Multiyear Risk of Credit Losses in SME Portfolios

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Abstract

We model multiyear loss distributions based on credit scores and macroeconomic risk drivers. In a two-step approach, we first model future default probabilities as functions of these risk factors and, second, model processes for the risk factors themselves. As an essential extension to one-year forecasts – used e.g. for calculating a bank’s regulatory capital charges under Basel II – we explicitly consider forecasting errors. These errors are introduced by forecasting future paths of the risk factors. We distinguish between idiosyncratic and systematic forecasting errors and show their effects on individual default probability and portfolio loss distribution forecasts using data provided by Deutsche Bundesbank. It turns out that default probability forecasts are much noisier than portfolio risk forecasts due to the possibility of diversification of idiosyncratic forecasting errors in the latter.
1 Introduction

As a response to improved modeling and measurement approaches for credit portfolio losses the Basel Committee on Banking Supervision has introduced a new standard for regulatory capital (‘Basel II’).

The technical model upon which the Basel II framework is built is a simplified variant of a Merton-type model where a firm default happens if some variable, e.g. the value of the firm’s assets, falls short of some threshold. Credit dependencies are introduced by correlations between the asset returns. Consistent with much of the practice in the banking industry regarding loans, the model is constructed in a ‘default-only mode’ where losses arrive during a given period (e.g. one year) only because of loan default events, since a large part of the entities in a loan portfolio typically consists of small and medium sized enterprises (SMEs) without traded debt.

Major default-only modeling approaches, e.g. the CreditRisk+ model developed by Credit Suisse, focus on a one-period forecasting horizon for credit losses, e.g. one-year ahead. Gordy (2000) provides a detailed comparison of the modeling techniques. Required ingredients are the default probabilities, exposures at default, losses given default, and correlations – all variables referring to a one-year horizon.

Given the nature of most SME portfolios, contract durations of the loans are usually much longer than only one year and for a couple of reasons multiyear loss forecasting is required. On the basis of a single loan, finding the adequate spread should make use of the entire term structure of its default probabilities. On a portfolio basis the risk in future periods beyond the next year should be assessed for dynamically adjusting economic capital. Here, the central question concerns the risk profile of the portfolio until maturity which is governed by the future credit qualities of the obligors as well as by correlation effect. Increasing correlations might lead to substantial risk exposure of the bank in future years. Another purpose is the evaluation and pricing of derivative contracts such as CDOs and CLOs. The future risk structure of the portfolio is the essential driver for the prices of these instruments.

1 The authors gratefully acknowledge helpful comments from Klaus Duellmann and Peter Raupach as well as from participants of the Financial Stability Workshop hosted by the Deutsche Bundesbank in November 2005 and the C. R. E. D. I. T. conference in September 2006 in Venice. We would also like to acknowledge useful comments from an anonymous referee.
In the light of these demands we provide a framework for forecasting multiyear portfolio loss distribution forecasts. The paper is closely related to former research in this area but extends it in three important ways. First, there is a long history in constructing credit ratings and estimating default probabilities, see e.g. Altman (1968), Altman/Saunders (1998), Shumway (2001), Engelmann/Hayden/Tasche (2003), Grunert/Norden/Weber (2005), Carling et al. (2007). While these papers focus on forecasting ratings or default probabilities for a fixed horizon, more recently Duffie/Saita/Wang (2007) provide an intensity model for forecasting multiyear default probabilities. As a necessary ingredient for multiyear portfolio loss distributions we also forecast default probabilities using a similar model with observable time-dependent co-variates such as distance-to-default and macroeconomic variables as in Duffie/Saita/Wang (2007). In contrast to these authors our approach is time-discrete but its probit type specification has the advantage that it is easy to include additional (asset) correlation effects between borrowers which are required for portfolio analysis. Moreover, it has the nice feature that it is a straightforward extension of the model used in Basel II for calculating regulatory capital charges; see e.g. Gordy (2000, 2003) and Heitfield (2005).

Second, we move from forecasts for individual default probabilities to portfolio forecasts and extend the existing approaches for forecasting loss distributions from a one-year horizon to a multiyear horizon. Popular models for distributions of one-year losses from defaults are of the ‘factor-model’-type and differentiate between systematic and idiosyncratic risk drivers; see, e.g., the model descriptions in Koyluoglu/Hickman (1998), Finger (1998), and Gordy (2000). Econometric specifications and an empirical comparison of the models are given in Hamerle/Rösch (2006). We provide econometric panel models for the observable risk factors and employ these models for forecasting multiyear loss distributions.

Third, we explicitly consider an additional important risk source in our multiyear portfolio model besides risks from systematic and idiosyncratic risk drivers. To our knowledge this source has not been analyzed in this context so far. Multiyear forecasting of portfolio losses requires forecasting of the risk factors which drive defaults. Given our econometric specifications for the risk drivers, errors due to forecasting are introduced.

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2 Here, we only focus on modelling the default event. Multiyear-modelling of exposures and losses given default is beyond the scope of the paper but might be an important topic for future research.
These forecasting errors generally increase with increasing time horizon and can be decomposed into systematic and idiosyncratic parts.

We show that the magnitude of the systematic components crucially affects benefits from portfolio diversification besides common systematic risk drivers and governs the multi-period risk profile of the portfolio.

We calibrate our models to a dataset provided by Deutsche Bundesbank which consists of a large panel of German small and medium sized companies from 1987 to 2003. We obtain the following main results:

- In line with parts of the previous literature e.g. Dietsch/Petey (2004) our estimates for asset correlations, i.e. the systematic part of portfolio risk, are rather low. This might be due to the fact that our sample consists of SME.

- When forecasting multi-period risk one should take forecasting errors into account. Risk from those errors may be substantial and can be divided into systematic and idiosyncratic parts.

- For our sample, idiosyncratic forecasting errors are approximately 97% of the entire forecasting risk for a two-year or three-year ahead forecast while systematic errors only make up to about 3%.

- The high proportion of idiosyncratic risk renders individual default probability forecasts very noisy.

- On the other hand, because idiosyncratic risk is diversifiable, forecasts for multi-year portfolio loss distributions are much less affected by forecasting risk. This result might encourage further attempts for calculating multiyear distributions.

The remainder of the paper is structured as follows. Section 2 provides a description of our models. Section 3 describes our data and shows the main estimation results. In section 4 some basic theoretical insights into forecasting risk factors and default probabilities are presented. Section 5 shows the impact of forecasting errors on individual and portfolio losses across time. Section 6 concludes.
2 The Models

2.1 Basic Structure

Credit default analysis makes use of information about loans or borrowers which is relevant for their credit quality. Credit ratings are one such type of information, representing aggregated views about default risk. A stylized model therefore might express default probabilities as functions of risk factors which drive the borrower’s credit quality.

Let $\lambda_{it}$ be the probability of borrower $i$ defaulting in time period $t$ ($i=1,...,n_i$, $t=1,...,T$). Then $\lambda_{it}$ is modeled as a function of risk factors $X_{it} = (X_{it1},...,X_{itK})'$ and $Z_t = (Z_{t1},...,Z_{tL})'$.

