr>
<tr>
<td>Experiment : Round\textsuperscript{2}</td>
<td>0.118</td>
<td>0.020</td>
</tr>
<tr>
<td>Random Intercept:</td>
<td>0.061</td>
<td></td>
</tr>
</tbody>
</table>

Table 11: Estimation results of the strong measure of predictive success for the incentive-compatible formation of networks.

According to the estimation results, the strong measure in the third experiment systematically increases over the course of the session. This increasing pattern has an increasing slope which leads to an estimated strong measure of about 75% in the last round. On the contrary, the estimated strong measure in the second experiment has an inverse U shape, i.e, it first increases and then decreases which leads to an estimated strong measure of 0.40% in the last round (the maximum is achieved in round 6 and 7 and it equals 8.29%). Estimation results are rather accurate as they nicely match the actual levels. Indeed, in rounds 1-3, 4-9, and 10-12 of the third experiment, the actual (estimated) strong measure equals respectively 33.33% (33.50%), 44.44% (45.55%), and 72.22% (69.32%). In rounds 1-3, 4-9, and 10-12 of the second experiment, the actual (estimated) strong measure equals respectively 0.00% (1.35%), 19.44% (6.58%), and 0.06% (1.34%). We conjecture that, in case subjects had interacted for more than twelve rounds in the third experiment, almost all

\textsuperscript{32} A 1-2-3 fixed effect leads qualitatively to the same estimation results as the coefficients for the first and second experiment do not differ significantly.

\textsuperscript{33} The model is again the outcome of a selection process which started with a full model including all interaction effects. The non-significant ones were sequentially dropped. Assuming a within-group first-order autoregressive form of the error term does not provide a substantially better fit of the data than the independent errors model.
actual networks would have been identical to the predicted one in the last round. This conjecture is supported by the fact that the estimated strong measure in round 20 of the third experiment equals 99.00%.

By reducing the complexity of the interactive environment we tested the second conjecture that the previously observed discrepancy between observed and predicted networks results from subjects’ limited capability to foresee the market outcomes. This explanation seems to be appropriate as the adequacy between actual and predicted networks is substantially increased in the third experiment.

6 Conclusion

The results expounded in this article directly tie up to results attained in the numerous industrial organization experiments (see Huck, Normann, and Oechssler, 2004 for a recent survey). One of the behavioral facts that have been established in this literature is that the observation of collusive behavior depends on the market size. While collusion tends to be observed in duopolies, it becomes far less likely in triopolies and basically disappears starting with $n = 4$. Our set of experiments provides a stress test of these previous results as the possibility to form collaboration links on a pre-competitive stage may have been expected to induce collusion. This is behaviorally not substantiated in our experiments which confirm the immunity of triopolies to collusion.

We move beyond the large part of the existing experimental industrial organization literature which standardly implements symmetric designs, not accounting for the actual importance of persisting intra-industry differences in profitability that have been stressed by empirical research already early on (see Demsetz, 1973; Hatten and Schendel, 1977; Pakes, 1987; Schmalensee, 1987). Noteworthy exceptions are Fonseca, Huck, and Normann (2005); Rassenti, Reynolds, Smith, and Szidarovsky (2000); Mason, Phillips, and Nowell (1992); and Mason and Phillips (1997) who exogenously induce cost asymmetries. Instead, our set of experiments investigates endogenously emerging asymmetries.

Taken together the set of experiments reported in this article suggest a robust discrepancy between the observed and predicted networks as well as the produced quantities, whenever the latter are asymmetric. By sequentially testing several possible conjectures to explain this result we can draw the following conclusions: the statistical analysis conducted for the last experiment suggests that the observed regularities seem to be largely accounted for by subjects’ limited capability to foresee the outcomes of the market interaction stage when forming networks rather than by inequity aversion as the econometric predictions for a higher number of rounds suggest full convergence to the predicted network. In any case, even if inequity aversion played some role, it clearly has been a minor one. This maintains the external validity of our results, as inequity aversion may be
a legitimate concern for experimental participants but cannot reasonably be assumed to inform executive decisions with respect to firms’ strategic partnering.

Our third experiment shows that a radical reduction in the complexity of the interactive environment almost fully dissolves the discrepancy. Consequently, the observed finding that subjects do not form the predicted asymmetric single-link network in the medium link formation cost treatments has to mainly be attributed to the complexity of the decision environment. Indeed, the decision environment of the collaborative networks game is very demanding requiring an anticipation of the outcomes of a subsequently played simultaneous market game. Nevertheless, the implemented environment is far less intricate than the actual one, in this sense giving best chances to the predicted networks. The laboratory decision environment exclusively features strategic uncertainty, abstracting from all other types of uncertainty. In the light of the evidence reported in several empirical studies, we believe that our results draw attention to a genuine point. Abstracting from unresolved methodological and measurement problems, some empirical studies find evidence for a negative impact of strategic alliance participation on profitability (Berg, Duncan, and Friedman, 1982; Vonortas, 1997). Moreover, the observed proliferation of strategic alliances in the last decades is known to conceal high levels of strategic alliance instability and dissolution with failure rates at or higher than 50% (Porter, 1987; Harrigan, 1988; Kogut, 1988; Dacin, Hitt, and Levitas, 1997). Taken together with our results, this indicates several gaps in our understanding of the forces driving alliance formation pointing out a need for concerted experimental, empirical as well as theoretical research efforts. To what extent and under what conditions alliance formation is strategic, and what biases and systematic deviations on the part of the relevant decision-makers organize the observed regularities, needs to be further investigated.

In this initial experimental study we relied on the most basic framework modeling the formation of inter-firm collaborative networks. Future research needs to consecutively incorporate additional essential features of naturally occurring alliance formation processes. For example, in our experimental implementation the formation of a collaboration link - in line with Goyal and Joshi’s baseline model - directly and deterministically translates into a production cost reduction. A more realistic model suggesting itself for future experimental research is the one suggested by Goyal and Moraga-González (2001). Here firms - after having decided to collaborate - decide how much they want to contribute to the joint endeavor. Consequently, firms’ effective production costs are derived from their actual investments (incorporating spillovers across connected links). In a series of theoretical and experimental studies Amaldoss and Rapoport have together with several co-authors obtained some relevant results concerning the investment behavior of partnering firms when they are competing against other strategic alliances.34 Moving beyond Goyal and Moraga-

34Concretely, these studies shed light on how individual firm’s resource commitments to joint projects depend
González’s framework, the authors have in some of their studies even accounted for the actual technological uncertainty incorporated in real-world product and market development by implementing patent races in which the winner is determined *probabilistically*. But different from the literature we relied on, all studies in this tradition assume an *exogenously given, fully symmetric market structure*. Clearly, future experimental research may benefit from trying to combine the mentioned two research strands, by 1) explicitly spelling out an investment stage and specifying a probabilistic link between investment and production cost reduction, while 2) reflecting the fact that alliance participation is a subject of choice meaning that the structure of the market needs to be derived endogenously. Both of these aspects are necessarily part of any explanation of the emergence, success or failure of strategic alliances in the field.

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35 Alliances are here conceptualized as coalitions rather than networks.

