Parameterizing Credit Risk Models

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Key Words: Credit Risk Models, Model Risk, Estimation Risk

JEL Classification: G20, G28, C51

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Abstract

The present paper shows how the parameters of three popular portfolio credit risk models can be empirically estimated by banks using a Maximum Likelihood framework. We apply the method to a database of German firms provided by Deutsche Bundesbank and analyze the inclusion of macroeconomic and borrower specific rating factors. Given the uniform ML estimation methodology, we compare the parameter estimates and the forecasted loss distributions for the credit risk models and find that they perform in very similar ways, in contrast to the differences found in some previous studies. We also propose an approach for addressing estimation errors. Our findings suggest that for a financial institution "model risk", i.e. the risk of choosing the "wrong" credit model, may be considerably reduced.

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1 Introduction

In recent years the needs for professional skills in the modeling and management of credit risk have rapidly increased and credit risk modeling has become an important topic in the field of finance and banking. While in the past most interests were in the assessments of the individual creditworthiness of an obligor, more recently a focus is on modeling the risk inherent in the entire banking portfolio.

In the banking industry there are some well-established paradigms for portfolio credit risk modeling. Among the most prominent models are JP Morgan’s CreditMetrics (see Gupton/Finger/Bhatia, 1997, and Finger, 1998, "CM" hereafter), CreditRisk+ ("CR+") by Credit Suisse (see Credit Suisse Financial Products, 1997), and CreditPortfolioView ("CPV", see Wilson 1997a,b). In these approaches credit quality and default of each firm is modeled via a factor structure. Correlations between defaults of different firms are induced due to the exposure to common systematic factors.

Koyluoglu/Hickman (1998a,b,) and Gordy (2000) show that the underlying mathematical structures of the models are very similar and that they primarily differ with respect to the presumed distributions of the risk factors. Due to these distributions the models are sometimes called the Gaussian (CM), the Logistic-Normal (CPV), and the Gamma-Exponential (CR+) model (see e.g. Frey/McNeil, 2003).

Because of the mathematical similarity one would expect the models to yield similar outputs when employed in practice by banks. Indeed, Koyluoglu/Hickman (1998a,b), Finger (1998), and Gordy (2000) show that once the parameters are given in one model, they can be mapped into the framework of the other models, thus leading to similar risk measures, such as the Value-at-Risk. Similar analyses are carried out by Crouhy/Galai/Mark (2000) with a bond portfolio and Bucay/Rosen (2001) using an artificial portfolio.

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1 See also Hamerle/Liebig/Rösch (2003b) for an empirical example of the reparameterizations using data from the G-7 countries.
However, in empirical applications the derivation – or estimation – of the relevant input parameters is a crucial part of any risk model. Here, the model developers suggest different proceedings. CM obtains estimates for the default probabilities from historical default rates available by rating agencies and the relevant correlation parameters from a linear multi-factor model of stock returns. CR+ also recommends using historical default rates as estimates for default probabilities but derives the co-movement parameters from the historical standard deviations of the default rates rather than from stock return data. CPV models in a first step default rates via a regression model with macroeconomic risk factors which themselves are modeled as autoregressive processes. In the second step default and migration rates of rating grades are then scaled due to the outcomes of the regression.

An extensive study with empirical analyses regarding the various models is due to Koyluoglu/Bangia/Garside (2000). Instead of mapping model parameters into each other model as in Koyluoglu/Hickman (1998a,b) the authors parameterize CM and CR+ as originally suggested in the model documentations for various portfolios. They find significant differences in portfolio risk levels even for the same underlying portfolio. As a main conclusion they state: "This study makes the case for a deeper analysis of credit risk parameter estimation technique." Elsewhere they conclude: "A 'naïve' comparison of the models, with parameters estimated from different data using different techniques, is quite likely to produce significantly different results for the same portfolio" (Koyluoglu/Hickman, 1998a,b).

Also, an important project was led by the Institute of International Finance and the International Swaps and Derivatives Association (IIF/ISDA, 2000). They provided portfolio managers from 25 banks in 10 countries with standard portfolios and asked them to report their resulting risk measure. In the end the study produced quite different outcomes and the IIF/ISDA concluded that the differences should be attributed to model inputs, particularly the correlations, the pre-processing of data, and different implementations rather than to the models themselves.

Given these substantial differences of model outcomes in practice, a bank seems to be exposed to large model risk, i.e. the risk that an applied model produces outputs which considerably deviate from those of other widely used models. Consequently, the Basel Committee on Banking Supervision (2004) proposed a revision of the standards for banks’ regulatory capital requirements which allows “internal” estimates only for the default probabilities of the obligors of a financial institution. The use of complete internal credit risk models for calculating regulatory capital charges will definitely not be intended in Basel II.
Therefore there is still a large gap between theoretical similarity and practical discrepancies. Given the empirical literature so far there is no evidence how this gap can be closed when a credit risk model is parameterized in practice. Here the present paper makes the following contributions. Firstly, we propose a simple econometric approach for estimating the respective input parameters of each model (default probabilities, dependency parameters) from default data. Our methodology is based on a non-linear panel model with random effects and uses Maximum-Likelihood technique.

Secondly, we extend the three popular credit risk model frameworks described in Koyluoglu/Hickman (1998a,b) and Gordy (2000) to dynamic settings similar to the one presented in Hamerle/Liebig/Rösch (2003a) and Heitfield (2005). The econometric specification which we use allows including individual and macroeconomic rating factors into each credit model and explaining defaults and default co-movements by observable co-variables. It is easy to conduct statistical significance tests for the risk factors and construct confidence intervals.

