Forecasting Retail Portfolio Credit Risk

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Abstract

A major topic in retail lending is the measurement of the inherent portfolio credit risk. Two important parameters are default probabilities (PDs) and correlations. Both are considered in the New Basel Accord. Due to limited empirical evidence on their magnitude, in particular for retail credit risk, the Basel Committee sets standard specifications for the asset correlations. Using the charge-off rates filed by all US commercial banks, we estimate in a first step default probabilities and asset correlations for the Basel II retail exposure classes and find that the asset correlations of the Basel proposal exceed the empirical estimates. The model is then extended by lagged macroeconomic risk drivers which explain the credit risk of retail exposures given the state of the business cycle. It is shown that many of the correlations can be explained by these factors. Finally, the findings on default probabilities and asset correlations are embedded in a portfolio model framework. We argue that taking lagged macroeconomic risk factors into account may lead to more accurate loss forecasts and may considerably reduce economic capital.

1 Introduction

A major topic in retail lending is the measurement of the inherent portfolio credit risk. The needs for a better understanding and dealing with default risky securities have been reinforced by the Basel Committee on Banking Supervision [1999a, 1999b, 2000, 2001a, 2001b, 2002] who has proposed a revision of the standards for banks' capital requirements.

Two important parameters in modeling retail credit portfolio risk problems are default probabilities (PDs) and correlations. They are input parameters to the proposals of the Basel Committee as well as for many credit risk models. Examples for the latter are CreditMetrics, CreditRisk+, CreditPortfolioManager or CreditPortfolioView. For outlines of these models see Gupton et al. [1997], Credit Suisse Financial Products [1997], Crosbie [1998] and Wilson [1997a, 1997b].

The main direction of modeling default probabilities and correlations has its origin in the seminal model due to Merton [1974, 1977] and Black/Scholes [1973]. It is assumed that a default event happens if the value of a borrower's assets falls short of the value of debt. The default probabilities are determined by a threshold model. Asset correlations are modeled as a measure of co-movement of the asset values of two borrowers. Note that default correlations can be analytically derived from the asset correlations.

The framework of the New Basel Capital Accord is built on such a threshold model which attributes these co-movements to one common random factor, whereas the factor itself remains unspecified. The exposure to the factor determines the asset correlation. Gordy [2000, 2001], Finger [2001] and Wilde [2001] provide overviews. The advantages of the model are its robustness, simplicity and the consequence of portfolio invariance of capital charges. However, the Basel Committee on Banking Supervision [1999a] states that databases for analyzing credit risk are frequently insufficient, in particular for non-traded obligors and retail loans. Thus asset correlations are specified in dependence of the default probability and exposure class and are not estimated by the bank. For residential mortgage exposures the correlations are set to 15% in the proposal as of October 2002. For qualifying revolving exposures it is assumed that asset correlations are a decreasing function of PDs and vary between 2% and 15%. For other retail exposures this range lies between 2% and 17%.

While much is known about default probabilities (see Altman/Saunders [1997] who have provided a survey on developments over the past two decades), empirical studies on asset correlations between borrowers are scarce. Most existing research focuses on corporate borrowers, where data is more easily available. Examples are KMV who approximate asset return correlations by equity return correlations or Lucas [1995], Gordy [2000], Gordy/Heitfield [2000], Rösch [2002] and Dietsch/Petey [2002] who treat asset returns as latent variables and estimate asset correlations implicitly by observable default data for rating grades or industries.

Recently, Iscoe et al. [1999] have proposed a general market and credit risk modeling framework. Bucay/Rosen [2001] presented an application of the model to a retail portfolio of a North American Financial Institution. An important part of their framework consists in the modeling of the joint behavior of defaults due to exposures to macroeconomic risk drivers. They use a sector-based and two factor-based models and show that they yield comparable results. Correlations are estimated using linear regressions of time series of logit or probit transformations of default rates on macroeconomic factors. Since the factors are stochastic, correlations between the sectors are derived from common exposures to these risk drivers. For the generation of future loss distributions, scenarios of the factors are simulated, their realizations in each scenario are plugged into the estimated functional relation between factors and default probabilities, the Expected Loss is calculated and finally the losses are aggregated over all scenarios.

While the general framework is maintained in the present paper, an alternative for modeling and estimating the joint behavior of defaults is presented. The approach differs from Bucay/Rosen [2001] in four ways. Firstly, while Bucay/Rosen [2001] or Wilson [1997a, 1997b] use linear regression of transformed aggregated default rates on macroeconomic factors we employ a probit model which is able to forecast default probabilities for individual borrowers as well as to estimate correlations between borrowers. Secondly, the model is an empirical application of the model which is used for the calibration of risk weights in the Basel II Capital Accord. Thus, an interpretation of the estimated correlations within Basel II is straightforward and capital requirements from the model and Basel II can be compared directly. To our knowledge, this paper is the first study to examine the Basel II asset correlations of individual retail exposures. Thirdly, Bucay/Rosen [2001] or Wilson [1997a, 1997b] attribute correlations to observable contemporaneous risk drivers which have to be simulated when loss distributions are modeled. In our model we find that a large part of co-movements can be attributed to lagged factors. Loss distributions can be forecasted, given the actual point of the business cycle and estimation uncertainty can be reduced. Fourthly, we use a longer time series of defaults which spans over more than one business cycle. This is an important requirement for the estimation of cyclical default probabilities and correlations.

The rest of the paper is organized as follows. After describing the model we estimate, in a first step, correlations for the exposure classes residential real estate, credit card and other consumer loans with a simple version of the Basel factor model using the charge-off rates filed by all US commercial banks. Then we extend the model by macroeconomic risk drivers which explain the credit risk of retail exposures given the state of the business cycle (point-in-time). Finally, we embed our findings in a portfolio model and show implications for the forecasted loss distributions of a bank's retail portfolio as well as for economic and regulatory capital.

The next section describes the modeling and estimation approach. Section 3 presents the empirical results for the three exposure classes. Section 4 shows how the findings can be integrated in a portfolio model and points out the implications on economic and regulatory capital. Section 5 provides a summary of the results and some comments.