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on several structural features such as the type of the strategic alliance (same-function versus parallel-development alliances), the intra-alliance profit-sharing rule, the size of the reward in case of competitive success (Amaldoss, Meyer, Raju, and Rapoport, 2000), on whether or not investments are alliance-specific such that they cannot be recovered in case the alliance loses the race, the size of alliances (Amaldoss and Rapoport, 2005b) and on the number of available technologies (Amaldoss and Rapoport, 2005a).
Appendix 1

In this appendix we first describe formally a sequential game of link formation between three firms. This sequential game of link formation corresponds to the last sequence of the first stage of the collaborative networks game when firms’ profits are given by Equation (2). We then prove Proposition 1 by characterizing the subgame perfect equilibria of the link formation game and showing that the resulting networks according to the subgame perfect equilibria of the link formation game are identical to the subgame perfect equilibrium networks of the collaborative networks game. Finally, we characterize the pairwise stable networks of the collaborative networks game.

A sequential game of link formation

$N = \{1, 2, 3\}$ denotes the set of firms. Initially, firms are connected in some collaborative network which we denote by $g_0$, $g_0 \in G$.

Over three periods, firms make contact with each other and determine whether to modify the initial network or not. Exactly one pair of firms meets each period and each pair of firms meets once and only once in the course of the game. In a given period, the two firms decide sequentially whether to modify the network or not. Each period is therefore characterized by two subperiods. $T = \{1, 2, 3, 4, 5, 6\}$ denotes the set of subperiods. Formally, an exogenously given and commonly known order of players is represented by two injective (not surjective) mappings $\rho_1 : N \to T$ and $\rho_2 : N \to T$ that assign to every firm $i \in N$ a first subperiod $\rho_1(i) \in T$ and a second subperiod $\rho_2(i) \in T$ such that $\rho_2(i) > \rho_1(i)$, $(\rho_1(i), \rho_2(i)) \notin \{(1, 2), (3, 4), (5, 6)\}$, and $\rho_1(i) \neq \rho_2(j)$, $\forall i, j \in N$, $i \neq j$. Thus, $T = \rho_1(N) \cup \rho_2(N)$, where $\rho_1(N) = \{t_1 \in T \mid (\exists i \in N) : t_1 = \rho_1(i)\}$ and $\rho_2(N) = \{t_2 \in T \mid (\exists i \in N) : t_2 = \rho_2(i)\}$. An inverse order of players is a surjective (not injective) mapping $\rho^{-1} : T \to N$ that assigns to every subperiod $t \in T$ a firm $i \in N$ such that $\exists \rho_x : \rho^{-1}(\rho_x(i)) = i$, $x \in \{1, 2\}$, $\forall i \in N$. By definition, $\rho^{-1}(1) \neq \rho^{-1}(2)$, $\rho^{-1}(1) \neq \rho^{-1}(4)$, and $\rho^{-1}(5) \neq \rho^{-1}(6)$. In a given period, if firm $i$ decides first when meeting firm $j$ then firm $i$’s set of actions is $A_i = \{y_i, n_i\}$ where $a_i = y_i$ means “form” (“keep”) the collaboration link $\{ij\}$ and $a_i = n_i$ means “not form” (“delete”) the collaboration link $\{ij\}$, $i, j \in N$. On the other hand, firm $j$’s set of actions depends on firm $i$’s chosen action: if firm $i$ chooses $y_i$ then $A_j = \{y_j, n_j\}$ whereas if firm $i$ chooses $n_i$ then firm $j$’s set of actions is empty. If firm $i$ and $j$ meet and $(a_i, a_j) \neq (y_i, y_j)$ then firm $k$ is only informed that the collaboration link has not been formed (has been deleted), and therefore firm $k$ only infers that $(a_i, a_j) \in \{(n_i), (y_i, n_j)\}$, $i, j, k \in N$.

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$g_0$ might be the empty network $\emptyset$. --

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For a given inverse order of players $\rho^{-1}$, a (feasible) history is a listing $h \in H(\rho^{-1}) = \bigcup_{z=0}^6 H_z(\rho^{-1})$, where

\[
H_0(\rho^{-1}) = \emptyset,
\]
\[
H_z(\rho^{-1}) = \{h_\rho^m\}_{m \in \{1,2,\ldots,2 \times 3^{(z-1)/2}\}} = H_{z-1}(\rho^{-1}) \times \{n_{\rho^{-1}(z)}, y_{\rho^{-1}(z)}\} \text{ if } z \in \{1,3,5\},
\]
\[
H_z(\rho^{-1}) = \{h_\rho^m\}_{m \in \{1,2,\ldots,3^z/2\}} = \left(\{h_{\rho-1}^m, h_z^m \times \{n_{\rho^{-1}(z)}, y_{\rho^{-1}(z)}\}\}_{m \in \{2,4,\ldots,2 \times 3^{(z-2)/2}\}}\right) \text{ if } z \in \{2,4,6\}.
\]

The history $h_z \in H_z(\rho^{-1})$ is said to have a length of $z$, $z \in \{0,1,2,3,4,5,6\}$, and $h_6$ is a terminal history. For each $i \in N$, firm $i$’s information partition is given by $I_i = \bigcup_{\{t \in T|\rho^{-1}(t)=i\}} T^t$ where

\[
\begin{align*}
I^1 & = \{I^1 = \{\emptyset\}\}; \\
I^2 & = \{I^2 = \{h_1^1\}\}; \\
I^3 & = \begin{cases}
\{I^3 = \{h_2^1, h_3^1\}, I^3_2 = \{h_3^3\}\} & \text{if } \rho^{-1}(3) \notin \{\rho^{-1}(1), \rho^{-1}(2)\}, \\
\{I^3 = \{h_1^1, I_3^3 = \{h_3^3\}\} & \text{otherwise};
\end{cases} \\
I^4 & = \begin{cases}
\{I^4 = \{h_3^2, h_3^3\}, I^4_2 = \{h_5^6\}\} & \text{if } \rho^{-1}(4) \notin \{\rho^{-1}(1), \rho^{-1}(2)\}, \\
\{I^4 = \{h_3^2, I_3^2 = \{h_3^3\}, I_3^3 = \{h_5^6\}\} & \text{otherwise};
\end{cases} \\
I^5 & = \begin{cases}
\{I^5 = \{h_3^2, I_3^2 = \{h_5^6\}, I_3^3 = \{h_5^6, h_5^{12}\}, \\
\{I_3^3 = \{h_3^2, I_3^3 = \{h_5^6, h_5^{12}\}, I_3^3 = \{h_3^2, h_5^6\}\} & \text{if } \rho^{-1}(5) \notin \{\rho^{-1}(1), \rho^{-1}(2)\}, \\
\{I_3^3 = \{h_3^2, I_3^3 = \{h_3^2, h_5^6\}, I_3^3 = \{h_3^2, h_5^6\}\} & \text{otherwise};
\end{cases} \\
I^6 & = \begin{cases}
\{I^6 = \{h_5^6, h_5^{12}\}, I^6_2 = \{h_5^{14}, h_5^{16}\}, I^6_3 = \{h_5^{14}, h_5^{16}\}, \\
\{I^6 = \{h_5^6, I^6_3 = \{h_5^{14}, h_5^{16}\}, I^6_3 = \{h_5^{14}, h_5^{16}\}\} & \text{if } \rho^{-1}(6) \notin \{\rho^{-1}(1), \rho^{-1}(2)\}, \\
\{I^6 = \{h_5^6, I^6_3 = \{h_5^6, h_5^{14}, h_5^{16}\}, I^6_3 = \{h_5^{14}, h_5^{16}\}\} & \text{otherwise}.
\end{cases}
\end{align*}
\]

The network $g(h_z) \in G$ corresponding to history $h_z \in H_z(\rho^{-1})$ is defined as the network that resulted from play till the end of subperiod $z$ in the link formation game with inverse order $\rho^{-1}$, where $z \in \{1,2,3,4,5,6\}$. Additionally, $g(h_0) = g(\emptyset) = g_0$.