Thirdly, we address the model developers’ suggestions for deriving the input parameters and the study due to Koyluoglu/Bangia/Garside (2000) who suspect that the significant differences might be attributed to the estimation approach using different datasets. However, empirical evidence on this suspect is missing. Thus, instead of estimating default probabilities and default correlations with different modules and data (e.g. the former by default data and the latter by equity returns), we apply our econometric approach to one single dataset. The three credit risk models are then compared with respect to their performance in terms of loss distributions and Value-at-Risk forecasts. Given the non-nested model structures of CM, CR+, and CPV we understand our paper as an open-minded empirical study of model comparison. Our results provide evidence for the similarity of the models when the input parameters are estimated from banks' default data using the same methodology.

Finally, we show how estimation risk, i.e. the errors induced by estimating input parameters from random sample data, can be addressed in credit models. Our econometric approaches and empirical model comparisons are demonstrated with a database provided by Deutsche Bundesbank.

The rest of the paper is organized as follows. Section two provides a brief theoretical outline of the models. Section three describes the estimation approach. Section four shows how to derive forecasts, given the parameter estimates. Section five contains the main empirical results. In section six estimation error is addressed and section seven concludes.
2 General Set-up of the Models

2.1 Standard Credit Risk Model Specifications

We use a default mode model framework with a discrete-time horizon. Borrowers are grouped into segments distinguished e.g. by country or industry sector, or a combination of them. In period \( t (t=1,...,T) \) - for example one year - a borrower \( i \)'s default status \( (i=1,...,N_t) \) can exhibit two states, namely “the borrower defaults” \( \{D_{it} = 1\} \), or “the borrower does not default” \( \{D_{it} = 0\} \). Default is modelled using a latent metric random variable \( R_{it} \) for each borrower \( i \) in the segment. The default event \( D_{it} \) is caused by \( R_{it} \) crossing some threshold \( c \) at time \( t \), i.e.

\[
R_{it} < c \iff D_{it} = 1.
\]

Now consider the credit risk models CM, CPV, and CR+ as described in Koyluoglu/Hickman (1998a,b) and Gordy (2000). All models use a factor model framework where a common and important feature is the separation between systematic and idiosyncratic risk. This idea is well known from capital market theory as the Capital Asset Pricing Model due to Sharpe (1964) or the Arbitrage Pricing Theory due to Ross (1978).

Systematic risk \( F_t \) is assumed to be a continuous random variable which affects all borrowers (or those within a homogenous segment) jointly at time \( t (t=1,...,T) \), whereas idiosyncratic risk \( U_{it} (i=1,...,N_t , t=1,...,T) \) drives them on an individual basis. For ease of exposition we consider a single common factor for a segment. This factor can also be comprised by a linear combination of multiple factors.

Following capital market theory it is assumed that idiosyncratic risks are independent across borrowers and independent from systematic risk. This gives a central building block of all credit risk models under consideration: Default events within a risk segment are thus conditionally independent, given a realization of the common systematic factor. Then, the models differ with respect to their assumptions about the distributions of the factors, the process for the latent variable and the default threshold, as given in Table 1 (see also Koyluoglu/Hickman, 1998a,b, Gordy, 2000, and Frey/McNeil. 2003).
The systematic risk factors are standard normally distributed in the CM and CPV model, and gamma distributed in the CR+ model. We denote their respective density by $h(f_i)$. Idiosyncratic risk factors are normally distributed in the CM specification, follow a logistic distribution in the CPV case, and an exponential distribution in CR+.

The respective exposures of the latent variable to the systematic factors are denoted by $\sqrt{\rho}$ and $b$ in the CM and CPV specification. In CR+ the impact of the common factor is determined by the parameter $\alpha$. The conditional default probability $\lambda(f_i)$ is the probability of default depending on the systematic risk factor. The unconditional default probability $\bar{\lambda}$ is its expectation with respect to the risk factor.

2.2 Dynamic Extensions

Once the parameters of the models are known, loss distributions can be generated, as described in Koyluoglu/Hickman (1998a,b). These authors also show that for given parameter values of one model, the parameters of the other models can be correspondingly calculated yielding similar loss distributions. Hence, for many practical purposes, the adequate determination of the parameters themselves, rather than the calculation of the loss distribution will be the crucial task. In most cases, portfolio credit risk models are natural extensions to single
borrower credit rating approaches. Therefore credit ratings, default probability estimates and portfolio loss distributions should be used in a consistent manner.

We suggest a simple approach of extending the above credit risk model specifications to the inclusion of rating factors. This extension is similar to the one chosen by Hamerle/Liebig/Rösch (2003a) and Heitfield (2005) and can be applied to each of the three models. The approach can directly be linked to a methodology for parameter estimation and thus be applied to observed default data.