The vector $X_{it}$ comprises all risk factors specific to borrower $i$, e.g. firm size, leverage, product strategy, management skills etc. The vector $Z_t$ contains risk factors which are common to all or a group of firms, e.g. economic climate, jurisdiction, etc. Moreover, we assume in the following that all risk drivers affect default probabilities with a time lag, i.e.

$$\lambda_{it} = g(X_{it1},...,X_{itK},Z_t).$$

Now, let us consider a bank facing the challenge of forecasting the default probabilities $\hat{\lambda}_{iT+1}$ of the loans in the portfolio at the beginning of some future time period $T+1$. It then needs a specification of the functional relationship between the risk drivers and the default probabilities and the values of the risk drivers. The parameters of a given specification might be estimated from a hazard rate model using historical default experiences. Because today’s values of the risk drivers are known, they can easily be inserted into the estimated specification, and forecasts can be derived from this information.

This proceeding can be extended straightforwardly to a multiyear setting. For a contract with a long-term maturity, the bank needs to forecast not only the one-year default probability but rather the entire term structure. Let $D_{it}$ be the default indicator of firm $i$ with
\[ D_{it} = \begin{cases} 1 & \text{borrower } i \text{ defaults in } t \\ 0 & \text{otherwise} \end{cases} \]

\((i \in N_t, N_t \text{ being the set of borrowers held in the portfolio at the beginning of period } t; t=1,\ldots,T)\).

Then we obtain a pattern akin to figure 1. In the case of a non-default, the contract persists for another period until maturity. Otherwise, the contract defaults and is terminated. The respective likelihood of the default event happening has to be predicted today for each year until maturity. To do so requires

- models describing the relationship between risk drivers and default probabilities (PD model), and
- models for the processes of the risk drivers themselves.

Suggestions for both are offered in the remainder of this section.
2.2 A Default Probability Model

For the multiyear default process, we use the model introduced by Hamerle/Liebig/Rösch (2003), Heitfield (2005), and Hamerle/Rösch (2005). A latent default propensity or credit quality index $R_{it}$ associated with each borrower $i$ in time period $t$ ($i \in N_i, t=1,\ldots,T$) is assumed to exist. $R_{it}$ is normally distributed and is a combination of observable and latent risk drivers:

\begin{equation}
R_{it} = \beta_0 + \beta' X_{it-1} + \gamma' Z_{t-1} + S_{it}.
\end{equation}

$X_{it-1}$ are the observable borrower-specific risk drivers and $Z_{t-1}$ are the observable macroeconomic drivers. $\beta$ and $\gamma$ are the respective factor loadings or exposures and $\beta_0$ is a constant. Note that (3) assumes that firms are homogeneous with respect to the exposures. Since this assumption might not be valid for an entire loan portfolio, we refer
(3) to be valid for a homogeneous risk segment, e.g. an industry sector. Moreover, as before, we include risk factors with a time lag. The factor distributions will be outlined in more detail below.

The unobservable component \( S_{it} \) is also composed of systematic and idiosyncratic risk drivers and is given by

\[
S_{it} = \sqrt{\rho} F_t + \sqrt{1-\rho} U_{it}
\]

\((0 \leq \rho \leq 1, \ i \in N_t, \ t=1,\ldots,T)\). \( F_t \sim N(0,1) \) is a standard normally distributed systematic risk factor, and \( U_{it} \sim N(0,1) \) contains the latent borrower-specific drivers in time period \( t \). All random variables are independent.

The default indicator is modeled according to (2), and a default occurs if the latent credit quality index crosses the default threshold 0, i.e.

\[
R_{it} < 0 \Leftrightarrow D_{it} = 1.
\]

\((i \in N_t, \ t = 1,\ldots,T)\). \( X_{it-1} \) and \( Z_{t-1} \) are assumed to be risk factors with observable realizations. The conditional default probability given the observable risk factors and the unobservable common risk factor is then given by

\[
\lambda_{it}(x_{it-1}, z_{t-1}, f_t) = P(D_{it} = 1 \mid X_{it-1} = x_{it-1}, Z_{t-1} = z_{t-1}, F_t = f_t)
\]

\[
= P(R_{it} < 0 \mid X_{it-1} = x_{it-1}, Z_{t-1} = z_{t-1}, F_t = f_t)
\]

\[
= P\left(\sqrt{1-\rho} U_{it} < -\left(\beta_0 + \beta' x_{it-1} + \gamma' z_{t-1} + \sqrt{\rho} f_t\right)\right)
\]

\[
= \Phi\left(-\frac{-\left(\beta_0 + \beta' x_{it-1} + \gamma' z_{t-1} + \sqrt{\rho} f_t\right)}{\sqrt{1-\rho}}\right)
\]

where \( \Phi(\cdot) \) denotes the standard normal cumulative distribution function (cdf). Conditional on the available information, all default events are independent. Then the likelihood can be established as a Bernoulli mixture distribution, and the parameters can be estimated via maximum likelihood (see Hamerle/Rösch, 2005 for details). In our paper the calculations are performed within the NLMIXED procedure using SAS.

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2.3 Models for the Risk Factors

In the next step, the observable risk factors are modeled. We use stylized discrete-time processes in which the risk factors $X_{it}$ are modeled via linear panel models with random effects. To keep things simple, we use an already aggregated risk score $X_{it}$ as opposed to modeling various risk drivers separately. Our study uses a credit quality score developed by the Deutsche Bundesbank and is defined as:

$$x_{it} = -7.74 + 2.85 \times \text{Liabilities/Total Assets}$$
$$-0.40 \times \text{Net Sales/Total Assets}$$
$$-12.18 \times \text{Ordinary Business Income/Total Assets}$$
$$+1.93 \times \text{Current Liabilities/Total Assets}$$

The process is considered to be an M-lag structure of the form:

$$X_{it} = \delta_0 + \delta_1 X_{it-1} + \ldots + \delta_M X_{it-M} + aV_i + bF_t^X + \sigma U_{it}^X$$

$i \in N_1$, $t=1,\ldots,T$, where $\delta_0$ is a constant and $\delta_1$ to $\delta_M$ are the coefficients for the respective time-lagged credit quality score. Note that this model also assumes the existence of macroeconomic ($F_t^X$) and firm-specific ($V_i$ and $U_{it}^X$) unobserved risk drivers. $V_i \sim N(0,1)$, $F_t^X \sim N(0,1)$ and $U_{it}^X \sim N(0,1)$ are standard normally distributed and independent.

$b$ is a coefficient for the time-specific random component; $\sigma$ is a coefficient for the residual error component $U_{it}$. $a$ is a coefficient for the borrower-specific random component. The linear panel model with random effects is estimated using the MIXED procedure within SAS.

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4 See Engelmann/Hayden/Tasche (2003).
Given the values $x_{it-1}$ to $x_{it-M}$ of the time-lagged risk factors, the variance of the credit score $X_{it}$ within a risk segment is

$$\text{Var}(X_{it}|x_{it-1},...,x_{it-M}) = a^2 + b^2 + \sigma^2 = \sigma^2$$

(i $\in N_i$) and is normally distributed according to

$$X_{it}|x_{it-1},...,x_{it-M} \sim N\left(\delta_0 + \delta_1 x_{it-1} + ... + \delta_M x_{it-M}, \sigma^2 \right)$$

(i $\in N_i$, $t=1,...,T$). The credit scores of two firms are correlated at time $t$ by

$$\rho^X = \frac{b^2}{\sigma^2}.$$ 

Secondly, we must also model the macroeconomic factor $Z_t$. Again, to keep things simple, we consider only one macroeconomic variable following an AR(P) process of the form

$$Z_t = \alpha_0 + \alpha_1 Z_{t-1} + ... + \alpha_p Z_{t-p} + e E_t$$

($t=1,...,T$), where $\alpha_0$ is a constant and $\alpha_1$ to $\alpha_p$ are the coefficients for the time-lagged variable. $E_t \sim N(0,1)$ is a standard normally distributed random variable, and $e$ is a parameter. The estimation is performed using the REG procedure within SAS.