A pure strategy of firm $i \in N$ is a mapping that assigns an action in $A_i = \{y_i, n_i\}$ to each information set $I_i \in I_i$. The strategy set of firm $i \in N$ is given by $S_i = \prod_{I_i \in I_i} A_i$. A strategy profile in the link formation game is given by $s = (s_1, s_2, s_3) \in S = \prod_{i \in N} S_i$. Obviously, for a given strategy profile, the outcome of the link formation game is independent of the initial network $g_0 \in G$. Therefore, with each strategy profile $s \in S$ we can define the resulting network as $g_s \in G$ independently of $g_0 \in G$. Firm $i$ receives a profit $\pi_i(g_s) = (q_i^E(g_s))^2 - f n_i(g_s)$ for every strategy profile $s \in S$, $i \in N$. For any inverse order of players $\rho^{-1}$, the above describes a
game tree $G_{\rho^{-1}}$ which implies that the link formation game $\Gamma_{\rho^{-1}}$ may be described by the 8-tuple $(N, G_{\rho^{-1}}, S_1, S_2, S_3, \pi_1, \pi_2, \pi_3)$. 

The extensive form of the sequential link formation game with the inverse order of players $\rho^{-1}(T) = \{1, 2, 3, 1, 2, 3\}$ is represented in Figure 1. Due to a lack of space, we only provide the resulting network below each terminal history instead of the firms’ profits.

Proof of Proposition 1

Subgame perfect equilibria of the sequential link formation game

Since the link formation game $\Gamma_{\rho^{-1}}$ is a well-defined extensive form game, we can use the concept of subgame perfection to analyze the evolution of networks. To investigate the nature of the subgame perfect equilibria of the link formation game, we first compare firms’ profits depending on the resulting network, for all possible values of the link formation cost $f > 0$. In each network architecture, the equilibrium profits are given by

- $\pi_i^E(g^0) = \frac{(\alpha - \lambda_0)^2}{16} = \pi_0^E$, $i \in N$,
- $\pi_i^E(\{\{jk\}\}) = \pi_0^E + \frac{\lambda(-\alpha + \lambda + \lambda_0)}{4}, \pi_j^E(\{\{jk\}\}) = \pi_k^E(\{\{jk\}\}) = \pi_0^E + \frac{\lambda(\alpha + \lambda - \lambda_0)}{4} - f$, $i, j, k \in N$,
- $\pi_i^E(\{\{ij\}, \{jk\}\}) = \pi_k^E(\{\{ij\}, \{jk\}\}) = \pi_0^E - f$, $\pi_j^E(\{\{ij\}, \{jk\}\}) = \pi_0^E + \frac{\lambda(\alpha + 2\lambda - \lambda_0)}{2} - 2f$, $i, j, k \in N$,
- $\pi_i^E(g^N) = \pi_0^E + \frac{\lambda(\alpha + \lambda - \lambda_0)}{4} - 2f$, $i \in N$.

Obviously, whatever the formation cost $f > 0$, the following inequalities hold: $\pi_i^E(g^0) > \pi_i^E(\{\{jk\}\})$, $\pi_i^E(g^0) > \pi_i^E(\{\{ij\}, \{jk\}\})$, $\pi_i^E(\{\{ij\}, \{ik\}\}) > \pi_i^E(g^N)$, $\pi_i^E(\{\{ij\}\}) > \pi_i^E(\{\{ij\}, \{jk\}\})$ where $i, j, k \in N$. Moreover, if $f < (\lambda(\alpha - \lambda - \lambda_0))/4$ then $\pi_i^E(g \cup \{ij\}) > \pi_i^E(g)$ where $\{ij\} \notin g$, $i, j \in N$. On the other hand, if $f > (\lambda(\alpha + \lambda - \lambda_0))/4$ then $\pi_i^E(g \cup \{ij\}) > \pi_i^E(g)$ where $g \neq \{ik\}$ and $\{ij\} \notin g$, $i, j, k \in N$. Finally, if $(\lambda(\alpha - \lambda - \lambda_0))/4 < f < (\lambda(\alpha + \lambda - \lambda_0))/4$ then $\pi_i^E(\{\{ij\}\}) > \pi_i^E(g^N) > \pi_i^E(\{\{ij\}, \{jk\}\})$, $\pi_i^E(\{\{ij\}, \{ik\}\}) > \pi_i^E(\{\{ij\}\}) > \pi_0^E$, and $\pi_i^E(\{\{jk\}\}) > \pi_i^E(\{\{ij\}, \{jk\}\})$ where $i, j, k \in N$.

We now characterize the subgame perfect equilibria of the link formation game $\Gamma_{\rho^{-1}}$ depending on the value of the link formation cost $f > 0$.

Case 1: $f < (\lambda(\alpha - \lambda - \lambda_0))/4$. Given such a low formation cost, each firm when asked to make a decision will choose to form (keep) the collaboration link. Consequently, whatever the inverse

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We will only consider equilibria in pure strategies.
order of players, the unique subgame perfect equilibrium of the link formation game is given by $\hat{s} = (\hat{s}_1, \hat{s}_2, \hat{s}_3)$ where $\hat{s}_i(I_i) = y_i$, $\forall I_i \in \mathcal{I}_i$, $i \in N$, and $g_{\hat{s}} = g^N$.

**Case 2:** $f > (\lambda(\alpha + 3\lambda - \lambda_0))/4$. Given such a high formation cost, each firm is worse off when forming (keeping) a collaboration link. The equilibrium outcome of the sequential link formation game is therefore the empty network. Whatever the inverse order of players, a subgame perfect equilibrium of the link formation game is given by $\hat{s} = (\hat{s}_1, \hat{s}_2, \hat{s}_3)$ where $\hat{s}_{\rho^{-1}(t)}(I_x^t) = n_{\rho^{-1}(t)}$ when $t \in \{2, 4, 6\}$, and $\hat{s}_{\rho^{-1}(t)}(I_x^t) \in \{n_{\rho^{-1}(t)}, y_{\rho^{-1}(t)}\}$ when $t \in \{1, 3, 5\}$, $\forall x$ such that $I_x^t \in \mathcal{I}^t$.