Consider the above specifications and furthermore assume that the default thresholds are functions of individual risk factors, such as balance sheet data, credit scores, etc., and macroeconomic variables, such as interest rates, GDP growth, business climate indices etc. In addition, we assume that these risk factors affect default probabilities with a time lag. Let $x_{lt-1} = \{x_{i,t-1,1}, ..., x_{i,t-1,L}\}$ an $L$-vector of individual observable risk factors, and $z_{t-1} = \{z_{t-1,1}, ..., z_{t-1,K}\}$ a $K$-vector of macroeconomic risk factors, where the index “$t-1$” denotes optional time-lags of one or more periods. Furthermore let $\beta = (\beta_1, ..., \beta_L)'$ denote an $L$-vector with exposures to the individual risk factors and $\gamma = (\gamma_1, ..., \gamma_K)'$ the exposures to the macroeconomic risk factors. Then the default threshold $c$ in the CM and the CPV model can be extended to

$$c_{lt} = \beta_0 + \beta' x_{lt-1} + \gamma' z_{t-1}$$

where $\beta_0$ is some constant. This parameterization of the threshold is inserted into the specifications in Table 1, yielding conditional default probabilities as functions of the risk factors as

$$\lambda_{it}(F_i) = \lambda(x_{it-1}, z_{t-1}, F_i).$$

While the integral for the unconditional probability in Table 1 has to be solved numerically in the CPV specification, in the CM model it is simply given by the probit link function

$$\lambda_{it} = E[\lambda_{it}(F_i)] = \int_{-\infty}^{\infty} \Phi\left(\beta_0 + \beta' x_{it-1} + \gamma' z_{t-1} - \sqrt{1-\rho} f_i\right) h(f_i) df_i.$$

Note that in the CM specification this is very similar to the specification in Heitfield (2005) except that we allow the individual risk factors to be dynamic.
In the CR+ model the threshold is the (unconditional) default probability itself, see Gordy (2000). While a simple linear representation is possible, it may lead to unconditional default probabilities $\lambda_{it}$ larger than one or lower than zero. In order to rule out these cases we exemplarily choose a probit link function as in the CM model for convenience, i.e.

$$\lambda_{it}(F_t) = \lambda_{it} \cdot F_t = \Phi(\beta_0 + \beta' x_{it-1} + \gamma' z_{t-1}) \cdot F_t$$

where $\lambda_{it} = \Phi(\beta_0 + \beta' x_{it-1} + \gamma' z_{t-1})$.

3 Model Estimation from Default Data

A crucial part is the estimation of the unknown parameters from observed data. For this we suggest a maximum likelihood approach which is a variant of the one used by Gordy/Heitfield (2000). Note the extension that it is not necessary to assume homogeneity of obligors within a segment with respect to the default probabilities, or to assume that default probabilities do not change over time. Moreover, it is not necessary to assume an infinitely large number of borrowers within the segment. The approaches suggested in this section allow borrower specific and time varying (unconditional) default probabilities and can be employed with any of the credit risk model specifications.

Consider time period $t$ for which a particular default pattern $(d_{1t}, \ldots, d_{N,t})$ has been observed. Because defaults are conditionally independent the conditional probability of observing this default pattern given the realization $f_t$ of the random factor is

$$P(D_{1t} = d_{1t}, \ldots, D_{N,t} = d_{N,t} \mid f_t) = \prod_{i=1}^{N_t} \lambda_{it}(f_t)^d_i [1 - \lambda_{it}(f_t)]^{1-d_i}.$$  

Thus, the unconditional likelihood is

$$P(D_{1t} = d_{1t}, \ldots, D_{N,t} = d_{N,t}) = \int_{-\infty}^{\infty} \prod_{i=1}^{N_t} \lambda_{it}(f_t)^d_i [1 - \lambda_{it}(f_t)]^{1-d_i} \cdot h(f_t) \, df_t.$$  

(6) is a function of the parameters constituting the conditional default probabilities $\lambda_{it}(F_t)$. By multiplying $T$ observations and taking the logarithm for convenience we finally obtain the log-likelihood.
In the CM and the CPV model the respective conditional default probability is inserted and the standard normal distribution for the random factor is used. In the case of the CR+ model the conditional default probability is multiplicative and the distribution over which is integrated is the gamma distribution. One obtains

\[
I = \sum_{t=1}^{T} \ln \left\{ \int_{-\infty}^{\infty} \prod_{i=1}^{N_t} \lambda_{it} (f_t)^{d_{it}} \left[ 1 - \lambda_{it} (f_t) \right]^{(1-d_{it})} h(f_t) df_t \right\}.
\]

where \(g(\cdot)\) denotes the density of the gamma distribution with parameters \(\alpha\) and \(1/\alpha\). As an extension of common logit or probit models an important part of this maximum likelihood method is the integral over the random effect. The integral approximation can for example be conducted by the adaptive Gaussian quadrature as it is described in Pinheiro/Bates (1995).

Usually the log-likelihood function (7) and (8) is numerically optimized with respect to the unknown parameters for which several algorithms, such as the Newton-Raphson method, exist and are implemented in many statistical software packages. Under usual regulatory conditions the resulting estimators asymptotically exist, are consistent and converge against normality. See Davidson/MacKinnon (1993), p. 243 for a detailed discussion.

4 Forecasting Default Probabilities and Loss Distributions

Once the parameters are estimated, forecasts for default probabilities for the next period (e.g. year) \(T+1\) are easily obtained. This is done by using the values \(x_{iT}\) and \(z_T\) of the individual and the macroeconomic risk factors which are known at time \(T\), and plugging them into the respective function for the conditional default probabilities

\[
\hat{\lambda}_{iT+1}(F_{T+1}) = \hat{\lambda}(x_{iT}, z_T, F_{T+1}).
\]

Since the future realization of the random risk factor is not known ex ante one can in a first step forecast the individual unconditional default probabilities as \(\hat{\lambda}_{iT+1} = E[\hat{\lambda}_{iT+1}(F_{T+1})]\), \((i=1,...,N_{T+1})\), where \(N_{T+1}\) is the number of borrowers in the portfolio at the beginning of period \(T+1\).
In the next step the goal is to forecast the distribution of potential portfolio losses for the next period (e.g. year) $T+1$. Based on the loss distribution, risk measures, such as expected loss, Value-at-Risk, unexpected loss or economic capital may be derived. In the present contribution we assume exposures at default (EAD) of 1 and losses given default (LGD) of 100% each, since our focus is not on these parameters but rather on the parameters of the default events.