2 The Model for PDs and Correlations

The model which we use is a variant of the individual two-state one factor Credit Metrics model which is employed in the framework of Basel II for calculating risk weights. The two states are referred to as "default" and "non-default". The discrete-time process for the normalized return R_{it} on the assets of borrower i at time t is assumed to follow a one-factor model of the form

$$R_{it} = b\,F_t + \sqrt{1-b^2}\,U_{it} \qquad (*\ 1)$$

where

$$F_t \sim N(0,1), \quad U_{it} \sim N(0,1)$$

(i=1,…,N_t, t=1,...,T) are normally distributed with mean zero and standard deviation one. Idiosyncratic shocks U_{it} are assumed to be independent from the systematic factor F_t and independent for different borrowers. All random variables are serially independent.

The exposure to the common factor is denoted by b. Under these assumptions the correlation ρ between the normalized asset returns of any two borrowers is b^2. We will refer to this correlation as asset correlation. The Basel Committee on Banking Supervision assumes in its October 2002 proposal that the asset correlation depends on the exposure class:

- 15% for residential mortgage exposures,
- depending on the PD between 2% and 15% for revolving exposures (e.g. credit card loans) and
- depending on the PD between 2% and 17% for other retail exposures.

Exhibit 1 shows the proposed asset correlation depending on the PD for the three exposure classes.

[---Insert Exhibit 1 about here---]

As in the Basel Accord we assume that borrowers can be grouped into homogenous segments. In each segment a borrower defaults at time t if the return on his assets falls short of some threshold β_0, i.e.

$$R_{it} < \beta_0 \quad \Leftrightarrow \quad Y_{it} = 1 \tag{* 2}$$

(i=1,…, N_t , t=1,…,T), where Y_{it} is an indicator variable with

$$Y_{it} = \begin{cases} 1 & \text{borrower } i \text{ defaults at time } t \\ 0 & \text{else} \end{cases}$$

The probability of default at time t for borrower i within a given segment is then

$$\lambda = P(Y_{it} = 1) = P(R_{it} < \beta_0) = P\left(b\, F_t + \sqrt{1-b^2}\, U_{it} < \beta_0\right) = \Phi(\beta_0) \tag{* 3}$$

where $\Phi(.)$ denotes the cumulative standard normal distribution function. This probability is actually a conditional probability, given the borrower has survived until time t. We skip the condition $y_{it-1} = 0$ for convenience. Conditional on a realization f_t of the common random factor at time t the default probability becomes

$$\lambda(f_t) = P\left(U_{it} < \frac{\beta_0 - b\, f_t}{\sqrt{1-b^2}}\right) = \Phi\left(\frac{\beta_0 - b\, f_t}{\sqrt{1-b^2}}\right) \tag{* 4}$$

The conditional default probability can also be expressed in terms of the unconditional probability of default and the asset correlation:

$$\lambda(f_t) = \Phi\left(\frac{\Phi^{-1}(\lambda) - \sqrt{\rho}\, f_t}{\sqrt{1-\rho}}\right) \tag{* 5}$$

where $\Phi^{-1}(.)$ denotes the inverse cumulative standard normal distribution function. As described in Finger [1998], the realization f_t of the factor can be interpreted as the state of the economy in *t*. The conditional default probabilities decrease in "good years" (positive factor realization) and increase in "bad years" (negative factor realization). Conditional on the realization of the random factor defaults are independent between borrowers. Then the number of defaults $D(f_t)$ at time *t* for a given number N_t of borrowers is (conditional) binomially distributed with probability $\lambda(f_t)$, i.e.

$$D(f_t) \sim B(N_t, \lambda(f_t))$$

where $B(.)$ denotes the Binomial distribution (see e.g. Gordy/Heitfield [2000]). The unconditional default probability can be obtained by $\int_{-\infty}^{+\infty} \lambda(f_t)\, \varphi(f_t)\, df_t$ where $\varphi(.)$ denotes the density function of the standard normal distribution.

Model (* 3) assumes that there is a default threshold which is time invariant and, thus, that the unconditional default probability is constant over the time period under consideration. A more advanced specification is to model time-varying default probabilities and explicitly take their fluctuation during the business cycle into account. This is done by including observable risk factors, i.e. macroeconomic risk factors. Let $\boldsymbol{z}_t = (z_{1t},\ldots,z_{Kt})'$ denote a *K*-vector of risk factors and $\boldsymbol{\beta} = (\beta_1,\ldots,\beta_K)'$ the vector of sensitivities with regard to these factors. Then within a segment the probability of default, conditional on the observable risk factors is

$$\lambda(z_t) = P\left(b\, F_t + \sqrt{1-b^2}\, U_{it} < \beta_0 + \boldsymbol{\beta}'\, z_t \right) = \Phi(\beta_0 + \boldsymbol{\beta}'\, z_t) \qquad (*\ 6).$$

Thus, the default probability depends on the state of the economy which is represented by the variables in the vector z_t. A positive sensitivity with respect to a factor leads to a higher default probability when the factor increases and vice versa. Again conditioning on a realization f_t of the random factor the default probability is

$$\lambda(f_t, z_t) = \Phi\left(\frac{\Phi^{-1}(\lambda(z_t)) - \sqrt{\rho}\, f_t}{\sqrt{1-\rho}} \right) \qquad (*\ 7).$$

The realization of the random factor captures the point-in-time effects of factors not included in the model or respectively the remaining asset correlation.

In this model the parameters can be estimated without the observation of asset returns. Only defaults have to be observed as dependent variables. The asset returns can then be treated as latent variables. This is especially convenient for retail credit risk where asset returns can not be observed. Note that these models are individual models. The asset correlations are the correlations between two borrowers within a risk segment. For a given time series of defaults and macroeconomic variables the parameters $\boldsymbol{\beta}$, β_0 and the asset correlation in model (* 3) and model (* 6) can be estimated by Maximum Likelihood using the threshold model (see e.g. Gordy/Heitfield [2000]). For the integral approximation we use the adaptive Gaussian quadrature as described in Pinheiro/Bates [1995] which is implemented in many statistical software packages.

The main difference between model (* 3) and model (* 6) is that the default probability can be modeled as being time-dependent as a function of observable covariates, i.e. the observable time-varying risk factors. Thus, credit cycles can be mapped. Moreover, in model (* 3) the time-variation of the default probabilities is captured by the variation of the random effect. If cyclical patterns can be identified and proxied by macroeconomic variables as in model (* 6) the influence of the random effect, i.e. the asset correlation should be diminished. In addition, we assume that the influence of macroeconomic factors is time lagged. That means that the defaults for a given period are explained by macroeconomic conditions in the past. Therefore, point-in-time default distributions can be forecasted without the need of forecasting the future state of the economy.