**Case 3:** $(\lambda(\alpha + 3\lambda - \lambda_0))/4 > f > (\lambda(\alpha + \lambda - \lambda_0))/4$.

**Subcase 3a:** If $\rho^{-1}(6) \notin \{\rho^{-1}(1), \rho^{-1}(2)\}$ then $\hat{s}_{\rho^{-1}(6)}(I_x^6) = n_{\rho^{-1}(6)}$ whenever $x \in \{1, 2, 4, 5, 6\}$ and $\hat{s}_{\rho^{-1}(6)}(I_x^6) = y_{\rho^{-1}(6)}$. As $\rho^{-1}(6) \notin \{\rho^{-1}(1), \rho^{-1}(2)\}$, $\rho^{-1}(5) \in \{\rho^{-1}(1), \rho^{-1}(2)\}$. By backwards induction, $\hat{s}_{\rho^{-1}(5)}(I_x^5) \in \{n_{\rho^{-1}(5)}, y_{\rho^{-1}(5)}\}$ whenever $x \in \{1, 3, 5, 6\}$ and $\hat{s}_{\rho^{-1}(5)}(I_x^5) = \hat{s}_{\rho^{-1}(5)}(I_x^4) = n_{\rho^{-1}(5)}$. Consequently, $\rho^{-1}(5) \rho^{-1}(6) \notin g_{\hat{s}}$ which entails that neither firm $\rho^{-1}(5)$ nor firm $\rho^{-1}(6)$ will agree to form (keep) a collaboration link with the third firm in a previous period. Henceforth, whatever the equilibrium, the outcome of the link formation game will be the empty network. We now pursue the precise characterization of the equilibrium strategies. Let assume that $\rho^{-1}(6) = \rho^{-1}(4)$. The equilibrium strategies in the previous subperiods are then given by: $\hat{s}_{\rho^{-1}(4)}(I_x^4) = n_{\rho^{-1}(4)}$, $\forall x \in \{1, 2\}$; $\hat{s}_{\rho^{-1}(3)}(I_x^3) \in \{n_{\rho^{-1}(3)}, y_{\rho^{-1}(3)}\}$, $\forall x \in \{1, 2, 3\}$; $\hat{s}_{\rho^{-1}(2)}(h_2^6) = n_{\rho^{-1}(2)}$; and $\hat{s}_{\rho^{-1}(1)}(\emptyset) \in \{n_{\rho^{-1}(1)}, y_{\rho^{-1}(1)}\}$. Let assume that $\rho^{-1}(6) = \rho^{-1}(3)$. The equilibrium strategies in the previous subperiods are then given by: $\hat{s}_{\rho^{-1}(4)}(I_x^4) = n_{\rho^{-1}(4)}$, $\forall x \in \{1, 2\}$ and $\hat{s}_{\rho^{-1}(4)}(I_x^4) = y_{\rho^{-1}(4)}$; $\hat{s}_{\rho^{-1}(3)}(I_x^3) \in \{n_{\rho^{-1}(3)}, y_{\rho^{-1}(3)}\}$ and $\hat{s}_{\rho^{-1}(3)}(I_x^3) = n_{\rho^{-1}(3)}$; $\hat{s}_{\rho^{-1}(2)}(h_2^5) = n_{\rho^{-1}(2)}$; and $\hat{s}_{\rho^{-1}(1)}(\emptyset) \in \{n_{\rho^{-1}(1)}, y_{\rho^{-1}(1)}\}$.

**Subcase 3b:** If $\rho^{-1}(6) \in \{\rho^{-1}(1), \rho^{-1}(2)\}$ then $\hat{s}_{\rho^{-1}(6)}(I_x^6) = n_{\rho^{-1}(6)}$ whenever $x \in \{1, 2, 3, 4, 6\}$ and $\hat{s}_{\rho^{-1}(6)}(I_x^6) = y_{\rho^{-1}(6)}$. As $\rho^{-1}(6) \notin \{\rho^{-1}(1), \rho^{-1}(2)\}$, $\rho^{-1}(5) \in \{\rho^{-1}(3), \rho^{-1}(4)\}$. By backwards induction, $\hat{s}_{\rho^{-1}(5)}(I_x^5) \in \{n_{\rho^{-1}(5)}, y_{\rho^{-1}(5)}\}$ whenever $x \in \{1, 2, 3, 6\}$ and $\hat{s}_{\rho^{-1}(5)}(I_x^5) = \hat{s}_{\rho^{-1}(5)}(I_x^4) = n_{\rho^{-1}(5)}$. Consequently, $\rho^{-1}(5) \rho^{-1}(6) \notin g_{\hat{s}}$ which entails that neither firm $\rho^{-1}(5)$ nor firm $\rho^{-1}(6)$ will agree to form (keep) a collaboration link with the third firm in a previous period. Again, whatever the equilibrium, the outcome of the link formation game will be the empty network. Moreover, if we assume that $\rho^{-1}(5) = \rho^{-1}(4)$ then the equilibrium strategies in the previous subperiods are identical to the above case where $\rho^{-1}(6) \notin \{\rho^{-1}(1), \rho^{-1}(2)\}$ and $\rho^{-1}(6) = \rho^{-1}(4)$. Similarly, if we assume that $\rho^{-1}(5) = \rho^{-1}(3)$ then the equilibrium strategies in the previous subperiods are identical to the above case where $\rho^{-1}(6) \notin \{\rho^{-1}(1), \rho^{-1}(2)\}$ and $\rho^{-1}(6) = \rho^{-1}(3)$. In conclusion, whatever the equilibrium $\hat{s}$, $g_{\hat{s}} = g^6$. 

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Case 4: \((\lambda (\alpha + \lambda - \lambda_0))/4 > f > (\lambda (\alpha - \lambda - \lambda_0))/4.\)

**Subcase 4a:** If \(\rho^{-1}(6) \not\in \{\rho^{-1}(1), \rho^{-1}(2)\}\) then \(\hat{s}_{\rho^{-1}(6)}(I^0_1) = y_{\rho^{-1}(6)}\) whenever \(x \in \{1, 2, 3, 6\}\) and \(\hat{s}_{\rho^{-1}(6)}(I^0_2) = \hat{s}_{\rho^{-1}(6)}(I^0_3) = n_{\rho^{-1}(6)}.\) Consequently, \(\hat{s}_{\rho^{-1}(5)}(I^5_3) \in \{n_{\rho^{-1}(5)}, y_{\rho^{-1}(5)}\}, \hat{s}_{\rho^{-1}(5)}(I^5_2) = y_{\rho^{-1}(5)}\) whenever \(x \in \{1, 3, 6\},\) and \(\hat{s}_{\rho^{-1}(5)}(I^5_2) = \hat{s}_{\rho^{-1}(5)}(I^5_3) = n_{\rho^{-1}(5)}.\) Firm \(\rho^{-1}(5)\) and \(\rho^{-1}(6)\) form (keep) a collaboration link if either no link has been formed in the past (all links have been deleted) or if all possible links have been formed (kept). Thus, the empty network and \(g^*\) are excluded from the possible outcomes of the link formation game.