In this case the loss distribution equals the distribution of the number of defaults $D_{T+1} = \sum_{i=1}^{N_{T+1}} D_{iT+1}$ in the portfolio for the next period $T+1$ which can be transformed into the distribution of the relative default frequency (default rate) $P_{T+1} = \frac{1}{N_{T+1}} D_{T+1}$.

Since the future value of the random risk factor is not known ex ante one forecasts the unconditional distribution of defaults

$$\hat{P}(D_{1T+1} = d_{1T+1}, \ldots, D_{N_{T+1}T+1} = d_{N_{T+1}T+1}) = \int \prod_{i=1}^{N_{T+1}} \tilde{\lambda}_{iT+1}(f_{iT+1})^{d_{iT+1}} \left[1 - \tilde{\lambda}_{iT+1}(f_{iT+1})\right]^{1-d_{iT+1}} \hat{h}(f_{iT+1}) df_{iT+1}$$

In general, this distribution has to be approximated via Monte-Carlo simulation since (10) requires the evaluation of $2^{N_{T+1}}$ probabilities which constitute the probability distribution of $D_{T+1}$. In the case of CR+ the distribution can be determined analytically if one makes use of some approximation possibilities. Details are given in the Appendix.

5 Empirical Comparison of Credit Risk Model Specifications

5.1 The Data

For the empirical analysis we use a database provided by Deutsche Bundesbank which contains yearly data for up to 53,280 firms in the years from 1991 to 2000. We considered all firms for which a unique classification into one of the two industry sectors “manufacturing” and “commerce” was possible. A default is noticed when a bankruptcy or insolvency procedure pursuant to sections 17, 18 or 19 of the German Insolvency Regulation has been filed. Altogether we obtained a database with 160,696 firm-years with 1,179 defaults in the years 1991 to 2000. The classification into the two industry sector is exhibited in Table 2.
Exhibit 1 shows the historical default rates of each sector.

Table 2
Dataset from Deutsche Bundesbank with two industry sectors

<table>
<thead>
<tr>
<th>Industry sector</th>
<th>Firm-periods</th>
<th>Defaults</th>
</tr>
</thead>
<tbody>
<tr>
<td>Manufacturing</td>
<td>88,869</td>
<td>773</td>
</tr>
<tr>
<td>Commerce</td>
<td>71,827</td>
<td>406</td>
</tr>
<tr>
<td>Sum</td>
<td>160,696</td>
<td>1,179</td>
</tr>
</tbody>
</table>

Exhibit 1
Default rates of the two industry sectors from 1991 to 2000

As borrower-specific risk factors we use a credit score provided by Deutsche Bundesbank. The score was estimated via Linear Discriminant Analysis separately for each sector and is composed by four balance sheet ratios each. The database was enriched by macroeconomic key figures which were collected by the Department of Statistics of the University of Regensburg. The variables cover areas such as income, prices, wages, economic growth, labour, capital markets and government activities. In addition, indices for the business climate were collected from the German IFO Institute for Economic Research.

For the analysis the database was divided into an estimation period (from 1991 to 1999) and a forecasting period (year 2000). Thus, in a first step the models are estimated for each industry sector with data from 1991 to 1999 (where we always used a time-lag for the score and the 3

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3 For a detailed description of the score and the database see Blochwitz/Liebig/Nyberg (2000).
macroeconomic data of one year). In a second step we forecast the default distributions for year 2000 using the estimation results up to year 1999.

5.2 Estimation Results

We compare three different ways of parameterizing each credit risk model. That is, we include different levels of information and estimate the respective parameters. The first model type treats all borrowers within a sector as homogenous. Furthermore it is assumed that default probabilities are constant through time. All temporal fluctuations are modeled via the systematic unobservable random factor. This naïve model serves as a kind of benchmark for the other two models.

Model type (II) includes borrower-specific information about their credit quality via the rating score and thus differentiates between their creditworthiness. This type of model is often found in internal rating systems of banks which incorporate several obligor-linked information such as balance sheet ratios. In the third (full) model (III) we additionally include a macroeconomic variable. Since balance sheet information reflects dynamic movements only to a certain extent, this final specification serves as a way of modeling credit risk dynamics by observable systematic factors. Hamerle/Liebig/Rösch (2003b) for instance find that these observable factors explain co-movements of defaults. Thereby implied correlations induced by the unobservable factor might be reduced.

The three information levels can be summarized as in Table 3. Each information level implies a different restriction for the default probability, or the default threshold $c_{it}$ respectively. Table 4 shows the estimation results for both industry sectors.