3 Estimation Results for PDs and Correlations

3.1 The Data

For the empirical analysis, we use the annual charge-off rates filed by all US commercial banks. These data are compiled from the quarterly Consolidated Reports of Condition and Income filed by all US commercial banks to the Federal Financial Institutions Examination Council (www.ffiec.gov). The charge-off rates are available for the Basel II exposure classes, including residential real estate, credit card and other consumer loans. We assume that the charge off rate is a good approximation of the default rate for a given year. To keep matters simple, we assume that the loss given default and exposure at default equal one for every loan.

In addition, we extend the data by US macroeconomic risk factors. They serve as proxies for the business cycle and were obtained from the Organization for Economic Cooperation and Development (www.oecd.org). They cover the areas of

- Demand and output
- Wages, costs, unemployment and inflation
- Supply side data
- Saving
- Fiscal balances and public indebtedness
- Interest rates and exchange rates
- External trade and payments
- Miscellaneous.

As it is common in econometrics, yearly changes of macroeconomic variables are taken as risk factors. All risk factors are lagged by one year. The risk factors used in the analysis are the

- change of the consumer price index in % (CPI),
- deposit interest rate in % (DIR),
- change of the gross domestic product in % (GDP), and
- change in the industrial production in % (IPI).

Tables 1 and 2 show descriptive statistics for the default rates of the retail exposure classes and the macroeconomic variables.

[---Insert Table 1 and Table 2 about here---]

For illustrative purposes, the empirical analysis is based on a retail portfolio with 100,000 borrowers in each exposure class. Note that the number of loans does not substantially change the empirical results.

3.2 Constant Default Probabilities

In a first step, we assume that a bank estimates the default probabilities with the average long-term default rate of the respective exposure class. The default probabilities are then constant over time. The correlations between the obligors can be estimated by model (* 3) for each exposure class. The results are depicted in Table 3.

[---Insert Table 3 about here---]

Table 3 shows that the constants and the respective default probabilities, as well as the random factor exposures and the respective asset correlation vary between the sectors. The default probability is highest for credit card loans with 4.03% and lowest for residential real estate loans with 0.15%. The asset correlation is again highest for credit card loans with 1.02% and lowest for other consumer loans with 0.73%. In every exposure class they are considerably lower than the ones assumed by the Basel Committee on Banking Supervision.

3.3 Point in Time Default Probabilities

The model in section 3.2 assumes constant default probabilities over time. In this case, any cyclical pattern is attributed to the asset correlations. In the next step we assume that default probabilities change during a business cycle and thus can be explained by observable macroeconomic risk drivers described in section 3.1. As a matter of fact, the risk factors represent the respective point in time of the business cycle and are not necessarily responsible for the default probabilities themselves. The exposures of the default probabilities to the risk factors and the random factor are estimated by model (* 6).

Table 4 shows the estimation results of model (* 6) for each exposure class. The models were estimated for residential real estate loans for the period 1991 to 2001 and for credit card and other consumer loans for the period 1985 to 2001.

[---Insert Table 4 about here---]

The macroeconomic variables change of consumer prices in % (CPI), deposit interest rate in % (DIR), growth in gross domestic product in % (GDP) and change in the industrial production in % (IPI) are statistically significant at the 6% level. The estimated parameters are used to estimate point in time default probabilities as well as to forecast the default probability for the year 2002. The exhibits 2 to 4 show the real default rates and the estimated and forecasted default probabilities (rates).

[---Insert Exhibits 2 to 4 about here---]

If the parameter estimates show a positive sign, the default probability increases with the respective variable and vice versa. Let us take a look at the percentage growth in the real gross domestic product (GDP). The variable is lagged by one year. Thus, the negative sign of the parameter indicates that an increase of GDP leads c. p. to a lower probability of default in the next year. The economic plausibility of the other macroeconomic variables can be assessed in a similar way.

With the inclusion of macroeconomic variables a considerable share of the fluctuation of default rates is explained. Therefore, the asset correlations of model (*6) have considerably decreased for every single exposure class in comparison to the model (*3) without risk factors.

4 Generating Loss Distributions

In the model framework due to Iscoe et al. [1999] and in the application due to Bucay/Rosen [2001] the Expected Loss is calculated for a given scenario using the parameter estimates from the model. Then a large number of scenarios are generated and the Expected Losses are aggregated over all scenarios. Here, we use a slightly different approach. Since the PDs and the correlations were estimated for each exposure class, we firstly derive the loss distributions separately for each exposure class. In a second step we aggregate these marginal distributions to a single portfolio distribution for each model. In both steps we compare the Expected Loss, Value at Risk and Unexpected Loss of

- the model with constant default probabilities and "Basel II"-correlations,
- the model with constant default probabilities and estimated correlations and
- the model with time-varying default probabilities and estimated correlations.

4.1 Marginal Loss Forecasts of Each Exposure Class

Given the parameters of the models, the default distribution i.e. the distribution of the potential numbers of defaulting borrowers for the next period $T+1$ (e.g. one year) can be calculated as it is shown in Vasicek [1987]. If model (* 3) with constant default probabilities is used the probability distribution for the number D_{T+1} of defaulting companies within a risk segment, given the number N_{T+1} of companies in this segment at the beginning of the period is

$$P(D_{T+1}) = \begin{cases} \binom{N_{T+1}}{D_{T+1}} \cdot \int\limits_{-\infty}^{+\infty} \left[\lambda(f_{T+1})^{D_{T+1}} \cdot [1-\lambda(f_{T+1})]^{N_{T+1}-D_{T+1}} \right] \varphi(f_{T+1}) df_{T+1} & D_{T+1} = 0,1,2,\ldots,N_{T+1} \\ 0 & \text{else} \end{cases} \quad (*\ 8)$$

where $\lambda(f_{T+1})$ is defined analogously to (* 4). This distribution depends on the point of the credit cycle only by N_{T+1} since the distribution of the random factor is standard normal at each point in time. The cyclical variation is captured by the asset correlation and introduces some uncertainty and skewness into the default distribution.