- Let assume that \(\rho^{-1}(6) = \rho^{-1}(4).\) The equilibrium strategies in the previous subperiods are then given by: \(\hat{s}_{\rho^{-1}(4)}(I^4_1) = \hat{s}_{\rho^{-1}(4)}(I^4_2) = y_{\rho^{-1}(4)}; \hat{s}_{\rho^{-1}(3)}(I^3_1) = \hat{s}_{\rho^{-1}(3)}(I^3_2) = y_{\rho^{-1}(3)}\) and \(\hat{s}_{\rho^{-1}(3)}(I^3_2) = n_{\rho^{-1}(3)};\) if \(\rho^{-1}(2) = \rho^{-1}(3)\) then \(\hat{s}_{\rho^{-1}(2)}(h^2_2) \in \{n_{\rho^{-1}(2)}, y_{\rho^{-1}(2)}\}\) and \(\hat{s}_{\rho^{-1}(1)}(\emptyset) = y_{\rho^{-1}(1)}\) whereas if \(\rho^{-1}(2) = \rho^{-1}(5)\) then \(\hat{s}_{\rho^{-1}(2)}(h^2_2) = y_{\rho^{-1}(2)}\) and \(\hat{s}_{\rho^{-1}(1)}(\emptyset) \in \{n_{\rho^{-1}(1)}, y_{\rho^{-1}(1)}\}.\) The equilibrium outcome is therefore \(g^1\) and the unique collaboration link is either formed (kept) in period 1 or 2.

- Let assume that \(\rho^{-1}(6) = \rho^{-1}(3).\) We have that \(\hat{s}_{\rho^{-1}(4)}(I^4_1) = \hat{s}_{\rho^{-1}(4)}(I^4_2) = y_{\rho^{-1}(4)}\) and \(\hat{s}_{\rho^{-1}(4)}(I^4_2) = n_{\rho^{-1}(4)},\) and \(\hat{s}_{\rho^{-1}(3)}(I^3_1) \in \{n_{\rho^{-1}(3)}, y_{\rho^{-1}(3)}\}, \forall x \in \{1, 2\}.\) The equilibrium strategies of the firms deciding in subperiod 1 and 2 depend on the tie-breaking rule we assume when a firm is indifferent between forming (keeping) and not forming (deleting). Indeed, if an indifferent firm forms (keeps) with probability 1 and \(\rho^{-1}(2) = \rho^{-1}(5)\) (respectively \(\rho^{-1}(1) = \rho^{-1}(5)\)) then \(\hat{s}_{\rho^{-1}(2)}(h^2_2) = y_{\rho^{-1}(2)}\) and \(\hat{s}_{\rho^{-1}(1)}(\emptyset) \in \{n_{\rho^{-1}(1)}, y_{\rho^{-1}(1)}\}\) (respectively \(\hat{s}_{\rho^{-1}(2)}(h^2_2) \in \{n_{\rho^{-1}(2)}, y_{\rho^{-1}(2)}\}\) and \(\hat{s}_{\rho^{-1}(1)}(\emptyset) = y_{\rho^{-1}(1)}\). The equilibrium outcome is therefore \(g^1\) and the unique collaboration link is formed (kept) either in period 1 or 2. If an indifferent firm does not form (deletes) with probability 1 and \(\rho^{-1}(2) = \rho^{-1}(5)\) (respectively \(\rho^{-1}(1) = \rho^{-1}(5)\)) then \(\hat{s}_{\rho^{-1}(2)}(h^2_2) \in \{n_{\rho^{-1}(2)}, y_{\rho^{-1}(2)}\}\) and \(\hat{s}_{\rho^{-1}(1)}(\emptyset) \in \{n_{\rho^{-1}(1)}, y_{\rho^{-1}(1)}\}\). The equilibrium outcome is therefore \(g^1\) and the unique collaboration link is formed (kept) either in period 1 or 3. Finally, if an indifferent firm forms (deletes) with a probability strictly positive but strictly lower than 1 then \(\hat{s}_{\rho^{-1}(2)}(h^2_2) = y_{\rho^{-1}(2)}\) and \(\hat{s}_{\rho^{-1}(1)}(\emptyset) = y_{\rho^{-1}(1)}\). The equilibrium outcome is therefore \(g^1\) and the unique collaboration link is formed (kept) in period 1.

**Subcase 4b:** If \(\rho^{-1}(6) \in \{\rho^{-1}(1), \rho^{-1}(2)\}\) then \(\hat{s}_{\rho^{-1}(6)}(I^0_1) = y_{\rho^{-1}(6)}\) whenever \(x \in \{1, 3, 5, 6\}\) and \(\hat{s}_{\rho^{-1}(6)}(I^0_2) = \hat{s}_{\rho^{-1}(6)}(I^0_3) = n_{\rho^{-1}(6)}.\) By backwards induction, \(\hat{s}_{\rho^{-1}(5)}(I^5_3) \in \{n_{\rho^{-1}(5)}, y_{\rho^{-1}(5)}\},\) \(\hat{s}_{\rho^{-1}(5)}(I^5_2) = y_{\rho^{-1}(5)}\) whenever \(x \in \{1, 2, 6\},\) and \(\hat{s}_{\rho^{-1}(5)}(I^5_2) = \hat{s}_{\rho^{-1}(5)}(I^5_3) = n_{\rho^{-1}(5)}.\) Firm \(\rho^{-1}(5)\) and \(\rho^{-1}(6)\) form (keep) a collaboration link if either no link has been formed in the past (all links have been deleted) or if all possible links have been formed (kept). Again, the empty network and \(g^*\) are excluded from the possible outcomes of the link formation game.

- Let assume that \(\rho^{-1}(5) = \rho^{-1}(4).\) The equilibrium strategies in the previous subperiods are identical to the case \(\rho^{-1}(6) \not\in \{\rho^{-1}(1), \rho^{-1}(2)\}\) and \(\rho^{-1}(6) = \rho^{-1}(4).\) The equilibrium outcome
is therefore $g^1$ and the unique collaboration link is either formed (kept) in period 1 or 2.

- Let assume that $\rho^{-1}(5) = \rho^{-1}(3)$. The equilibrium strategies in the previous subperiods are identical to the case $\rho^{-1}(6) \notin \{\rho^{-1}(1), \rho^{-1}(2)\}$ and $\rho^{-1}(6) = \rho^{-1}(3)$. The equilibrium outcome is therefore $g^1$. If an indifferent firm forms (keeps) with probability 1 then the unique collaboration link is formed (kept) either in period 1 or 2. If an indifferent firm does not form (deletes) with probability 1 then the unique collaboration link is formed (kept) either in period 1 or 3. If an indifferent firm forms (deletes) with a probability strictly positive but strictly lower than 1 then the unique collaboration link is formed (kept) in period 1.

Given the provided characterization of the subgame perfect equilibria, we can now summarize the possible equilibrium outcomes of the sequential link formation game depending on the link formation cost $f > 0$: if $f < (\lambda(\alpha - \lambda - \lambda_0))/4$ then the unique equilibrium outcome is the complete network; if $f > (\lambda(\alpha + \lambda - \lambda_0))/4$ then the unique equilibrium outcome is the empty network; if $(\lambda(\alpha + \lambda - \lambda_0))/4 > f > (\lambda(\alpha - \lambda - \lambda_0))/4$ then the equilibrium outcome is $g^1$, and, depending on the inverse order of players and on the assumed tie-breaking rule, the single collaboration link is formed (kept) in either period 1, 2 or 3.