Given the realizations of the variables of former periods, $Z_t$ is distributed according to

$$Z_t|z_{t-1},...,z_{t-p} \sim N\left(\alpha_0 + \alpha_1 z_{t-1} + ... + \alpha_p z_{t-p}, e^2 \right)$$

($t=1,...,T$).
3 Empirical Results

3.1 The Data

Our empirical analysis uses data provided by the Deutsche Bundesbank. It contains default experiences and balance sheet data for German corporates from 1987 until 2003. A default is noticed when a bankruptcy or insolvency procedure pursuant to sections 17, 18 or 19 of the German Insolvency Regulation has been filed. We use data from the manufacturing industry in western Germany. The data have been enriched by macroeconomic panels provided by the Department of Statistics of the University of Regensburg.

Figure 2 shows the time series of default rates of our sample. Altogether, the data consists of 143,647 observation points for balance sheet data and 1,344 defaults.

[Graph showing default rates over years]

To estimate the credit score model, we used all firms for which a complete time series of scores was available.\footnote{We checked for a “survivorship bias” by performing the analysis with defaulted firms as well. The results were qualitatively similar.} Since the database has only a small number of observations in 1987 and 2003, those years were excluded from the estimation. We arrived at a total number of
2,362 firms with a complete history of credit scores from 1988 to 2002. Some descriptive statistics are given in Table 1. Note that a lower value of the score corresponds to higher credit quality, e.g. -7 is better than -5. As can be seen from Table 1, the credit quality declines slightly on average, and the standard deviation in the cross-section is rather large.

Table 1: Descriptive statistics of the credit quality score

<table>
<thead>
<tr>
<th>YEAR</th>
<th>$\bar{x}_t$</th>
<th>STD</th>
<th>Minimum</th>
<th>Maximum</th>
</tr>
</thead>
<tbody>
<tr>
<td>1988</td>
<td>-6.72758</td>
<td>1.82114</td>
<td>-12.69071</td>
<td>-1.89798</td>
</tr>
<tr>
<td>1989</td>
<td>-6.70122</td>
<td>1.81653</td>
<td>-12.59773</td>
<td>-1.88222</td>
</tr>
<tr>
<td>1990</td>
<td>-6.73333</td>
<td>1.82092</td>
<td>-12.60678</td>
<td>-1.83278</td>
</tr>
<tr>
<td>1991</td>
<td>-6.70478</td>
<td>1.90229</td>
<td>-12.62246</td>
<td>-2.27995</td>
</tr>
<tr>
<td>1992</td>
<td>-6.54338</td>
<td>1.92829</td>
<td>-12.38394</td>
<td>-2.27507</td>
</tr>
<tr>
<td>1993</td>
<td>-6.29403</td>
<td>1.92915</td>
<td>-12.31808</td>
<td>-2.17604</td>
</tr>
<tr>
<td>1994</td>
<td>-6.38553</td>
<td>1.83920</td>
<td>-12.38399</td>
<td>-2.10886</td>
</tr>
<tr>
<td>1995</td>
<td>-6.36236</td>
<td>1.76952</td>
<td>-12.57937</td>
<td>-2.37479</td>
</tr>
<tr>
<td>1996</td>
<td>-6.35697</td>
<td>1.78320</td>
<td>-12.84968</td>
<td>-2.14300</td>
</tr>
<tr>
<td>1997</td>
<td>-6.53869</td>
<td>1.74841</td>
<td>-14.65338</td>
<td>-2.15549</td>
</tr>
<tr>
<td>1998</td>
<td>-6.61541</td>
<td>1.78524</td>
<td>-12.51027</td>
<td>-2.00773</td>
</tr>
<tr>
<td>1999</td>
<td>-6.55397</td>
<td>1.70233</td>
<td>-12.09349</td>
<td>-2.19750</td>
</tr>
<tr>
<td>2000</td>
<td>-6.43865</td>
<td>1.69844</td>
<td>-12.60414</td>
<td>-2.18667</td>
</tr>
<tr>
<td>2001</td>
<td>-6.36917</td>
<td>1.73476</td>
<td>-13.10068</td>
<td>-2.01111</td>
</tr>
<tr>
<td>2002</td>
<td>-6.31294</td>
<td>1.73551</td>
<td>-12.18090</td>
<td>-2.16415</td>
</tr>
</tbody>
</table>

As our macroeconomic variable, we used the Ifo business climate index for West Germany, which reflects how some 7,000 German firms view the current business situation.7

7 For further information about this index, see www.ifo.de.
Figure 3 shows the time series from 1980 to 2004.

![Figure 3: Ifo business climate index for manufacturing (West Germany)](image)

3.2 Estimation Results for the Default Probability Model

We begin by estimating the default probability model and applying model (6) to the default data. We estimated the model in three stages regarding the included information:

Model 1: without observables

\[
\lambda_{it}(f_t) = \Phi\left(-\left(\beta_0 + \sqrt{\rho f_t}\right) / \sqrt{1-\rho}\right)
\]

Model 2: credit quality score only

\[
\lambda_{it}(f_t) = \Phi\left(-\left(\beta_0 + \beta_1 x_{it-1} + \sqrt{\rho f_t}\right) / \sqrt{1-\rho}\right)
\]

Model 3: credit quality score and macroeconomic variable

\[
\lambda_{it}(f_t) = \Phi\left(-\left(\beta_0 + \beta_1 x_{it-1} + \gamma_1 z_{t-1} + \sqrt{\rho f_t}\right) / \sqrt{1-\rho}\right)
\]
<table>
<thead>
<tr>
<th>Coefficient</th>
<th>Model 1</th>
<th>Model 2</th>
<th>Model 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta_0$</td>
<td>-2.3500***</td>
<td>0.7943***</td>
<td>0.0581</td>
</tr>
<tr>
<td></td>
<td>(0.02693)</td>
<td>(0.0502)</td>
<td>(0.3774)</td>
</tr>
<tr>
<td>$\beta_1$</td>
<td>-0.2939***</td>
<td>-0.2940***</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.0086)</td>
<td>(0.0086)</td>
<td></td>
</tr>
<tr>
<td>$\gamma_1$</td>
<td></td>
<td></td>
<td>0.0076*</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(0.0038)</td>
</tr>
<tr>
<td>$\sqrt{\rho}$</td>
<td>0.09645***</td>
<td>0.1102***</td>
<td>0.0971**</td>
</tr>
<tr>
<td></td>
<td>(0.0206)</td>
<td>(0.0231)</td>
<td>(0.0211)</td>
</tr>
</tbody>
</table>

Table 2: parameter estimates (standard deviations in parentheses) for default probability models 1 to 3, *** significant at 1%, ** significant at 5%, * significant at 10%

As Table 2 shows, the parameter estimates for the credit score and the macroeconomic indicator are significantly different from zero at the 1% and 10% level, respectively. The sign for the credit score is negative, meaning that a higher score is associated with a lower default probability. The sign for the business climate index is positive, indicating that a higher value corresponds to higher average credit quality in the subsequent year.