If model (* 6) is assumed the probability distribution is

$$P(D_{T+1}) = \begin{cases} \binom{N_{T+1}}{D_{T+1}} \cdot \int\limits_{-\infty}^{+\infty} \left[\lambda(f_{T+1}, z_{T+1})^{D_{T+1}} \cdot [1-\lambda(f_{T+1}, z_{T+1})]^{N_{T+1}-D_{T+1}} \right] \varphi(f_{T+1}) df_{T+1} & D_{T+1} = 0,1,2,\ldots,N_{T+1} \\ 0 & \text{else} \end{cases}$$

(* 9)

where $\lambda(f_{T+1}, z_{T+1})$ is defined analogously to (* 7). The distribution (* 9) explicitly depends on the state of the economy by the macroeconomic factors.

Note that if a loss given default and exposure at default of one are assumed, the distribution of potential defaults becomes the distribution of potential losses. Otherwise, the loss distribution can be simulated as described in Bucay/Rosen [2001].

We showed in the previous sections 3.2 and 3.3 that the estimated asset correlations for the observed default rates of the retail exposure classes are lower than the ones proposed by the Basel Committee on Banking Supervision [2002]. In a first step, we will use the forecasted default probabilities for 2002 from model (* 3) separately for each retail exposure class and compare the resulting loss distributions based on the estimated asset correlations and the ones proposed by the Basel Committee on Banking Supervision [2002]. For ease of exposition we assume an Exposure and a Loss Given Default of one for each borrower. Thus, the distributions can be calculated due to (* 8) and (* 9). These assumptions can easily be relaxed, whereas in general the distributions should then be simulated. Exhibits 5 to 7 show the distributions for the three classes. The loss is given as a percentage of the portfolio value.

[---Insert Exhibits 5 to 7 about here---]

As the Value at Risk quantiles (VaR0 and VaR1) in Table 5 show, the regulatory capital charge under Basel II exceeds the economic capital charge by far in each exposure segment due to the high asset correlation. This is also the case if it is assumed that Expected Losses are covered by future margin income, as it is provided for the Credit Card Loans under the October 2002 proposal. Basel II allows a provision for 0.9*Expected Loss in the case of Credit Card Loans, which leads to an Unexpected Loss or a capital charge of 8.428%. Then, the UnExpected Loss constitutes the capital requirement. In Table 5 we used 1*EL in calculating the Unexpected Loss for consistency (i.e. 8.025%).

[---Insert Table 5 about here---]

The conservative assumptions about asset correlations under Basel II may be justifiable, because we have not taken maturity adjustments or estimation and model risks in our analysis into account. Nevertheless, a future adjustment of regulatory capital to economic capital could consider empirical values for the asset correlations.

Section 3.3 showed that the estimated asset correlation decreases if the business cycle is modeled. In a second step we use the forecasted default probabilities for 2002 and the estimated asset correlation and compare the resulting loss distributions of model (* 3) to the one of model (* 6). Exhibits 8 to 10 show the results for the different retail exposure classes and Table 5 contains the Expected Loss and Value at Risk quantiles.

[---Insert Exhibits 8 to 10 about here---]

Since loss distributions depend on the default probabilities, model (* 6) leads to a point in time loss distribution. With this model, we forecasted default rates for 2002 that are higher than average for all exposure classes. The loss distributions induced by model (* 3) average over the potential states of the business cycle. As a result the point in time loss distribution is, generally speaking, more narrow and the forecasted retail portfolio risk is more accurate. Note that the loss distributions tend to broaden with higher point in time default probabilities, i.e. Expected Loss.

4.2 Aggregated Loss Forecasts

In the last step of the portfolio modeling approach we aggregate the three exposure classes and calculate a single overall loss distribution. To do this model (* 1) is defined for each asset return $R_{it}^{(l)}$ within exposure class *l* (*l*=1,…,3)

$$R_{it}^{(l)} = b^{(l)}\, F_t^{(l)} + \sqrt{1 - b^{(l)2}}\, U_{it}^{(l)} \qquad (* \; 10)$$

where

Fehler! Es ist nicht möglich, durch die Bearbeitung von Feldfunktionen Objekte zu erstellen.,

$U_{it}^{(l)} \sim N(0{,}1)$

(*i*=1,…, N_t , *t*=1,…,*T*). The correlation between two asset returns is then

$$Corr\left(R_{it}^{(l)}, R_{jt}^{(s)}\right) = \begin{cases} b^{(l)2} & l = s, i \neq j \\ b^{(l)}\, b^{(s)}\, \rho_{ls} & l \neq s, i \neq j \end{cases} \qquad (* \; 11)$$

where

$$\rho_{ls} \equiv Corr\left(F_t^{(l)}, F_t^{(s)}\right) \qquad (* \; 12)$$

denotes the correlation between the random factors of two different segments. Table 6 contains the estimated correlations for the models (* 3) and (* 6).

[---Insert Table 6 about here---]

In both models the correlation between the effects of the residential real estate and the credit card loans and between the effects of the residential real estate and other consumer loans are negative whereas the correlation between the effects of credit card and other consumer loans is positive.

Using these estimates the loss distributions can be calculated by integrating over the joint distribution of the three random effects. While the one-dimensional integral from section 4.1 was numerically tractable, in general a higher-dimensional integral requires sophisticated software packages. To keep things realizable we approximate the loss distributions by Monte-Carlo simulation. 10,000 simulations are run for each distribution. 100,000 borrowers are assumed within each exposure class for expository purposes. In practice, any user-defined numbers can be employed as well as user-defined exposures and recovery rates. Exhibits 11 and 12 show these distributions as well as the actual default rate. In Exhibit 11 the Basel II distribution is compared to the distribution using constant PDs. Exhibit 12 compares the distributions under constant PDs and point in time PDs.

[---Insert Exhibits 11 and 12 about here---]

The Value at Risk quantiles in Table 7 demonstrate again that the regulatory capital charge under Basel II exceeds the economic capital charge for the aggregated loss of model (* 3) The comparison of model (* 3) and (* 6) with the estimated asset correlations shows that the Expected Loss is closer to the actual loss and the Unexpected Loss is lower when macroeconomic variables are taken into account.

[---Insert Table 7 about here---]

5 Summary

The present paper describes an alternative methodology for modeling and estimating retail portfolio credit risk. Within the general model framework suggested by Iscoe et al. [1999] and applied by Bucay/Rosen [2001] our approach suggests several modifications:

- individual default probabilities can be forecasted and asset (or default) correlations can be estimated.