<table>
<thead>
<tr>
<th>Information level</th>
<th>Model name</th>
<th>Restriction on default threshold</th>
</tr>
</thead>
<tbody>
<tr>
<td>(I)</td>
<td>“Naïve” model</td>
<td>$c_{it} = \beta_0$</td>
</tr>
<tr>
<td>(II)</td>
<td>Rating model</td>
<td>$c_{it} = \beta_0 + \beta' x_{it-1}$</td>
</tr>
<tr>
<td>(III)</td>
<td>Dynamic rating model</td>
<td>$c_{it} = \beta_0 + \beta' x_{it-1} + \gamma' z_{t-1}$</td>
</tr>
</tbody>
</table>
Table 4

Estimation results for credit risk models from various parameterizations; as borrower specific rating variable the Deutsche Bundesbank score is used in models (II) and (III); in the manufacturing industry a business climate index (manufacturing) from the German IFO institute is used as proxy for the economic cycle in model (III); for commerce we used debit interest rates; all variables enter with 1 year time-lag; standard errors are in parentheses

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>CM specification</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\beta_0$</td>
<td>-2.3725*** -2.0547*** -1.2441***</td>
<td>-2.5236*** -2.2831*** -2.5596***</td>
</tr>
<tr>
<td></td>
<td>(0.0299) (0.0269) (0.2415)</td>
<td>(0.0232) (0.0265) (0.1449)</td>
</tr>
<tr>
<td>$\sqrt{\rho}$</td>
<td>0.0763* 0.0621** 0.0207 0.0371 0.0393 0.0117</td>
<td>0.0295 (0.0284) (0.0246) (0.0626)</td>
</tr>
<tr>
<td></td>
<td>(0.0242) (0.0237) (0.0295)</td>
<td></td>
</tr>
<tr>
<td>$\gamma$</td>
<td>-0.8621***</td>
<td>2.5497*</td>
</tr>
<tr>
<td></td>
<td>(0.2557)</td>
<td>(1.3503)</td>
</tr>
<tr>
<td>$\beta$</td>
<td>-0.5343*** -0.5346*** -0.2989*** -0.3044***</td>
<td>-0.0393</td>
</tr>
<tr>
<td></td>
<td>(0.0191) (0.0191) (0.0165)</td>
<td>(0.0167)</td>
</tr>
<tr>
<td>CPV specification</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\beta_0$</td>
<td>-4.7415*** -3.9893*** -1.9801***</td>
<td>-5.1459*** -4.5513*** -5.2990***</td>
</tr>
<tr>
<td></td>
<td>(0.08127) (0.0652) (0.5868)</td>
<td>(0.0673) (0.0672) (0.3862)</td>
</tr>
<tr>
<td>$\beta$</td>
<td>-2.1277***</td>
<td>6.9660*</td>
</tr>
<tr>
<td></td>
<td>(0.6232)</td>
<td>(3.6051)</td>
</tr>
<tr>
<td>$\beta$</td>
<td>-1.3090*** -1.3082*** -0.7798*** -0.7902***</td>
<td>-0.0368</td>
</tr>
<tr>
<td></td>
<td>(0.0448) (0.0448) (0.0401)</td>
<td>(0.0405)</td>
</tr>
<tr>
<td>CR+ specification</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\beta_0$</td>
<td>-2.3710*** -2.0498*** -1.2185***</td>
<td>-2.5226*** -2.2821*** -2.5542***</td>
</tr>
<tr>
<td></td>
<td>(0.0297) (0.0276) (0.2356)</td>
<td>(0.0233) (0.0258) (0.1356)</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>23.7358 45.797 601.80*** 89.537* 100.26*** 22816***</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(19.3340) (30.081) (168.68) (43.6908) (37.026) (1844.52)</td>
<td></td>
</tr>
<tr>
<td>$\gamma$</td>
<td>-0.8799***</td>
<td>2.5408*</td>
</tr>
<tr>
<td></td>
<td>(0.2492)</td>
<td>(1.2755)</td>
</tr>
<tr>
<td>$\beta$</td>
<td>-0.5356*** -0.5442*** -0.2999*** -0.3061***</td>
<td>-0.0368</td>
</tr>
<tr>
<td></td>
<td>(0.0196) (0.0197) (0.0167)</td>
<td>(0.0170)</td>
</tr>
</tbody>
</table>

Estimations are done with SAS. We employed a quasi-Newton optimization technique. As integral approximation we used the adaptive Gaussian quadrature with 128 quadrature points. Running time for full model (III) is about 30 minutes on a Pentium 4 machine.
The first analysed restriction (I) is the naïve case of homogenous borrowers with constant default probability. It is assumed that borrowers do not differ with respect to their creditworthiness (within an industry sector) and that fluctuations of default rates are due to random shocks of the common stochastic risk factor – or the default correlation respectively. Thus, in each industry segment two parameters are estimated for each credit risk model, i.e. the default threshold and the dispersion parameter of the random factor.

Firstly, it can be seen from Table 4 that the parameter estimates for the default threshold of the CM and the CR+ correspond well since the same link function (probit specification) is underlying. The threshold model can be directly translated into the unconditional default probability by (3) with constant $\beta_0$ only. For example in the manufacturing industry an estimate for the default probability of approximately 0.89% is obtained. In the CPV model the default probability must be calculated by solving the integral in Table 1 and yields the same value. Regarding the dispersion parameters of the random factors we obtain in the CM model an estimate for the asset correlation $\rho$ of 0.0763$^2 = 0.00582$, i.e. 0.582%. Compared to the high values which are presumed in the New Basel Accord our empirical estimates are quite lower, even if their standard errors are taken into account. Similar conclusions hold for the other two credit risk models if we note that the three models can be transformed into each other as shown by Koyluoglu/Hickman (1998a,b).