**Subgame perfect equilibrium networks of the collaborative networks game**

In a subgame perfect equilibrium of the collaborative networks game, given the resulting network $g_{spe} \in G$, firm $i$’s profits are equal to $(q_i^{E}(g_{spe}))^2 - f n_i(g_{spe}), \forall i \in N, f > 0$. That is, when they shape the collaborative network during the first stage of the game, firms anticipate that equilibrium quantities will be produced at the second stage of the game. Consequently, the last sequence of the first stage of the collaborative networks game is identical to the sequential game of link formation. Moreover, whatever the decisions firms make in the previous sequences of the first stage of the collaborative networks game, those decisions have no impact on the network resulting from equilibrium play as, for a given strategy profile, the outcome of the sequential game of link formation is independent of the initial network $g_0 \in G$. We can therefore conclude that, for a given link formation cost $f > 0$, the subgame perfect equilibrium networks of the collaborative networks game are identical to the networks resulting from equilibrium play in the sequential link formation game.

**Pairwise stable networks of the collaborative networks game**

We now characterize the pairwise stable networks under the assumption that myopic firms foresee that they will produce equilibrium quantities at the second stage of the collaborative networks game, and depending on the value of the link formation cost $f > 0$. Straightforward computations
show that a pair of myopic firms finds it profitable to add a link to: i) the empty network if
and only if $f < (\lambda(\alpha + \lambda - \lambda_0))/4$ (transition condition 1); ii) a single-link network if and only if
$f < (\lambda(\alpha - \lambda - \lambda_0))/4$ (transition condition 2); iii) a star network if and only if $f < (\lambda(\alpha + \lambda - \lambda_0))/4$
(transition condition 3). Consequently, in the first sequence of the first stage of the game, if
$f < (\lambda(\alpha - \lambda - \lambda_0))/4$ then all three transition conditions are fulfilled and myopic firms form the
complete network, if $f > (\lambda(\alpha + \lambda - \lambda_0))/4$ then no transition condition is fulfilled and myopic
firms do not form any collaboration link, and if $(\lambda(\alpha + \lambda - \lambda_0))/4 > f > (\lambda(\alpha - \lambda - \lambda_0))/4$ then
the first and third transition conditions are fulfilled but not the second transition condition and
therefore only the first pair of myopic firms according to the order of players forms a collaboration
link. In a latter sequence, myopic firms have of course no incentives to modify the network formed
at the end of sequence 1 as it is pairwise stable by definition. We can therefore conclude that for
$f < (\lambda(\alpha - \lambda - \lambda_0))/4$ or $f > (\lambda(\alpha + \lambda - \lambda_0))/4$, the pairwise stable network is identical to the
subgame perfect equilibrium network whereas for $(\lambda(\alpha - \lambda - \lambda_0))/4 < f < (\lambda(\alpha + \lambda - \lambda_0))/4$, the
pairwise stable network is isomorphic to the subgame perfect equilibrium network.

As already mentioned, Goyal and Joshi (2003) extend the standard definition of pairwise stabil-
ity by additionally requiring that a firm should find it profitable to maintain its collaboration links
in the network rather than not having any links. Formally, the individual rationality condition
for participation in the network $g \in G$ is given by $\forall i \in N$, $\pi_i(g) \geq \pi_i(g^{-i})$ where $g^{-i}$ denotes
the network in which all of firm $i$’s collaboration links are deleted. That is, either $g^{-i} = \{ \{jk\} \}$
or $g^{-i} = g^0$ where $j, k \in N$. The participation constraint is identical to the first transition con-
dition in the single-link network and it is given by $f < (\lambda(\alpha - \lambda_0))/4$ in the complete network.
If $f < (\lambda(\alpha - \lambda - \lambda_0))/4$ then $f < (\lambda(\alpha - \lambda_0))/4$ and therefore the “extended” pairwise stable
networks are identical to the standard pairwise stable networks.

Appendix 2

In this appendix, for the parametrization implemented in the laboratory, we characterize the sub-
game perfect equilibria of the link formation game when firms collude on the market stage. In the
following, we therefore assume that $f \in \{5, 20, 35\}$ and $\rho^{-1}(T) \in$
$\{\{1, 2, 2, 3, 3, 1\}, \{2, 3, 1, 2, 3, 1\}, \{3, 1, 1, 2, 2, 3\}, \{1, 2, 3, 1, 2, 3\}, \{2, 3, 3, 1, 1, 2\}, \{3, 1, 2, 3, 1, 2\}\}$.

We characterize the subgame perfect equilibria of the sequential link formation game $\Gamma_{\rho^{-1}}^C = (N, G_{\rho^{-1}}, S_1, S_2, S_3, \pi_1^C, \pi_2^C, \pi_3^C)$ where
$\pi_1^C(g) = 65 - 2f$, $\pi_1^C(g^*) = 135 - 2f$, $\pi_2^C(g^*) = \pi_3^C(g^*) = 18 - f$, $\pi_i^C(g^1) = \pi_j^C(g^1) = 49 - f$, $\pi_k^C(g^1) = 1$, and $\pi_i^C(g^0) = 16$, with $g^* = \{ \{ij\}, \{ik\} \}$,
$g^1 = \{ \{ij\} \}$, $i, j, k \in N$, $i \neq j \neq k$. 37
Case 1: \( f = 5 \). Given such a low formation cost, each firm when asked to make a decision will choose to form (keep) the collaboration link. Consequently, whatever the inverse order of players, the unique subgame perfect equilibrium of \( \Gamma_{\rho^{-1}}^C \) is given by \( \hat{s} = (\hat{s}_1, \hat{s}_2, \hat{s}_3) \) where \( \hat{s}_i(I_t) = y_i, \forall I_t \in \mathcal{I}_t, i \in N, \) and \( g_s = g^N \).

Case 2: \( f = 35 \). Given such a high formation cost, each firm is worse off when forming (keeping) a collaboration link. The unique subgame perfect equilibrium outcome of \( \Gamma_{\rho^{-1}}^C \) is therefore the empty network. Whatever the inverse order of players, a subgame perfect equilibrium of \( \Gamma_{\rho^{-1}}^C \) when \( t \) is strictly smaller than 1.

Case 3: \( f = 20 \).