- the Basel II model is used. The estimated parameters can be compared to the ones proposed by the Basel Committee on Banking Supervision or used to calibrate future proposals.
- systematic risk is forecasted by observable time lagged macroeconomic variables.
- a longer time period is used, improving estimates and forecasts of systematic risk parameters.

The main empirical results of our model are that

- the asset correlations proposed by the Basel Committee on Banking Supervision [2002] are much higher than the ones empirically observed for residential real estate loans, credit card loans and other consumer loans.
- the inclusion of variables which are correlated with the business cycle improves forecasts of default probabilities, loss distributions and economic capital. The uncertainty of the forecasts is diminished.
- asset correlations depend on the factors used to model default probabilities. Thus, asset correlations and default probabilities should always be estimated simultaneously.

Two directions of future research should be mentioned. While our paper focused more on modeling correlations and PDs in the business cycle, individual borrower characteristics could also be easily employed in our model leading to a more detailed discrimination and segmentation. Furthermore, estimation risk was not taken into account. Although the standard errors of the estimates are rather small, a methodology for incorporating estimation risk into the forecasts of loss distributions may amend our approach.

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Appendix A: Tables

Table 1: Summary Statistics of Variables

Variable	Mean	Std Dev	Minimum	Maximum
Residential	0.0015	0.0005	0.0008	0.0024
Credit Card	0.0419	0.0111	0.0252	0.0697
Other	0.0093	0.0023	0.0051	0.0142
CPI	3.1244	1.0121	1.5580	5.4030
DIR	5.9783	1.5469	3.1700	9.0900
GDP	3.1076	1.1593	-0.2140	4.5040
IPI	2.9562	2.0226	-1.9980	6.0090

Table 2: Pearson Correlations between Variables

Variable	Residential	Credit Card	Other	CPI	DIR	GDP	IPI
Residential	1.0000	-0.1423	-0.0545	0.6666	-0.6608	-0.5794	-0.4909
Credit Card	-0.1423	1.0000	0.8214	-0.3762	-0.4441	-0.1949	0.0402
Other	-0.0545	0.8214	1.0000	0.0035	-0.0025	-0.4141	-0.3835
CPI	0.6666	-0.3762	0.0035	1.0000	0.6201	-0.4190	-0.4435
DIR	-0.6608	-0.4441	-0.0025	0.6201	1.0000	0.0888	-0.2917
GDP	-0.5794	-0.1949	-0.4141	-0.4190	0.0888	1.0000	0.7221
IPI	-0.4909	0.0402	-0.3835	-0.4435	-0.2917	0.7221	1.0000

Table 3 : Estimation results for constant default probabilities

Parameter	Estimate	Std. Error	p-Value	Unconditional PD	Asset Correlation
Residential Real Estate Loans					
β_0	-2.9845	0.0311	<.0001	0.0015	0.0098
b	0.0996	0.0227	0.0014		
Credit Card Loans					
β_0	-1.7564	0.0247	<.0001	0.0403	0.0102
b	0.1015	0.0175	<.0001		
Other Consumer Loans					
β_0	-2.3751	0.0210	<.0001	0.0090	0.0073
b	0.0855	0.0150	<.0001		

Table 4 : Estimation results for point in time default probabilities

The time-specific unconditional default probabilities are displayed in exhibit 1 to 3

Parameter	Estimate	Standard Error	p-Value	Asset Correlation
Residential Real Estate Loans				
β_0	-2.8695	0.0292	<.0001	
IPI	-0.0368	0.0077	0.0007	0.0028
b	0.0526	0.0138	0.0035	
Credit Card Loans				
β_0	-1.5095	0.0858	<.0001	
CPI	-0.0391	0.0193	0.0599	0.0066
GDP	-0.0351	0.0135	0.0021	
b	0.0813	0.0141	<.0001	
Other Consumer Loans				
β_0	-2.3415	0.0709	<.0001	
CPI	-0.0617	0.0255	0.0281	
GDP	-0.0409	0.0145	0.0121	0.0044
DIR	0.0484	0.0155	0.0065	
b	0.0663	0.0118	<.0001	

Table 5: Value-at-Risk quantiles of forecasted loss distributions for year 2002 for the retail segments

The number of borrowers is 100,000; losses are in % of portfolio value; Value-at-Risk is defined as a quantile of the loss distribution; Unexpected Loss is defined as the difference between the Value-at-Risk quantile and the Expected Loss; EL0 (VaR0, UL0) is the Expected Loss (Value-at-Risk, Unexpected Loss) with constant PD and Basel II asset correlation, EL1 (VaR1, UL1) is the Expected Loss (Value-at-Risk, Unexpected Loss) with constant PD and estimated model correlation, EL2 (VaR2, UL2) is the Expected Loss (Value-at-Risk, Unexpected Loss) with point in time PD and estimated model correlation.

Exposure Class	**Residential Real Estate Loans**			**Credit Card Loans**			**Other Consumer Loans**		
Confidence Level (%)	99	99.5	99.9	99	99.5	99.9	99	99.5	99.9
EL0	0.149	0.149	0.149	4.028	4.028	4.028	0.898	0.898	0.898
VaR0	1.242	1.621	2.724	9.295	10.139	12.053	5.061	6.145	8.943
UL0	1.093	1.472	2.575	5.267	6.111	8.025	4.163	5.247	8.045
EL1	0.149	0.149	0.149	4.028	4.028	4.028	0.898	0.898	0.898
VaR1	0.299	0.323	0.377	6.426	6.751	7.460	1.482	1.564	1.745
UL1	0.150	0.174	0.228	2.398	2.723	3.432	0.584	0.666	0.847
EL2	0.161	0.161	0.161	5.223	5.223	5.223	1.142	1.142	1.142
VaR2	0.242	0.252	0.275	7.509	7.802	8.434	1.681	1.752	1.906
UL2	0.081	0.091	0.114	2.286	2.579	3.211	0.539	0.610	0.764
Actual Loss 2002		0.153			6.973			1.423	

Table 6: Pearson correlations between empirical Bayes estimates for the random effects of model (* 3) and model (* 6)

Model (* 3)	Residential Real Estate	Credit Card Loans	Others
Residential Real Estate	1	-0.259	-0.123
Credit Card Loans		1	0.715
Others			1

Model (* 6)	Residential Real Estate	Credit Card Loans	Others
Residential Real Estate	1	-0.586	-0.393
Credit Card Loans		1	0.896
Others			1

Table 7: Value-at-Risk quantiles of forecasted loss distributions for year 2002 for aggregated loss distributions

The number of borrowers is 100,000 within each of the three segments; losses are in % of portfolio value; Value-at-Risk is defined as a quantile of the loss distribution; Unexpected Loss is defined as the difference between the Value-at-Risk quantile and Expected Loss; EL0 (VaR0, UL0) is the Expected Loss (Value-at-Risk, Unexpected Loss) with constant PD and Basel II asset correlation, EL1 (VaR1, UL1) is the Expected Loss (Value-at-Risk, Unexpected Loss) with constant PD and the estimated model correlation, EL2 (VaR2, UL2) is the Expected Loss (Value-at-Risk, Unexpected Loss) with point in time PD and estimated model correlation.