3.3 Estimation Results for the Risk Factor Models

In the next step, we estimate the following models for the credit score:

Model 1: $X_{it} = \delta_0 + aV_i + bF_t^X + \sigma U_{it}^X$

Model 2: $X_{it} = \delta_0 + \delta_1 x_{it-1} + aV_i + bF_t^X + \sigma U_{it}^X$

Model 3: $X_{it} = \delta_0 + \delta_1 x_{it-1} + \delta_2 x_{it-2} + aV_i + bF_t^X + \sigma U_{it}^X$
<table>
<thead>
<tr>
<th>Coefficient</th>
<th>Model 1</th>
<th>Model 2</th>
<th>Model 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\delta_0$</td>
<td>-6.5092***</td>
<td>-1.8567***</td>
<td>-1.1841***</td>
</tr>
<tr>
<td></td>
<td>(0.0493)</td>
<td>(0.0373)</td>
<td>(0.0390)</td>
</tr>
<tr>
<td>$\delta_1$</td>
<td></td>
<td>0.7108***</td>
<td>0.6665***</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.0039)</td>
<td>(0.0057)</td>
</tr>
<tr>
<td>$\delta_2$</td>
<td></td>
<td></td>
<td>0.1463***</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(0.0057)</td>
</tr>
<tr>
<td>$a^2$</td>
<td>1.7097***</td>
<td>0.0774***</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.0528)</td>
<td>(0.0096)</td>
<td></td>
</tr>
<tr>
<td>$b^2$</td>
<td>0.0249***</td>
<td>0.0096**</td>
<td>0.0118**</td>
</tr>
<tr>
<td></td>
<td>(0.0097)</td>
<td>(0.0040)</td>
<td>(0.0050)</td>
</tr>
<tr>
<td>$\sigma^2$</td>
<td>1.5391***</td>
<td>1.1690***</td>
<td>1.2146***</td>
</tr>
<tr>
<td></td>
<td>(0.0120)</td>
<td>(0.0105)</td>
<td>(0.0098)</td>
</tr>
</tbody>
</table>

Table 3: parameter estimates (standard deviations in parentheses) for credit score models 1 to 3, *** significant at 1%, ** significant at 5%, * significant at 10%

Model 1 only includes a constant and the three variance components. As we see from Table 3, both borrower-specific components ($a^2$ and $\sigma^2$) are much larger in size than the time-specific variance component, indicating that most of the credit quality is driven by borrower-specific risk. The reason might be that the score is constructed from balance sheet data which are rather slow to react to macroeconomic changes. However, the time-specific variance effect is significantly different from zero. In model 2, a one-year lagged rating score is included as an AR(1) process which is significantly different from zero. This means that today's credit quality is a good predictor of next year's credit quality. The other effects are still significant. Model 3 includes credit scores from two time-lagged
periods as a stationary AR(2) process, both of which are significant.\textsuperscript{8} As a result, the borrower-specific variance component turned out to be insignificant and was therefore excluded from the estimation. The time-specific and the remaining error component are still significant, though of smaller size. Calculating the implied correlation of the credit scores according to (10) yields approximately 0.96%.

The last estimation step concerns the macroeconomic variable for which we used the AR processes:

Model 1: \( Z_t = \alpha_0 + \alpha_1 z_{t-1} + e_{t} \)

Model 2: \( Z_t = \alpha_0 + \alpha_1 z_{t-1} + \alpha_1 z_{t-2} + e_{t} \)

<table>
<thead>
<tr>
<th>Coefficients</th>
<th>Model 1</th>
<th>Model 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \alpha_0 )</td>
<td>50.9681***</td>
<td>66.8280***</td>
</tr>
<tr>
<td></td>
<td>(17.8557)</td>
<td>(20.4072)</td>
</tr>
<tr>
<td>( \alpha_1 )</td>
<td>0.4752**</td>
<td>0.5314**</td>
</tr>
<tr>
<td></td>
<td>(0.1852)</td>
<td>(0.2116)</td>
</tr>
<tr>
<td>( \alpha_2 )</td>
<td>-0.2172</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.2075)</td>
<td></td>
</tr>
<tr>
<td>( e^2 )</td>
<td>43.1121***</td>
<td>41.0001***</td>
</tr>
<tr>
<td></td>
<td>(12.9988)</td>
<td>(12.9654)</td>
</tr>
</tbody>
</table>

Table 4: parameter estimates (standard deviations in parentheses) for macroeconomic variables, models 1 and 2, *** significant at 1%, ** significant at 5 %, * significant at 10%

As Table 4 shows, the AR(2) process turns out to be insignificant; we therefore limit our following analyses to the AR(1) process (Model 1).

\textsuperscript{8} We did not include longer time lags because of the small time series sample size.
It should be noted, however, that it is not the purpose of the paper to find the best process for the IFO business climate index. Here, more complex models such as VAR processes might be more adequate. All we are looking for is simple processes which enable us to gain an insight into the effects of forecasting errors on default probability and loss distribution forecasts.

4 Multiyear Forecasts

In this section we describe how to get multiyear forecasts for default probabilities and loss distributions in our model framework. To obtain these we have to forecast the risk factors credit score and IFO business climate index. We demonstrate this in subsection 4.2. One key advantage of the present model is that we can distinguish between idiosyncratic and systematic forecasting errors. In subsection 4.3 we show explicit the separation between this two risk components for period \( T+2 \) and \( T+3 \). Another advantage is that we can derive explicit formulae for the density of individual default probability forecasts. The last two subsections illustrate the forecasting of multiyear loss distributions and the impact of forecasting errors on the key risk figures Expected Loss, Value at Risk and Expected Shortfall.

4.1 Forecasts for the Default Probabilities

Once the parameters of model (6) have been estimated using data from periods 1 to \( T \), default probability forecasts can be made for the next time period \( T+1 \), given the information up to period \( T \). In our case, this is the current credit score \( x_{iT} \) and the current business climate condition \( z_T \).

The latent random effect in (6) is not observable for the future time period; hence it might be a good idea to forecast the unconditional default probabilities. Let \( \hat{\beta}_0 \), \( \hat{\beta}_1 \) and \( \hat{\gamma}_1 \) be the parameter estimates from (6). Then we can forecast the default probabilities by

\[
\begin{align*}
\hat{\lambda}_{iT+1}(x_{iT}, z_T) &= \hat{\lambda}(D_{iT+1} = 1 \mid X_{iT} = x_{iT}, Z_T = z_T) \\
&= \phi(- (\hat{\beta}_0 + \hat{\beta}_1 x_{iT} + \hat{\gamma}_1 z_T)),
\end{align*}
\]
For multiyear forecasts beyond year $T + 1$, the risk drivers themselves have to be forecast. Let $\hat{x}_{iT+1}$ and $\hat{z}_{iT+1}$ be forecasts for $T + 1$; we can then forecast the default probability for period $T + 2$ according to

$$\hat{\lambda}_{iT+2} (\hat{x}_{iT+1}, \hat{z}_{iT+1})$$

$$= \hat{p}(D_{iT+2} = 1 | X_{iT+1} = \hat{x}_{iT+1}, Z_{iT+1} = \hat{z}_{iT+1})$$

$$= \phi(- (\beta_0 + \beta_1 \hat{x}_{iT+1} + \gamma_1 \hat{z}_{iT+1}))$$

An analogous approach can be used for subsequent years.