Subcase 3a: If \( \rho^{-1}(6) \notin \{\rho^{-1}(1), \rho^{-1}(2)\} \). As the equilibrium strategies in some subperiods depend on the assumed tie-breaking rule used by firms which are indifferent between forming (keeping) and not forming (deleting), we will differentiate three further subcases:

- 1) Let assume that if a firm is indifferent between forming (keeping) and not forming (deleting) a collaboration link, the firm forms (keeps) the collaboration link with probability 1. Then, \( \hat{s}_{\rho^{-1}(6)}(I_2^5) = y_{\rho^{-1}(6)} \) whenever \( x \in \{1,2,3,6\} \), and \( \hat{s}_{\rho^{-1}(6)}(I_2^5) = n_{\rho^{-1}(6)} \) whenever \( x \in \{4,5\} \). By backwards induction, \( \hat{s}_{\rho^{-1}(5)}(I_2^5) = y_{\rho^{-1}(5)} \) whenever \( x \in \{1,3,5,6\} \), and \( \hat{s}_{\rho^{-1}(5)}(I_2^5) = n_{\rho^{-1}(5)} \) whenever \( x \in \{2,4\} \). Firms \( \rho^{-1}(5) \) and \( \rho^{-1}(6) \) form (keep) a collaboration link if either no link has been formed (all links have been deleted) or if all links have been formed (kept) in the past. Thus, the empty network and the star network are excluded from the possible outcomes of the link formation game. Consequently, \( \hat{s}_{\rho^{-1}(4)}(I_2^5) = y_{\rho^{-1}(4)} \) whenever \( x \in \{1,2\} \) and \( \hat{s}_{\rho^{-1}(4)}(I_2^5) = n_{\rho^{-1}(4)} \), and \( \hat{s}_{\rho^{-1}(3)}(I_2^5) = y_{\rho^{-1}(3)} \) for all \( x \in \{1,2\} \). Firms \( \rho^{-1}(3) \) and \( \rho^{-1}(4) \) form (keep) a collaboration link only if no link has been formed (all links have been deleted) in the past. This leads to the following subgame perfect equilibrium strategies in subperiods 1 and 2: \( \hat{s}_{\rho^{-1}(3)}(h_2^2) = y_{\rho^{-1}(2)} \), and \( \hat{s}_{\rho^{-1}(1)}(\emptyset) = y_{\rho^{-1}(1)} \). The unique subgame perfect equilibrium outcome is therefore the single-link network where the single collaboration link is formed in period 1.

- 2) Let assume that if a firm is indifferent between forming (keeping) and not forming (deleting) a collaboration link, the firm forms (keeps) the collaboration link with a strictly positive probability which is strictly smaller than 1. Then, \( \hat{s}_{\rho^{-1}(6)}(I_2^5) = y_{\rho^{-1}(6)} \) whenever \( x \in \{1,2,3,6\} \), and \( \hat{s}_{\rho^{-1}(6)}(I_2^5) = n_{\rho^{-1}(6)} \) whenever \( x \in \{4,5\} \). By backwards induction, \( \hat{s}_{\rho^{-1}(5)}(I_2^5) = y_{\rho^{-1}(5)} \) whenever \( x \in \{1,3,6\} \), \( 0 < Pr(\hat{s}_{\rho^{-1}(5)}(I_2^5) = y_{\rho^{-1}(5)}) < 1 \), and \( \hat{s}_{\rho^{-1}(5)}(I_2^5) = n_{\rho^{-1}(5)} \) whenever \( x \in \{2,4\} \). Firms \( \rho^{-1}(5) \) and \( \rho^{-1}(6) \) form (keep) a collaboration link if either no link has been formed (all
links have been deleted) or if all links have been formed (kept) in the past. Thus, the empty network and the star network are excluded from the possible outcomes of the link formation game. Consequently, \( \hat{s}_{\rho^{-1}(4)}(I_x^4) = y_{\rho^{-1}(4)} \) whenever \( x \in \{1, 2\} \) and \( \hat{s}_{\rho^{-1}(5)}(I_x^5) = n_{\rho^{-1}(4)} \), and \( 0 < Pr(\hat{s}_{\rho^{-1}(3)}(I_x^3) = y_{\rho^{-1}(3)}) < 1 \) for all \( x \in \{1, 2\} \). Firms \( \rho^{-1}(3) \) and \( \rho^{-1}(4) \) form (keep) a collaboration link with some strictly positive probability (strictly smaller than 1) if no link has been formed (all links have been deleted) in the past. Finally, \( \hat{s}_{\rho^{-1}(2)}(h_1^2) = y_{\rho^{-1}(2)} \), \( \hat{s}_{\rho^{-1}(1)}(\emptyset) = y_{\rho^{-1}(1)} \), and the unique subgame perfect equilibrium outcome is the single-link network where the single collaboration link is formed in period 1.

- 3) Let assume that if a firm is indifferent between forming (keeping) and not forming (deleting) a collaboration link, the firm does not form (deletes) the collaboration link with probability 1. Then, \( \hat{s}_{\rho^{-1}(6)}(I_x^6) = y_{\rho^{-1}(6)} \) whenever \( x \in \{1, 2, 3, 6\} \), and \( \hat{s}_{\rho^{-1}(6)}(I_x^6) = n_{\rho^{-1}(6)} \) whenever \( x \in \{4, 5\} \). By backwards induction, \( \hat{s}_{\rho^{-1}(5)}(I_x^5) = y_{\rho^{-1}(5)} \) whenever \( x \in \{1, 3, 6\} \) and \( \hat{s}_{\rho^{-1}(5)}(I_x^5) = n_{\rho^{-1}(5)} \) whenever \( x \in \{2, 4, 5\} \). Firms \( \rho^{-1}(5) \) and \( \rho^{-1}(6) \) form (keep) a collaboration link if either no link has been formed (all links have been deleted) or if all links have been formed (kept) in the past. Thus, the empty network and the star network are excluded from the possible outcomes of the link formation game. Consequently, \( \hat{s}_{\rho^{-1}(4)}(I_x^4) = y_{\rho^{-1}(4)} \) whenever \( x \in \{1, 2\} \), \( \hat{s}_{\rho^{-1}(5)}(I_x^5) = n_{\rho^{-1}(4)} \), and \( \hat{s}_{\rho^{-1}(3)}(I_x^3) = n_{\rho^{-1}(3)} \) for all \( x \in \{1, 2\} \). Firm \( \rho^{-1}(3) \) never forms (always deletes) the respective collaboration link, meaning that no link is formed (an existing link is deleted) in period 2. This leads to \( \hat{s}_{\rho^{-1}(2)}(h_1^2) = n_{\rho^{-1}(2)} \), and \( \hat{s}_{\rho^{-1}(1)}(\emptyset) = n_{\rho^{-1}(1)} \). The unique subgame perfect equilibrium outcome is the single-link network where the single collaboration link is formed in period 3.