The second parameterization differs from the first by dropping the assumption of homogenous default probabilities in the cross-section. To allow for individual differentiation of creditworthiness the Deutsche Bundesbank score with one year time lag is included into the model (without macroeconomic variables). As can be seen from Table 4 in all models and both sectors the lagged creditworthiness score is statistically significant with a negative sign, that is, a higher score corresponds with a lower default probability in the next year. In comparison to model (I) the weights of the random effects are slightly reduced in sector 1 (e.g. the asset correlation in the CM model is 0.386%) whereas they remain nearly unchanged in sector 2. Although in the current model time-variant information is included via $x_{t-1}$, the Bundesbank score does not explain all cyclical variations. This is clear since the Bundesbank score is comprised by balance sheet ratios. These are only slightly affected by macroeconomic changes due to business policies of the firms which often try to avoid cyclical peaks in their financial statements. Thus, the cyclical variation of default probabilities is still captured by the random factor.

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5 See Hamerle/Liebig/Rösch (2003b) for analogous results for the G7 countries and Rösch (2003) for Germany in particular.
In the final parameterization we additionally include macroeconomic factors. The macroeconomic variables were chosen as follows. Firstly, their economic significance for the respective sector was taken into account. Variables which were economically not meaningful were excluded. Then, a variable was chosen which was statistically significant, at least at the 10% level. As a matter of fact, these variables should not be understood to be responsible for the default rates themselves. Rather, we interpret them as proxies for the state of the cycle. Therefore, some variable may be interchangeable by other variables if they evolve similarly through the economic cycle. In sector 1 we found the IFO business climate index economically and statistically significant, in sector 2 we found the debit interest rates (both one-year time lagged). Thus, a higher index (more positive climate) in sector 1 corresponds with lower default probabilities of the following year, higher interest rates are followed up by higher default probabilities of sector 2 in the next year, et vice versa. Moreover, in both sectors the creditworthiness score is still statistically significant.

Note that in this parameterization the weights of the random factors are fairly reduced compared to the previous models. For example, the estimate for the asset correlation in the CM model is reduced from 0.582% to only 0.04% in the manufacturing sector. In the CR+ model the alpha parameter accordingly increases from 23.7 to 602. Thus, the observable variables are able to economically “explain” the unobservable random factor and replace its exposure. However, this does not mean that the co-movements vanish; rather large parts of the co-movements are now modelled by time-dependent and co-moving default probabilities as functions of observable risk drivers instead of attributing them to the unobservable factor.

5.3 Model Comparison

We consider two ways of comparing the estimation results. First, from a statistical perspective we calculate goodness of fit measures for our models. Common measures for model fit (particularly for non-nested models) are the entropy based information criteria; see e.g. Burnham/Anderson (1998) for details. A finite sample corrected version of Akaike’s information criterion is given by

\[ AICC = 2f(\hat{\theta}) + 2pn/(n-p-1) \]

where \( f(\cdot) \) is the negative of the marginal log-likelihood function, \( \hat{\theta} \) is the vector of parameter estimates, \( p \) is the number of parameters in the respective model, and \( n \) is the number of

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6 See Rösch (2003) for detailed analyses of this effect with German bankruptcies.
7 We additionally calculated other measures, such as the Bayesian information criterion (BIC), and obtained qualitatively similar results.
observations. A smaller value indicates a better model fit. The respective AICCs for each of the nine models are shown in Table 5.

**Table 5**

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Model I II III</td>
<td>I II III</td>
</tr>
<tr>
<td>CM</td>
<td>7942.8 7051.4 7046.0</td>
<td>4258.5 3897.7 3897.1</td>
</tr>
<tr>
<td>CPV</td>
<td>7942.9 7087.7 7082.1</td>
<td>4258.4 3904.5 3903.5</td>
</tr>
<tr>
<td>CR+</td>
<td>7942.5 7052.4 7043.1</td>
<td>4258.4 3897.9 3896.6</td>
</tr>
</tbody>
</table>

In both sectors and each credit risk model an information gain by including risk factors (credit scores, macroeconomic variable) can be observed. The information criteria decrease when additional covariates enter the models. The improvement is particularly obvious when the individual credit scores are included. A comparison of the three model specifications reveals that the CPV specification performs worse than both the CM and the CR+ specification (with a slight advantage for CR+) when risk factors enter the model. From a statistical perspective, thus, either CR+ or CM would be slightly preferred.

Next, we compare the models from a practical perspective. Consider a bank has parameterized one of the three credit model specifications (CM, CR+, CPV). Given the parameter estimates, the next step consists in deriving a loss distribution for the portfolio for the following time period, e.g. year. Since a lot of decisions might be tied with this loss distribution, e.g. determining economic capital based on the Value-at-Risk (VaR) or Expected Shortfall, attributing capital to business segments, or evaluating spreads for credit derivatives such as CDOs, an important question is about model risk. In other words, do the distributions strongly change when switching from one credit model to another?\(^8\)

Based on these parameter estimates we therefore forecast the loss distributions out-of-time for year 2000. Since approach (III) is the most comprehensive one with most information about ratings and macroeconomic variables we use this model type. Because of lack of data on exposure size and losses given default we forecast the unconditional distribution of default

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\(^8\) Of course, another question is about the adequateness of either of these loss distributions. However, backtesting approaches for credit portfolio models require lots of time series data and the data scarceness on hand does not allow statistically sound backtests, see e.g. Rösch (2005). Therefore we focus on model risk in a sense of choosing among three model alternatives.
events, divided by number of borrowers, i.e. the distribution of the default rate. Exhibit 2 shows the forecast distribution for both industries (graphs on the left) and the distribution tails in particular (graphs on the right). Due to the weak estimated dependence the distributions are rather narrow and exhibit only a moderate degree of skewness. Moreover, the distributions generated by each credit risk model specification are very similar. Even if we consider the tails, the differences are not substantial.