### 4.2 Forecasts for the Risk Factors

As obviously follows from (14), multiyear default probabilities require forecasts for the risk factors. In our setting, we use models (7) and (11) for this purpose. For the score, we obtain for period $T + 1$

$$X_{iT+1} | x_{iT}, \ldots, x_{iT-M+1} = \delta_0 + \delta_1 x_{iT} + \ldots + \delta_M x_{iT-M+1} + aV_i + bF_{iT+1}^X + \sigma U_{iT+1}^X.$$  

Regarding period $T + 2$, using the information up to $T$ yields

$$X_{iT+2} | x_{iT}, \ldots, x_{iT-M+1} = (1 + \delta_1) \delta_0 + \sum_{s=1}^{M-1} (\delta_1 \delta_s + \delta_{s+1}) x_{iT-s+1} + \delta_1 \delta_M x_{iT-M+1}$$

$$+ aV_i (1 + \delta_1) + b(F_{IT+2}^X + \delta_1 F_{IT+1}^X) + \sigma (U_{IT+2}^X + \delta_1 U_{IT+1}^X).$$
We proceed by means of successively insertion for the subsequent years as well.

Given the values \( x_{iT} \) to \( x_{iT-M+1} \) of the score, we obtain the expectation of \( X_{iT+1} \) and \( X_{iT+2} \) by

\[
E\left(X_{iT+1}|x_{iT}, \ldots, x_{iT-M+1}\right) = \delta_0 + \delta_1 x_{iT} + \ldots + \delta_M x_{iT-M+1}
\]

\[
E\left(X_{iT+2}|x_{iT}, \ldots, x_{iT-M+1}\right) = (1 + \delta_1) \delta_0 + \sum_{s=1}^{M-1} (\delta_1 \delta_s + \delta_{s+1}) x_{iT-s+1} + \delta_1 \delta_M x_{iT-M+1}.
\]

The respective variances are given by

\[
\text{Var}\left(X_{iT+1}|x_{iT}, \ldots, x_{iT-M+1}\right) = \sigma^2
\]

\[
\text{Var}\left(X_{iT+2}|x_{iT}, \ldots, x_{iT-M+1}\right) = a^2 (1 + \delta_1)^2 \text{Var}(V_i) + b^2 \text{Var}\left(F_{T+2}^X + \delta_1 F_{T+1}^X\right)
\]

\[
+ \sigma^2 \text{Var}\left(U_{iT+2}^X + \delta_1 U_{iT+1}^X\right)
\]

\[
= \sigma^2 + \delta_1^2 \sigma^2 + 2\delta_1 a^2
\]

\[
= \text{Var}(X_{iT+1}|x_{iT}, \ldots, x_{iT-M+1}) + \delta_1^2 \sigma^2 + 2\delta_1 a^2
\]

Note that the variance increases through transition from \( T+1 \) to \( T+2 \) by \( \delta_1^2 \sigma^2 + 2\delta_1 a^2 \).

For subsequent years \( T+3, T+4, \ldots \) we proceed similarly, thereby further increasing the variance.

Analogously, we obtain forecasts for the macroeconomic variable by

\[
Z_{T+1}|z_T, \ldots, z_{T-P+1} = \alpha_0 + \alpha_1 z_T + \ldots + \alpha_P z_{T-P+1} + eE_{T+1}
\]

\[
Z_{T+2}|z_T, \ldots, z_{T-P+1} = (1 + \alpha_1) \alpha_0 + \sum_{s=1}^{P-1} (\alpha_1 \alpha_s + \alpha_{s+1}) z_{T-s+1} + \alpha_1 \alpha_P z_{T-P+1} + e\left(E_{T+2} + \alpha_1 E_{T+1}\right)
\]

for given values \( z_T \) to \( z_{T-P+1} \).
Their conditional expectations are

\[ E(Z_{T+1}|z_T, \ldots, z_{T-p+1}) = \alpha_0 + \alpha_1 z_T + \ldots + \alpha_p z_{T-p+1} \]

(23)

\[ E(Z_{T+2}|z_T, \ldots, z_{T-p+1}) = (1 + \alpha_1)\alpha_0 + \sum_{s=1}^{P-1} (\alpha_1 \alpha_s + \alpha_{s+1})z_{T-s+1} + \alpha_1 \alpha_p z_{T-p+1}. \]

(24)

The variances are obtained by

\[ \text{Var}(Z_{T+1}|z_T, \ldots, z_{T-p+1}) = e^2 \]

(25)

\[ \text{Var}(Z_{T+2}|z_T, \ldots, z_{T-p+1}) = e^2 \text{Var}(E_{T+2} + \alpha_1 E_{T+1}) \]

\[ = e^2 + \alpha_1^2 e^2 \]

\[ = \text{Var}(Z_{T+1}|z_T, \ldots, z_{T-p+1}) + \alpha_1^2 e^2 \]

(26)

Note that the variance increases by \( \alpha_1^2 e^2 \) from \( T+1 \) to \( T+2 \). Subsequent years are obtained analogously.

4.3 Forecasting Errors in Future Periods

As shown in section 4.1, given our models and model parameters, forecasting default probabilities (and hence loss distributions, given values for Exposures and Recovery Rates) requires only knowledge of current risk factors, whereas for subsequent years forecasts for the risk factors themselves are required. In our models, forecasting risk for the risk factors in \( T+1 \) can be measured by the variances given in (19) and (25).

For ease of exposition, we assume that the parameters are known, i.e. \( \hat{\beta}_1 = \beta_1 \) and \( \hat{\gamma}_1 = \gamma_1. \)

Otherwise, estimation error for the parameters is additionally induced; see e.g. Loeffler (2003), Hamerle/Knapp/Liebig/Wildenauer (2005), and Hamerle/Rösch (2005).
Then forecasting risk for period \( T+2 \) is given by

\[
Risk_{pr}(T+2) = \text{Var}(\beta_1 \hat{x}_{iT+1} + \gamma_1 \hat{z}_{T+1})
\]

\[
= \beta_1^2 \sigma^2 + \gamma_1^2 \epsilon^2
\]

\[
= b_{T+2}^* + c_{T+2}^*
\]

where \( b_{T+2}^* = \beta_1^2 b^2 + \gamma_1^2 \epsilon^2 \), \( c_{T+2}^* = \beta_1^2 (a^2 + \omega^2) \).

\( b_{T+2}^* \) is the variance component which only depends on the time effects and thus affects all borrowers within the segment. Therefore it is the non-diversifiable part of the variance. On the other hand, \( c_{T+2}^* \) contains the borrower-specific variance components and is therefore diversifiable.

Moreover, \( b_{T+2}^* \) and the asset correlation \( \rho \) determine the correlation of default probability forecasts.

For period \( T+3 \), forecasting risk is given by using (20) and (26) as

\[
Risk_{pr}(T+3) = \text{Var}(\beta_1 \hat{x}_{iT+2} + \gamma_1 \hat{z}_{T+2})
\]

We obtain

\[
Risk_{pr}(T+3) = \beta_1^2 \left( \left( 1 + \delta_1^2 \right) \sigma^2 + 2 \delta_1 a^2 \right) + \gamma_1^2 \left( 1 + \alpha_1^2 \right) \epsilon^2
\]

\[
= b_{T+3}^* + c_{T+3}^*
\]

where \( b_{T+3}^* = \beta_1^2 \left( 1 + \delta_1^2 \right) b^2 + \gamma_1^2 \left( 1 + \alpha_1^2 \right) \epsilon^2 \), \( c_{T+3}^* = \beta_1^2 \left( 1 + \delta_1^2 \right) (a^2 + \omega^2) + 2 \delta_1 a^2 \).