Confidence Level (%)	99	99.5	99.9
EL0	1.69	1.69	1.69
VaR0	5.45	6.29	7.75
UL0	1.93	4.60	6.06
EL1	1.69	1.69	1.69
VaR1	2.67	2.78	3.07
UL1	0.98	1.09	1.38
EL2	2.17	2.17	2.17
VaR2	3.09	3.18	3.47
UL2	0.92	1.01	1.30
Actual Loss 2002		2.85	

Appendix B: Exhibits

Exhibit 1: Basel II PDs and asset correlations for different retail exposure classes

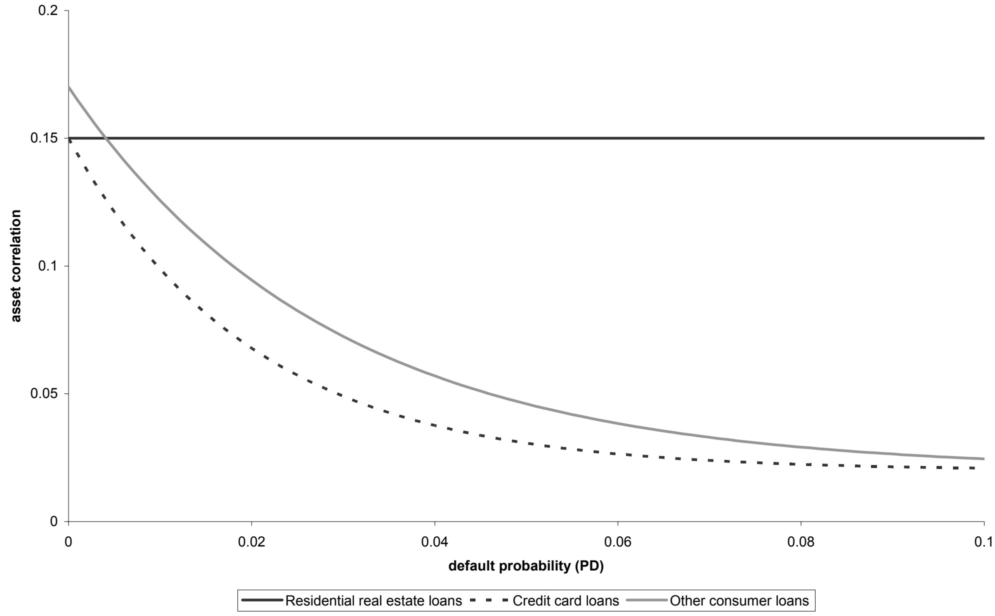

Exhibit 2: Real, fitted and forecasted default rates Residential Real Estate Loans, 1991-2002

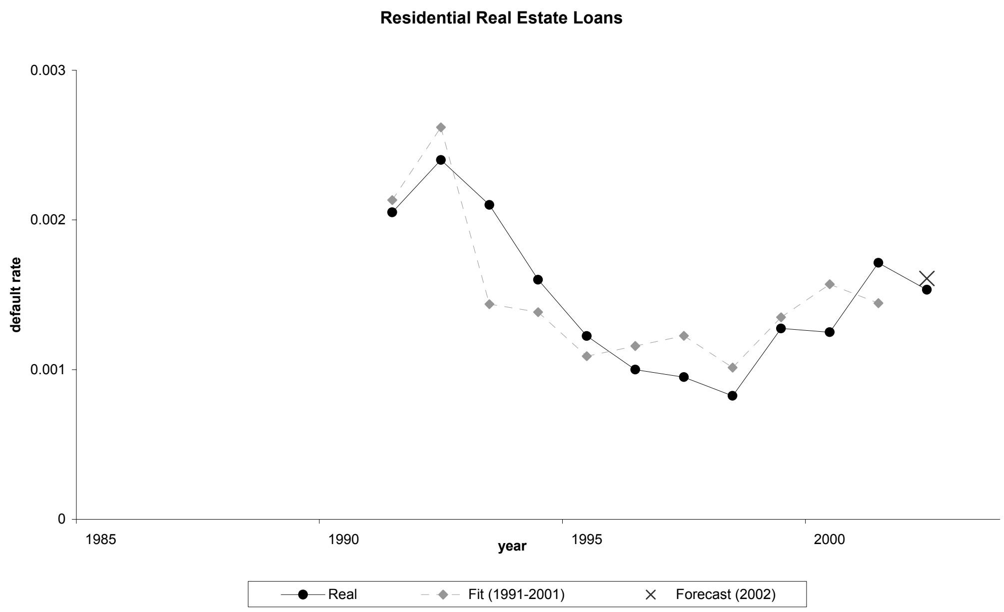

Exhibit 3: Real, fitted and forecasted default rates Credit Card Loans, 1985-2002