**Exhibit 2**

Forecasted loss distributions (left) and distribution tails (right) for year 2000 using the estimates of the model with Deutsche Bundesbank score and macroeconomic variable (dynamic rating model)

Upper graph: Manufacturing sector, N=8734 borrowers at the beginning of year 2000
Lower graph: Commerce sector, N=12009 borrowers at the beginning of year 2000
EAD of 1 and LGD of 100% is used for each borrower

As a measure of congruence we calculate the "degree of agreement" (DoA) which has been used for comparison in Koyluoglu/Hickman (1998a). The DoA between two distributions is defined as

\[
\Psi_q(f,g) = 1 - \frac{\int_\infty^\infty |f(x) - g(x)| \, dx}{\int_\infty^\infty f(x) \, dx + \int_\infty^\infty g(x) \, dx}
\]

(12)
where \( f(\cdot) \) and \( g(\cdot) \) are the respective densities and \( q \) is the lower bound of the "area" which is compared. We choose \( q \) as the value equal to two standard deviations above the mean, see Koyluoglu/Hickman (1998a). The resulting statistics are given in Table 6. All values are higher than 80\%, and compared to the findings in Koyluoglu/Hickman (1998a) they indicate reasonable congruence, particularly in sector 2. The CM and the CPV specification seem to be more similar than CR+ which might be because of the different distributional assumptions for the systematic factor and parameter estimation uncertainty.

Table 6

Degree of agreement statistics for the three credit risk model loss distributions; model (III) is used; \( q \) equals the value of two standard deviations above the mean

<table>
<thead>
<tr>
<th>Degree of Agreement</th>
<th>CR+</th>
<th>CPV</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sector 1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>CM</td>
<td>86.10%</td>
<td>95.97%</td>
</tr>
<tr>
<td>CR+</td>
<td>81.98%</td>
<td></td>
</tr>
<tr>
<td>Sector 2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>CM</td>
<td>92.38%</td>
<td>96.75%</td>
</tr>
<tr>
<td>CR+</td>
<td>89.46%</td>
<td></td>
</tr>
</tbody>
</table>

Finally, we compare the Value-at-Risk quantiles of the distributions which are given in Table 7. Again as obvious from the quantiles the differences between the distributions are only small. Therefore, the bank is free to choose among either of the specifications given that the parameters are estimated in the described consistent way. If risk measures derived from the distributions, or the whole distributions, are the basis for decisions of the bank, it is virtually not exposed to model risk at all.
Table 7

Value-at-Risk (VaR) quantiles at different confidence levels; model (III) is used

<table>
<thead>
<tr>
<th>Sector</th>
<th>CM</th>
<th>CR+</th>
<th>CPV</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sector 1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>VaR 95%</td>
<td>0.0090</td>
<td>0.0092</td>
<td>0.0090</td>
</tr>
<tr>
<td>VaR 99%</td>
<td>0.0098</td>
<td>0.0098</td>
<td>0.0097</td>
</tr>
<tr>
<td>VaR 99.5%</td>
<td>0.0101</td>
<td>0.0102</td>
<td>0.0101</td>
</tr>
<tr>
<td>VaR 99.9%</td>
<td>0.0106</td>
<td>0.0106</td>
<td>0.0105</td>
</tr>
</tbody>
</table>

| Sector 2 |      |      |      |
| VaR 95% | 0.0063 | 0.0063 | 0.0063 |
| VaR 99% | 0.0068 | 0.0068 | 0.0068 |
| VaR 99.5% | 0.0069 | 0.0069 | 0.0069 |
| VaR 99.9% | 0.0074 | 0.0073 | 0.0074 |

6 Assessing Estimation Risk

Besides model risk, another important issue in empirical applications of risk models is the consideration of estimation risk, i.e. the errors induced by estimating unknown model parameters from random sample data. As one of the first, Jorion (1996) assesses this problem in the context of market risk. See Löffler (2003) for examples on possible impacts of estimation risk on credit portfolio analyses. While we used the point estimates for forecasting the loss distributions in the previous examination, in a statistical sense these point estimates are realizations of random variables which are affected by uncertainty. This may particularly be substantial for short time series as usually the standard errors decrease with increasing sample size.

To develop a simple strategy for implementing an assessment of estimation risk we assume asymptotic normal distributions for the estimators according to Maximum Likelihood theory, because the true small sample distribution is unknown. Under this approximation we derive a distribution of potential VaR measures in the following way:

- Step 1: For each of the three estimated model specifications we generate a set of “pseudo-parameters” from a joint normal distribution with mean vector equal to the point estimates of the parameters and the matrix of covariances equal to the estimated matrix of covariances. By doing this we simulate one possible set of values for the “true” unknown parameters.
- Step 2: Using this set of pseudo-parameters we generate a loss distribution and calculate a VaR quantile.
- Step 3: Steps 1 to 2 are repeated 1,000 times.

As a result we obtain a distribution of simulated VaR quantiles. Each of these values can be interpreted as the VaR derived under the assumptions that the underlying simulated potential set of parameters is the true one. Exhibit 3 shows the results for 95%, 99%, 99.5%, and 99.9% confidence levels for sector 1 and model type (III); sector 2 can be shown to perform in a similar way. We see that estimation risk is not negligible and the "true" VaR quantiles may be much higher than those generated by using the point estimates for the parameters. Therefore, estimation risk should always be considered in practice by giving confidence bounds around the estimated VaR, see e.g. Jorion (1996). Moreover, we see that CM and CPV behave similar with respect to estimation error while the CR+ distribution is narrower. The discrepancy between the distributions can also be seen from the DoA statistics in Table 8 where we compare the areas under the entire distributions. While the values for CM vs. CPV are throughout approximately 90%, the degree of agreement between CM and CR+ is roughly only between 70% and 81% and between CPV and CR+ between 66% and 78%.