### 4.4 Distribution of Individual Default Probability Forecasts

Due to results in Gordy (2000) and Koyluoglu/Hickman (1998) we can state explicit formulae for addressing forecasting risk of default probabilities for periods beyond \( T+1 \).

Conditional on a particular realization of the risk factors \( \hat{x}_{iT+1} \) and \( \hat{z}_{T+1} \) we obtain the default probability forecast for \( T+2 \) as

\[
\hat{x}_{iT+2}(\hat{x}_{iT+1}, \hat{z}_{T+1}) = \Phi(- (\beta_0 + \beta_1 \hat{x}_{iT+1} + \gamma_1 \hat{z}_{T+1})).
\]
These forecasts are subject to errors because of the random nature of the risk factor forecasts. Its mean is given by

\[
\hat{\lambda}_{iT+2} = E\left[\hat{\lambda}_{iT+2} \left(\hat{X}_{iT+1}, \hat{Z}_{T+1}\right)\right] = \Phi\left(\frac{-\beta_0 + \beta_1 E\left(\hat{X}_{iT+1}\right) + \gamma_1 E\left(\hat{Z}_{T+1}\right)}{\sqrt{1 + b_{T+2}^* + c_{T+2}^*}}\right)
\]

where the expectations of the risk factor forecasts are given analogously to (17) and (23). For ease of exposition we skip the notation that these forecasts depend on values of year \(T\).

According to Gordy (2000) we obtain the variance

\[
\text{Var}\left[\hat{\lambda}_{iT+2} \left(\hat{X}_{iT+1}, \hat{Z}_{T+1}\right)\right] = E\left[\Phi\left(-\left(\beta_0 + \beta_1 \hat{X}_{iT+1} + \gamma_1 \hat{Z}_{T+1}\right)\right)^2\right] - \bar{\lambda}_{iT+2}^2
\]

\[
= \phi_2\left(\Phi^{-1}\left(\hat{\lambda}_{iT+2}\right), \Phi^{-1}\left(\hat{\lambda}_{iT+2}\right), \xi\right) - \bar{\lambda}_{iT+2}^2
\]

Where \(\phi_2\left(\cdot, \cdot, \xi\right)\) denotes the distribution function of the two dimensional standard normal distribution with correlation \(\xi\), and \(\xi = \frac{b_{T+2}^* + c_{T+2}^*}{1 + b_{T+2}^* + c_{T+2}^*}\).

Its density at point \(p\) can also be given due to a result in Koyluoglu/Hickman (1998) as

\[
g(p) = \frac{\sqrt{1 - \xi} \phi\left(\frac{\Phi^{-1}\left(\hat{\lambda}_{iT+2}\right) - \phi^{-1}(p) \sqrt{1 - \xi}}{\sqrt{\xi}}\right)}{\sqrt{\xi} \phi\left(\phi^{-1}(p)\right)}
\]

where \(\phi(\cdot)\) denotes the standard normal density function.

### 4.5 Forecasting Default Rates and Loss Distributions

Within a credit risk model, we are not only interested in default probabilities but also in future portfolio loss distributions from which we might derive risk measures such as Expected Loss, Value at Risk or Expected Shortfall.
For ease of exposition, we assume an exposure unit of one for each borrower and a homogeneous loss (rate) given default (LGD) of 45%. Then the portfolio loss within a future period \( T+k, k=1,2,\ldots \) is given by

\[
L_{T+k} = \sum_{i \in N_{T+k}} 0.45 \cdot D_{iT+k}
\]

where \( N_{T+k} \) is the set of borrowers held in the portfolio at the beginning of period \( T+k \).

We might also define the percentage loss (relative to total exposure) as

\[
L^*_{T+k} = \frac{1}{N_{T+k}} L_{T+k}.
\]

Due to given fixed exposures and LGDs, we only need the distribution of defaults in order to derive the distribution of portfolio losses.

For the next period \( T+1 \), we can again insert the known values for \( x_{iT} \) and \( z_T \). Given a realization of the latent systematic factor \( f_{T+1} \), the defaults are independent. Mixture across the distribution of the latent factor yields the joint distribution of defaults

\[
P(D_{iT+1} = d_{iT+1}; i \in N_{T+1}) =
\int_{-\infty}^{+\infty} \prod_{i \in N_{T+1}} \hat{\lambda}_{iT+1}(x_{iT}, z_T, f_{T+1})^{d_{iT+1}} \left[1 - \hat{\lambda}_{iT+1}(x_{iT}, z_T, f_{T+1})\right]^{1-d_{iT+1}} \varphi(f_{T+1}) df_{T+1}
\]

where \( d_{iT+1} \in \{0,1\} \) and \( \hat{\lambda}_{iT+1}(x_{iT}, z_T, f_{T+1}) \) according to (6) with \( t=T+1 \). Given the parameters, we evaluate this distribution by simulation as described in section 5.2

Moving one period ahead to \( T+2 \), we have to forecast again the risk factors \( x_{iT+1} \) und \( z_{T+1} \) as described in the previous section.
For given forecasts $\hat{x}_{T+1}$ and $\hat{z}_{T+1}$ we obtain the joint distribution of defaults for period $T+2$

\begin{equation}
P(D_{iT+2} = d_{iT+2}; i \in N_{T+2} | \hat{x}_{iT+1}, \hat{z}_{T+1}) = \int \prod_{i \in N_{T+2}} \hat{\lambda}_{iT+2}(\hat{x}_{iT+1}, \hat{z}_{T+1}, f_{T+2}) \text{d}f_{T+2} \left[ -d_{iT+2}(\hat{x}_{iT+1}, \hat{z}_{T+1}, f_{T+2}) \right]^{-d_{iT+2}} \phi(f_{T+2}) \text{d}f_{T+2}
\end{equation}

with conditional default probabilities

\begin{equation}
\hat{\lambda}_{iT+2}(\hat{x}_{iT+1}, \hat{z}_{T+1}, f_{T+2}) = \Phi \left( \frac{-\beta_0 + \beta_1 \hat{x}_{iT+1} + \gamma_1 \hat{z}_{T+1} + \sqrt{\rho} f_{T+2}}{\sqrt{1 - \rho}} \right).
\end{equation}

For a given pair of forecasts for the risk factors, we come up with a whole loss distribution. Simulating many pairs of risk factors yields a distribution of loss distributions.

### 4.6 Forecasts for Key Figures of the Loss Distributions

Basically, we could aggregate the distributions which we obtain for each simulated pair of risk factors to one single unconditional loss distribution. However, in most cases we are mainly interested in key risk figures of the distributions, such as Expected Loss, Value at Risk or Expected Shortfall.

Recall that, for period $T+1$, the values of the risk factor are known and we come up with one single loss distribution and its risk figures.

In the next period, $T+2$, we once again have to generate pairs of forecasts $\hat{x}_{iT+1}$ and $\hat{z}_{T+1}$. Given a pair of forecasts, we obtain a loss distribution and calculate the key figures of the distribution given $\hat{x}_{iT+1}$ and $\hat{z}_{T+1}$. We then repeat this for many pairs of risk factor forecasts and investigate what the distribution of the risk figures looks like.