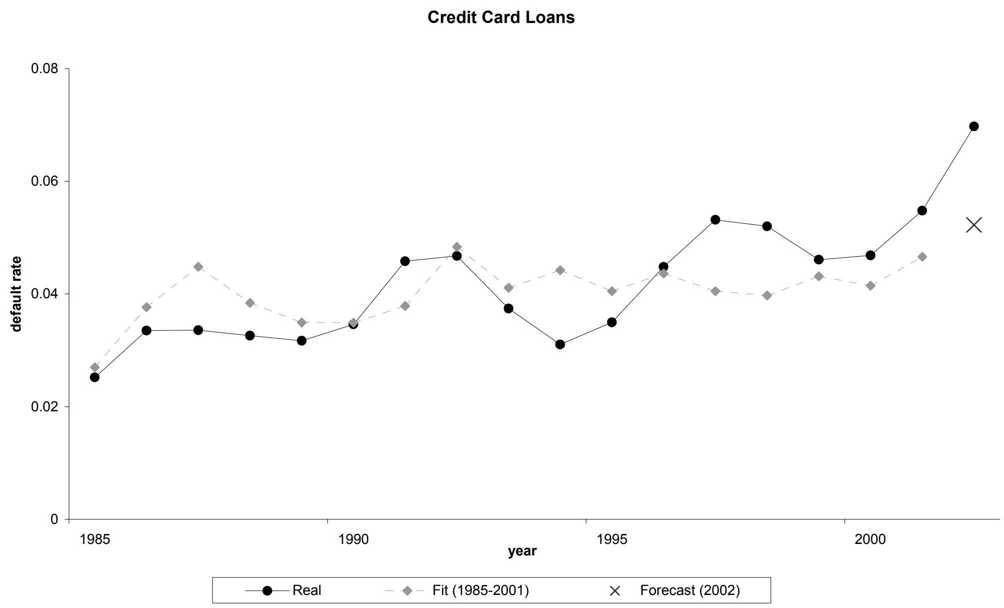

Exhibit 4: Real, fitted and forecasted default rates Other Consumer Loans, 1985-2002

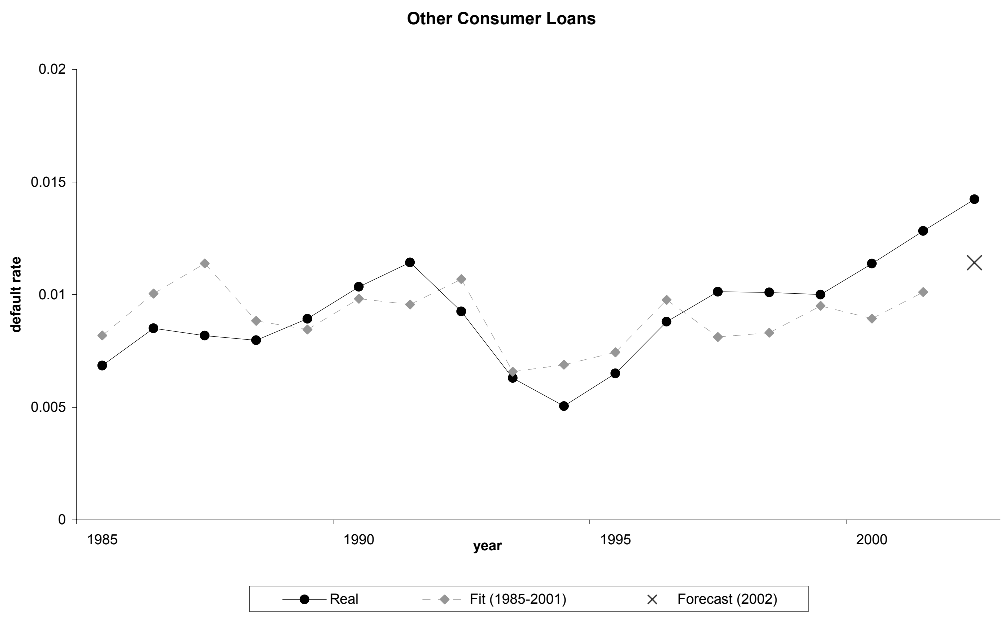

Exhibit 5: Forecasted distributions of potential losses for year 2002; Residential Real Estate Loans

N=100,000 borrowers each; losses are given as a percentage of portfolio value; Basel II correlations and estimates for correlations from model (3) are used*

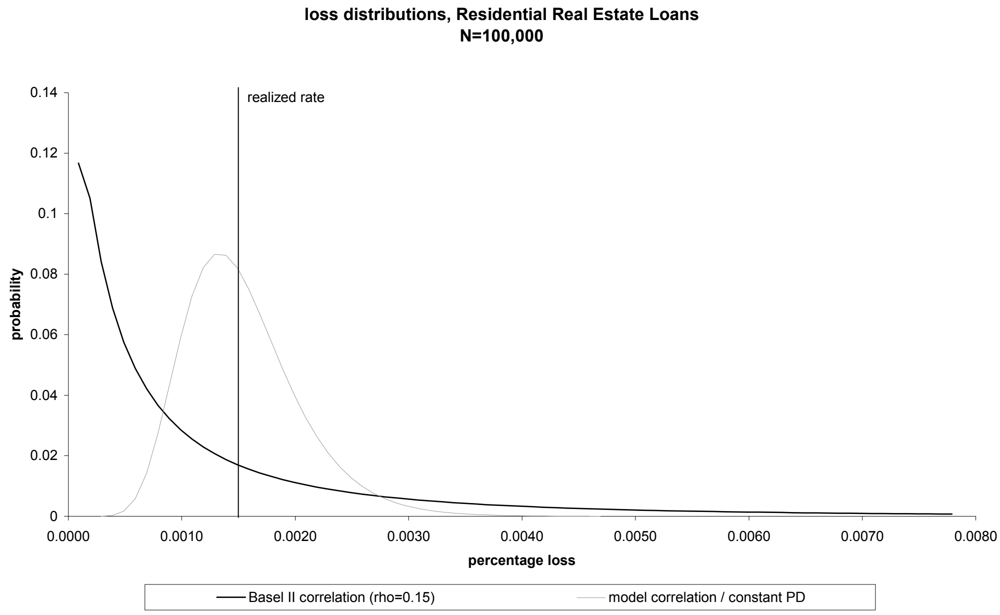

Exhibit 6: Forecasted distributions of potential losses for year 2002; Credit Card Loans

N=100,000 borrowers each; losses are given as a percentage of portfolio value; Basel II correlations and estimates for correlations from model (3) are used*