Exhibit 3
Distributions of Value-at-Risk quantiles for various confidence levels and the three credit risk model specifications in sector 1 and model (III)
Table 8
Degree of agreement statistics for distributions of the Values-at-Risk with estimation error; model (III) is used;
Comparison of the entire densities

<table>
<thead>
<tr>
<th>Degree of Agreement</th>
<th>CR+</th>
<th>CPV</th>
</tr>
</thead>
<tbody>
<tr>
<td>95%-VaR</td>
<td>CM</td>
<td>81.0%</td>
</tr>
<tr>
<td></td>
<td>CR+</td>
<td>77.9%</td>
</tr>
<tr>
<td>99%-VaR</td>
<td>CM</td>
<td>80.4%</td>
</tr>
<tr>
<td></td>
<td>CR+</td>
<td>71.8%</td>
</tr>
<tr>
<td>99.5%-VaR</td>
<td>CM</td>
<td>75.7%</td>
</tr>
<tr>
<td></td>
<td>CR+</td>
<td>69.3%</td>
</tr>
<tr>
<td>99.9%-VaR</td>
<td>CM</td>
<td>70.7%</td>
</tr>
<tr>
<td></td>
<td>CR+</td>
<td>66.4%</td>
</tr>
</tbody>
</table>

This result is explainable by the high alpha parameter (weak dependence) in the CR+ specification and the assumption of approximate normal distributions of parameter estimates. Simulating confidence bounds around the parameter estimates results only in very rare realizations of low alphas which imply high dependence. Exhibit 4 illustrates the impact of various dependency parameters on the VaR of a portfolio.\(^9\) The upper graph shows the 99.5% VaR as a function of the square root of the asset correlation. Moving from a given value of, say 0.05, two standard deviations of 0.02 to the left and to the right results in a significant alteration of the VaR from about 1% to almost 2%. The lower graph shows the CR+ case. If we move here from a value of alpha of e.g. 600 two standard deviations of 170 to the left and to the right, the VaR remains nearly unchanged. Thus, in most cases alpha is high and the VaR is rather small. From this point of view the other two model specifications are more conservative when estimation errors are considered in the VaR in the way we did.

\(^9\) For ease of exposition a large uniform portfolio is assumed in the illustration with a homogenous default probability of 1% each.
7 Conclusion

In the present paper we showed how the input parameters for three credit risk models can be estimated by observed default data. The estimations were conducted for a database provided by Deutsche Bundesbank which may be interpreted as a portfolio of an exemplary bank. The most important results are that the three credit risk models lead to analogous results for forecasted loss distributions and that various kinds of information about the creditworthiness of borrowers and the state of the credit cycle may easily be included and used for forecasting. We also showed how to handle estimation risk in the various models.

Until evidence on other banking portfolios is found, these results should not be interpreted as final. However, we tried to close some gaps between the practical use of credit risk models and the discussion about model risk due to potential different outcomes they produce, and last
but not least about the use of internal credit risk models not only for risk management but also for regulatory purposes. On this way however, there still seems a lot of work to do.

One point is the assumed dependency structure which we simply adopted from the three popular credit risk model specifications under consideration. We do not claim that these dependence structures, or the models in general, are the best ones. Recent literature has analysed more general structures such as copulas, see e.g. Frey/McNeil/Nyfeler (2001) and Bluhm/Overbeck/Wagner (2003), and their effects on loss distributions. They show that even for small linear correlations the loss distributions can exhibit fat tails. Future work may develop methodologies for choosing among alternative copulas and empirically estimating their respective parameterizations, see also Hamerle/Rösch (2005).

Secondly, we assumed Losses Given Default of 100% and Exposures of 1 for each borrower. While these assumptions were originated by data restrictions, different values for these parameters may have some effects on our results regarding the comparability of the credit risk models, see e.g. Bluhm/Overbeck/Wagner (2001). Because considering different assumptions is beyond the scope of the present paper, this area may be an important field for future research.

**Appendix**

In the CR+ model, the conditional distribution of $D_{T+1}$ for a given value of $f_{T+1}$ is approximated by the Poisson distribution with parameter

$$\sum_{i=1}^{N_{T+1}} \hat{\lambda}_{i,T+1} f_{T+1} = \lambda_{T+1}^* f_{T+1}$$

where $\lambda_{T+1}^* = \sum_{i=1}^{N_{T+1}} \hat{\lambda}_{i,T+1}$ and $\hat{\lambda}_{i,T+1} = E(\hat{\lambda}_{i,T+1}(F_{T+1}))$. Since the future realization of the random factor is unknown, one has to use the unconditional distribution of $D_{T+1}$ again which is

$$\hat{P}(D_{T+1} = d_{T+1}) = \begin{cases} \exp(-\lambda_{T+1}^* f_{T+1}) \frac{\hat{g}(f_{T+1})}{d_{T+1}!} & d_{T+1} = 0,1,\ldots \\ 0 & \text{else} \end{cases}.$$
References


Wilson, T., 1997b. Portfolio Credit Risk II, Risk, October.