Diversification effects might affect the dispersion of the distribution of these risk figures. If forecasting errors can be diversified, the distribution will be rather narrow. Otherwise, ignoring systematic forecasting risk might lead to an underestimation of the portfolio risk.
We now return to the data and empirically analyze the pattern of the effects of systematic risks in a multiyear setting.

5 Multiyear Forecasting Results

5.1 Multiyear Default Probability Forecasts

What we need for forecasting each borrower’s default probability for $T+1$ is the current credit score $x_{iT}$, the lagged credit score $x_{iT-1}$ (because we used an AR(2) process) and the current value $z_T$ of the macroeconomic variable. These values for three exemplary borrower classes (good, medium, minor credit quality) are shown in Table 5.

<table>
<thead>
<tr>
<th>Risk factor</th>
<th>Minor credit quality</th>
<th>Medium credit quality</th>
<th>Good credit quality</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x_{iT}$</td>
<td>-5.0</td>
<td>-6.5</td>
<td>-8.0</td>
</tr>
<tr>
<td>$x_{iT-1}$</td>
<td>-4.5</td>
<td>-6.5</td>
<td>-8.5</td>
</tr>
<tr>
<td>$z_T$</td>
<td>94.75</td>
<td>94.75</td>
<td>94.75</td>
</tr>
</tbody>
</table>

Table 5: Risk factor values for three credit quality classes

Therefore, given our parameter estimates, we obtain a point forecast for each borrower in the first year. For the second and third year, $T+2$ respectively $T+3$, the distribution functions for each borrower can be evaluated using numerical methods such as the Gauss-Hermite procedure.

Figure 4 shows the distributions, and Tables 6 through 8 show some descriptive statistics for the three borrowers. While the default probability forecasts for the first year are given, the outcomes for the second and third year are affected by forecasting error. Note first that the proposed processes for the score and the macroeconomic variable cause their forecasts to move to their long-term average. Thus, the PD forecasts for the minor as well as for the
good credit quality borrower move to medium quality in an intermediate macroeconomic surrounding. The median of the forecasts for the minor quality borrower decreases form 124 bp in $T+1$ to 90 bp in $T+3$, whereas the good quality borrower deteriorates from 9 bp to 13 bp. The median of the medium quality borrower remains constant throughout.

Secondly, note that the impact of forecasting error is remarkably high. For example, the mean forecast for the medium quality borrower in year $T+2$ is approximately 50 bp, the 99% quantile of the forecast distribution is more than 270 bp, meaning that there is a 1% chance that the probability in the second year increases to more than 270 bp. This uncertainty even increases if we go one year ahead to $T+3$ and is considerable for all three borrowers.

<table>
<thead>
<tr>
<th>Year</th>
<th>Mean forecast PD</th>
<th>Median</th>
<th>95% percentile</th>
<th>99% percentile</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T+1$</td>
<td>0.0124</td>
<td>0.0124</td>
<td>0.0124</td>
<td>0.0124</td>
</tr>
<tr>
<td>$T+2$</td>
<td>0.0143</td>
<td>0.0106</td>
<td>0.0389</td>
<td>0.0618</td>
</tr>
<tr>
<td>$T+3$</td>
<td>0.0139</td>
<td>0.0090</td>
<td>0.0431</td>
<td>0.0740</td>
</tr>
</tbody>
</table>

Table 6: Statistics of the PD forecast distribution for a minor credit quality borrower
<table>
<thead>
<tr>
<th>Year</th>
<th>Mean forecast PD</th>
<th>Median</th>
<th>95% percentile</th>
<th>99% percentile</th>
</tr>
</thead>
<tbody>
<tr>
<td>T+1</td>
<td>0.0036</td>
<td>0.0036</td>
<td>0.0036</td>
<td>0.0036</td>
</tr>
<tr>
<td>T+2</td>
<td>0.0054</td>
<td>0.0036</td>
<td>0.0160</td>
<td>0.0275</td>
</tr>
<tr>
<td>T+3</td>
<td>0.0063</td>
<td>0.0036</td>
<td>0.0210</td>
<td>0.0388</td>
</tr>
</tbody>
</table>

Table 7: Statistics of the PD forecast distribution for a medium credit quality borrower

<table>
<thead>
<tr>
<th>Year</th>
<th>Mean forecast PD</th>
<th>Median</th>
<th>95% percentile</th>
<th>99% percentile</th>
</tr>
</thead>
<tbody>
<tr>
<td>T+1</td>
<td>0.0009</td>
<td>0.0009</td>
<td>0.0009</td>
<td>0.0009</td>
</tr>
<tr>
<td>T+2</td>
<td>0.0018</td>
<td>0.0011</td>
<td>0.0058</td>
<td>0.0107</td>
</tr>
<tr>
<td>T+3</td>
<td>0.0026</td>
<td>0.0013</td>
<td>0.0093</td>
<td>0.0187</td>
</tr>
</tbody>
</table>

Table 8: Statistics of the PD forecast distribution for a high credit quality borrower
Figure 4: PD Forecast Distributions
5.2 Multiyear Forecasts of Portfolio Risk Characteristics

As we move from the single borrower forecast to the portfolio forecast, we make use of our database, which contains 2,362 firms for which a credit score has been calculated for the years 2002 and 2003. That is, the year 2003 equals the end of our estimation sample $T$.

Analogously to the previous section, we want to investigate the impact of forecasting risk on multiyear forecasts for Expected Losses, Value at Risk and Expected Shortfall. Those multiyear figures are affected by forecasting risk as well, so we are therefore interested in the distributions of these figures. We proceed as follows:

- **Step 1:** We simulate future paths for credit quality scores of each firm and the macroeconomic variable. Note that the random draws of the systematic components $f_{T+1}^X, f_{T+2}^X$ and $e_{T+1}$ are identical for all firms, whereas the idiosyncratic components $u_{iT+1}^X$ and $u_{iT+2}^X$ are drawn independently for all firms. Thus, we obtain forecasts $\hat{z}_{T+1}$ and $\hat{z}_{T+2}$, and $\hat{x}_{iT+1}$ and $\hat{x}_{iT+2}$ respectively ($i=1,...,2,362$).

- **Step 2:** Given the path from step 1, we randomly draw realizations of the unobservable random factor $f_{T+k}^X$, $k=1,2,3$. Together with $x_{iT}$ and $z_T$ and the forecasts $\hat{x}_{iT+1}$, $\hat{x}_{iT+2}$, $\hat{z}_{T+1}$ and $\hat{z}_{T+2}$, we calculate conditional default probabilities for each year.

- **Step 3:** Due to conditional independence of defaults, we draw random Bernoulli defaults using the conditional default probabilities. Assuming again EAD=1 and LGD=0.45, we obtain loss realizations for year $T+1$, $T+2$ and $T+3$.

- **Step 4:** We repeat steps 2 and 3 10,000 times for each period. This generates a loss distribution for the given path of the risk factor forecasts for $T+1$, $T+2$ and $T+3$. From the loss distributions, we calculate the Expected Loss, the Value at Risk and the Expected Shortfall.

- **Step 5:** We continue to repeat steps 1 to 4 for 1,000 times. This generates a sample of 1,000 loss distributions, and draws for Expected Losses, Value at Risks, and Expected Shortfalls.

---

10 Defaulting borrowers are replaced by “twins”. Thus, the portfolio consists of 2,362 borrowers each year.
The following tables recapitulate the simulation results using the parameter estimates of our models from Tables 2 to 4, referring to Case 1.