**Subcase 3b:** If \( \rho^{-1}(6) \in \{\rho^{-1}(1), \rho^{-1}(2)\} \).

- 1) Let assume that if a firm is indifferent between forming (keeping) and not forming (deleting) a collaboration link, the firm forms (keeps) the collaboration link with probability 1. Then, \( \hat{s}_{\rho^{-1}(6)}(I_x^6) = y_{\rho^{-1}(6)} \) whenever \( x \in \{1, 3, 5, 6\} \) and \( \hat{s}_{\rho^{-1}(6)}(I_x^6) = n_{\rho^{-1}(6)} \) whenever \( x \in \{2, 4\} \). By backwards induction, \( \hat{s}_{\rho^{-1}(5)}(I_x^5) = y_{\rho^{-1}(5)} \) whenever \( x \in \{1, 2, 3, 6\} \) and \( \hat{s}_{\rho^{-1}(5)}(I_x^5) = n_{\rho^{-1}(5)} \) whenever \( x \in \{4, 5\} \). Firms \( \rho^{-1}(5) \) and \( \rho^{-1}(6) \) form (keep) a collaboration link if either no link has been formed (all links have been deleted) or if all links have been formed (kept) in the past. Thus, the empty network and the star network are excluded from the possible outcomes of the link formation game. Therefore, \( \hat{s}_{\rho^{-1}(4)}(I_x^4) = y_{\rho^{-1}(4)} \) for all \( x \in \{1, 2\} \), \( \hat{s}_{\rho^{-1}(3)}(I_x^3) = y_{\rho^{-1}(3)} \) whenever \( x \in \{1, 2\} \) and \( \hat{s}_{\rho^{-1}(3)}(I_x^3) = n_{\rho^{-1}(3)} \). Firms \( \rho^{-1}(3) \) and \( \rho^{-1}(4) \) form (keep) a collaboration link only if no link has been formed (all links have been deleted) in the past. Finally, \( \hat{s}_{\rho^{-1}(2)}(h_1^2) = y_{\rho^{-1}(2)} \) and \( \hat{s}_{\rho^{-1}(1)}(\emptyset) = y_{\rho^{-1}(1)} \). The unique subgame perfect equilibrium outcome is the single-link network where the single collaboration link is formed in period 1.

- 2) Let assume that if a firm is indifferent between forming (keeping) and not forming (deleting) a collaboration link, the firm forms (keeps) the collaboration link with a strictly positive probabil-
ity which is strictly smaller than 1. Then, \( \hat{s}_{\rho^{-1}(6)}(I_5^6) = y_{\rho^{-1}(6)} \) whenever \( x \in \{1, 3, 5, 6\} \) and \( \hat{s}_{\rho^{-1}(6)}(I_5^6) = n_{\rho^{-1}(6)} \) whenever \( x \in \{2, 4\} \). By backwards induction, \( \hat{s}_{\rho^{-1}(5)}(I_4^5) = y_{\rho^{-1}(5)} \) whenever \( x \in \{1, 2, 6\} \), \( \hat{s}_{\rho^{-1}(5)}(I_2^5) = n_{\rho^{-1}(5)} \) whenever \( x \in \{4, 5\} \), and \( 0 < Pr(\hat{s}_{\rho^{-1}(5)}(I_3^5) = y_{\rho^{-1}(5)}) < 1 \).

Firms \( \rho^{-1}(5) \) and \( \rho^{-1}(6) \) form (keep) a collaboration link if either no link has been formed (all links have been deleted) or if all links have been formed (kept) in the past. Thus, the empty network and the star network are excluded from the possible outcomes of the link formation game. Consequently, \( 0 < Pr(\hat{s}_{\rho^{-1}(4)}(I_4^1) = y_{\rho^{-1}(4)}) < 1 \), \( \hat{s}_{\rho^{-1}(4)}(I_2^4) = y_{\rho^{-1}(4)} \), and \( \hat{s}_{\rho^{-1}(3)}(I_3^5) = n_{\rho^{-1}(3)} \) whenever \( x \in \{1, 2\} \) and \( \hat{s}_{\rho^{-1}(3)}(I_3^5) = n_{\rho^{-1}(3)} \). Firms \( \rho^{-1}(3) \) and \( \rho^{-1}(4) \) form (keep) a collaboration link with some strictly positive probability (strictly smaller than 1) only if no link has been formed (all links have been deleted) in the past. Finally, \( \hat{s}_{\rho^{-1}(2)}(h_2^1) = y_{\rho^{-1}(2)} \) and \( \hat{s}_{\rho^{-1}(1)}(\emptyset) = y_{\rho^{-1}(1)} \). The unique subgame perfect equilibrium outcome is the single-link network where the single collaboration link is formed in period 1.

- 3) Let assume that if a firm is indifferent between forming (keeping) and not forming (deleting) a collaboration link, the firm does not form (deletes) the collaboration link with probability 1. Then, \( \hat{s}_{\rho^{-1}(6)}(I_5^6) = y_{\rho^{-1}(6)} \) whenever \( x \in \{1, 3, 5, 6\} \) and \( \hat{s}_{\rho^{-1}(6)}(I_5^6) = n_{\rho^{-1}(6)} \) whenever \( x \in \{2, 4\} \). By backwards induction, \( \hat{s}_{\rho^{-1}(5)}(I_4^5) = y_{\rho^{-1}(5)} \) whenever \( x \in \{1, 2, 6\} \) and \( \hat{s}_{\rho^{-1}(5)}(I_2^5) = n_{\rho^{-1}(5)} \) whenever \( x \in \{3, 4, 5\} \). Firms \( \rho^{-1}(5) \) and \( \rho^{-1}(6) \) form (keep) a collaboration link if either no link has been formed (all links have been deleted) or if all links have been formed (kept) in the past. Thus, the empty network and the star network are excluded from the possible outcomes of the link formation game. Consequently, \( \hat{s}_{\rho^{-1}(4)}(I_4^1) = n_{\rho^{-1}(4)} \), \( \hat{s}_{\rho^{-1}(4)}(I_2^4) = y_{\rho^{-1}(4)} \), and \( \hat{s}_{\rho^{-1}(3)}(I_3^5) = n_{\rho^{-1}(3)} \) for all \( x \in \{1, 2, 3\} \). Firm \( \rho^{-1}(3) \) never forms (always deletes) the respective collaboration link, meaning that no link is formed (an existing link is deleted) in period 2. Finally, \( \hat{s}_{\rho^{-1}(2)}(h_2^1) = y_{\rho^{-1}(2)} \) and \( \hat{s}_{\rho^{-1}(1)}(\emptyset) = n_{\rho^{-1}(1)} \). The unique subgame perfect equilibrium outcome is the single-link network where the single collaboration link is formed in period 3.

We can now summarize the possible subgame perfect equilibrium outcomes of \( \Gamma^C_{\rho^{-1}} \): if \( f = 5 \) the unique outcome is the complete network; if \( f = 35 \) the unique outcome is the empty network; if \( f = 20 \) then the outcome is \( g^1 \). If a firm is indifferent between forming (keeping) and not forming (deleting) and forms with a strictly positive probability, the single collaboration link is formed (kept) in period 1. If a firm is indifferent between forming (keeping) and not forming (deleting) and forms with a null probability, the single collaboration link is formed (kept) in period 3. Moreover, for a given link formation cost \( f > 0 \), the collusive networks of the collaborative networks game are identical to the networks resulting from equilibrium play in \( \Gamma^C_{\rho^{-1}} \).

Given our parametrization, one can show that, for a given link formation cost \( f > 0 \), the subgame perfect equilibrium networks of the collaborative networks game are identical to the networks
resulting from equilibrium play in $\Gamma^C_{\rho^{-1}}$ (details are omitted).
Figure 1: The extensive form of the sequential link formation game with $\rho^{-1}(T) = \{1, 2, 3, 1, 2, 3\}$. 
References