For period $T+1$ there again exist given values $x_{iT}$ and $z_{i}$ for the risk factors. We therefore come up with a single loss distribution with the risk figures Expected Loss, VaR (99%) and Expected Shortfall (99%). In other words, using our models, there is no one-year forecasting risk.

Regarding years $T+2$ and $T+3$, we see that the distributions of the forecasted Expected Losses, and even for the VaR and the Expected Shortfalls (for given $\hat{z}_{T+1}$, $\hat{z}_{T+2}$, $\hat{x}_{T+1}$ and $\hat{x}_{T+2}$), are remarkably narrow, despite the major uncertainty of forecasts for individual default probability of section 5.1. The reason behind that is that idiosyncratic risk constitutes the main part of forecasting risk to credit quality. As the estimation results of the panel model from section 3.3 revealed, the systematic component in the score evolution is rather small. While individual credit quality forecasts are affected by systematic and idiosyncratic components, a huge part of the idiosyncratic risk might be diversified when forecasting loss distributions for large portfolios. More precisely, we obtain the following from Tables 2 to 4:

\[
\begin{align*}
\hat{\beta}_1 &= 0.2940 & \hat{a} &= 0 & \hat{\sigma}^2 &= 43.1121 \\
\hat{\gamma}_1 &= 0.0076 & \hat{b}^2 &= 0.0118 & \hat{\omega}^2 &= 1.2146 \\
\end{align*}
\]

We then calculate

\[
\begin{align*}
\hat{b}_{T+2}^* &= 0.00351 & \hat{c}_{T+2}^* &= 0.10499 \\
Risk_{pr}(T + 2) &= 0.1085 .
\end{align*}
\]

This shows that the systematic portion of the entire forecasting risk in period $T+2$ is $0.00351/0.1085 = 0.0324 = 3.24\%$, which means that idiosyncratic risk makes up nearly 97\% of all forecasting risk. For period $T+3$, the respective values are $\hat{b}_{T+3}^* = 0.00452$, $\hat{c}_{T+3}^* = 0.15163$ and $Risk_{pr}(T + 3) = 0.15615$. The systematic part here is just 2.89\%. Therefore, in future forecasts for loss distributions, many diversifiable risk sources are
included, and the distributions of Expected Losses, VaRs and Expected Shortfalls are narrow.

To investigate the effect of diversification in more detail, consider (an artificial) Case 2 with a considerably higher systematic risk proportion. For example, let us specify

\[
\hat{\beta}^2 = 0.5199, \quad \hat{\epsilon}^2 = 6.520 \quad \text{and} \quad \hat{\omega}^2 = 0.731.
\]

Then we have

\[
b^*_{T+2} = 0.0453 \quad \text{and} \quad c^*_{T+2} = 0.0632.
\]

Note that the total forecasting risk in \(T+2\) remains unchanged. However, the proportion of systematic risk in \(T+2\) is now \(0.0453/0.1085 = 41.76\%\). A similar result holds for \(T+3\), where we obtain \(b^*_{T+2} = 0.0653 \quad \text{and} \quad c^*_{T+2} = 0.0913\) without changing total forecasting risk. The proportion of systematic risk is now 41.73\%.

Tables 9 to 11 and Figures 5 to 7 show that the higher proportion of non-diversifiable risk translates into considerably wider distributions of the key risk figures. For example, the 95\% percentile of the VaR (99\%) distribution increases in year \(T+2\) by 55 per cent from 1.197 to 1.856, and in year \(T+3\) by 65 per cent from 1.217 to 2.035. Similar statements hold for the other figures as well.

<table>
<thead>
<tr>
<th>Case 1</th>
<th>Case 2</th>
</tr>
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<tbody>
<tr>
<td>(T+1)</td>
<td>(T+2)</td>
</tr>
<tr>
<td>Mean</td>
<td>0.516</td>
</tr>
<tr>
<td>Median</td>
<td>0.516</td>
</tr>
<tr>
<td>90% percentile</td>
<td>0.516</td>
</tr>
<tr>
<td>95% percentile</td>
<td>0.516</td>
</tr>
<tr>
<td>99% percentile</td>
<td>0.516</td>
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</table>

Table 9: Descriptive statistics of Expected Loss distributions
<table>
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<th></th>
<th>Case 2</th>
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<td></td>
<td>$T+1$</td>
<td>$T+2$</td>
<td>$T+3$</td>
<td>$T+1$</td>
</tr>
<tr>
<td>Mean</td>
<td>0.958</td>
<td>0.979</td>
<td>0.966</td>
<td>0.958</td>
</tr>
<tr>
<td>Median</td>
<td>0.958</td>
<td>0.958</td>
<td>0.958</td>
<td>0.958</td>
</tr>
<tr>
<td>90% percentile</td>
<td>0.958</td>
<td>1.137</td>
<td>1.157</td>
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<tr>
<td>95% percentile</td>
<td>0.958</td>
<td>1.197</td>
<td>1.217</td>
<td>0.958</td>
</tr>
<tr>
<td>99% percentile</td>
<td>0.958</td>
<td>1.317</td>
<td>1.357</td>
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Table 10: Descriptive statistics of VaR (99%) distributions

<table>
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<th>Case 2</th>
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<td>$T+1$</td>
<td>$T+2$</td>
<td>$T+3$</td>
<td>$T+1$</td>
</tr>
<tr>
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<td>1.064</td>
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<tr>
<td>Median</td>
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<tr>
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<td>1.244</td>
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<tr>
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<td>1.309</td>
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<tr>
<td>99% percentile</td>
<td>1.043</td>
<td>1.436</td>
<td>1.456</td>
<td>1.043</td>
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Table 11: Descriptive statistics of Expected Shortfall (99%) distributions
Figure 5: Empirical distributions of Expected Losses conditional on the forecasts of the risk factors
<table>
<thead>
<tr>
<th>VaR (99%)</th>
<th>Case 1</th>
<th>Case 2</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image1" alt="Graph" /></td>
<td><img src="image2" alt="Graph" /></td>
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<tr>
<td><img src="image3" alt="Graph" /></td>
<td><img src="image4" alt="Graph" /></td>
<td></td>
</tr>
<tr>
<td><img src="image5" alt="Graph" /></td>
<td><img src="image6" alt="Graph" /></td>
<td></td>
</tr>
</tbody>
</table>

Figure 6: Empirical distributions of VaRs (99%) conditional on the forecasts of the risk factors
Figure 7: Empirical distributions of expected shortfalls (99%) conditional on the forecasts of the risk factors
6 Summary

The present paper provides a method for forecasting multiyear default probabilities and loss distributions. Default probabilities are related to risk factors, for which we specify econometric processes using data provided by the Deutsche Bundesbank.

We find that individual default probability forecasts are affected by a huge amount of noise. However, the bulk of this noise (or uncertainty) is idiosyncratic and therefore diversifiable if loss distributions are forecasted for larger portfolios. An additional risk diversification might therefore not be obtained by changing the single exposure sizes, unless a few exposures are dominant within the portfolio.\textsuperscript{11}

Since our models are kept rather simple, further research could focus on more sophisticated processes and other datasets. The extension to multi-sector models and the possibilities and benefits therein might be topics for future research as well.

7 References


\textsuperscript{11} See Hanson/Pesaran/Schuermann (2005)