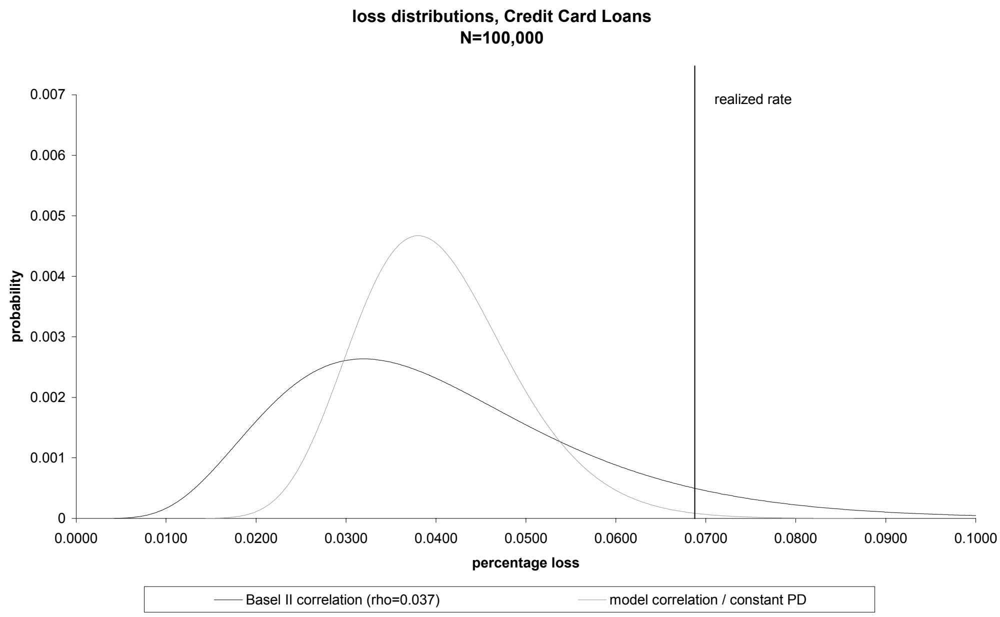

Exhibit 7: Forecasted distributions of potential losses for year 2002; Other Consumer Loans

N=100,000 borrowers each; losses are given as a percentage of portfolio value; Basel II correlations and estimates for correlations from model (3) are used*

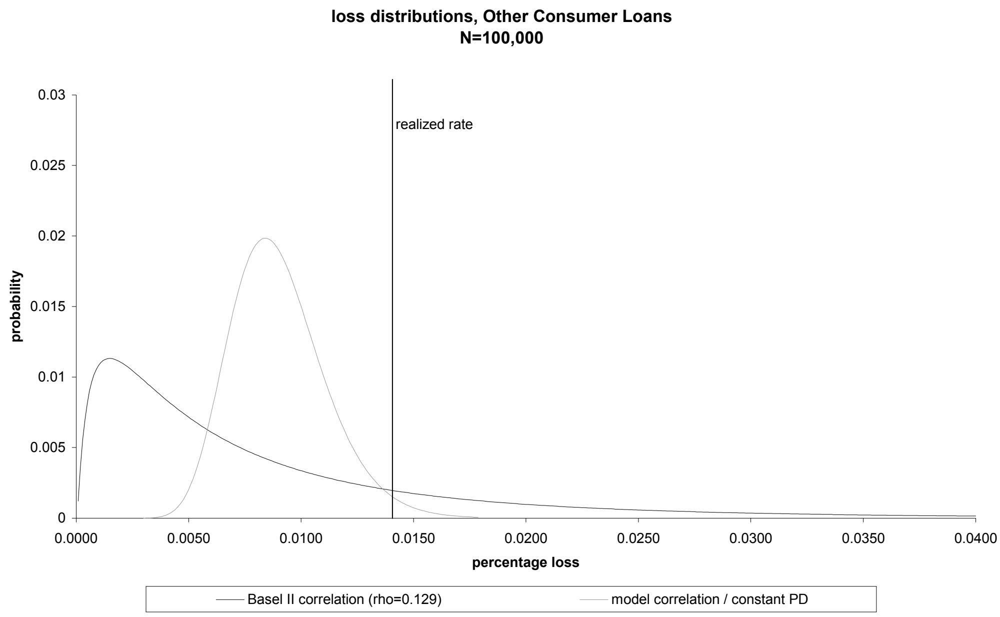

Exhibit 8: Forecasted distributions of potential losses for year 2002; Residential Real Estate Loans

N=100,000 borrowers each; losses are given as a percentage of portfolio value; estimates for PD and correlations from model (3) and model (* 6) are used*

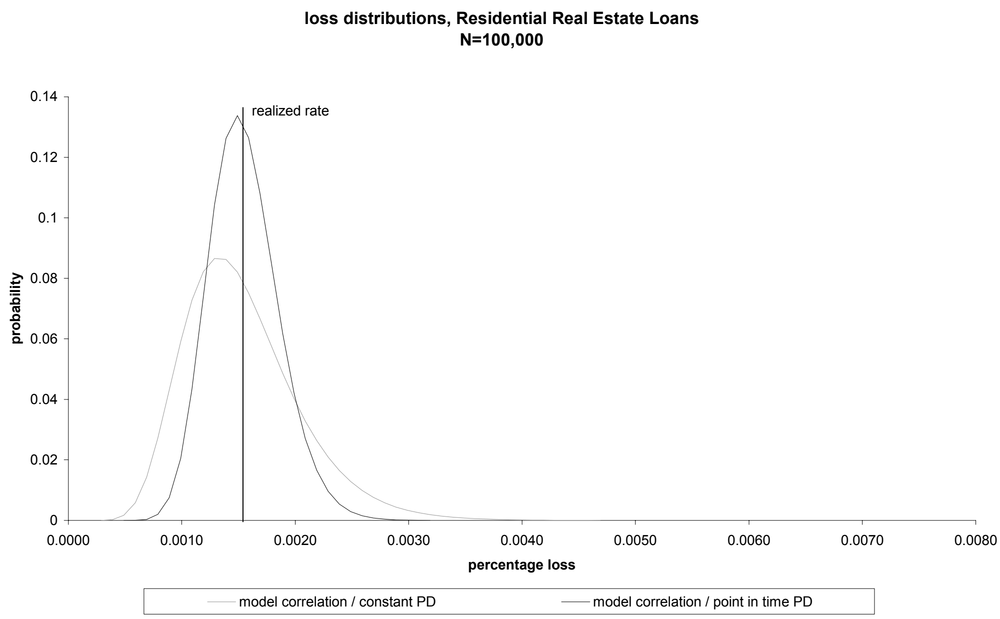

Exhibit 9: Forecasted distributions of potential losses for year 2002; Credit Card Loans

N=100,000 borrowers each; losses are given as a percentage of portfolio value; estimates for PD and correlations from model (3) and model (* 6) are used*

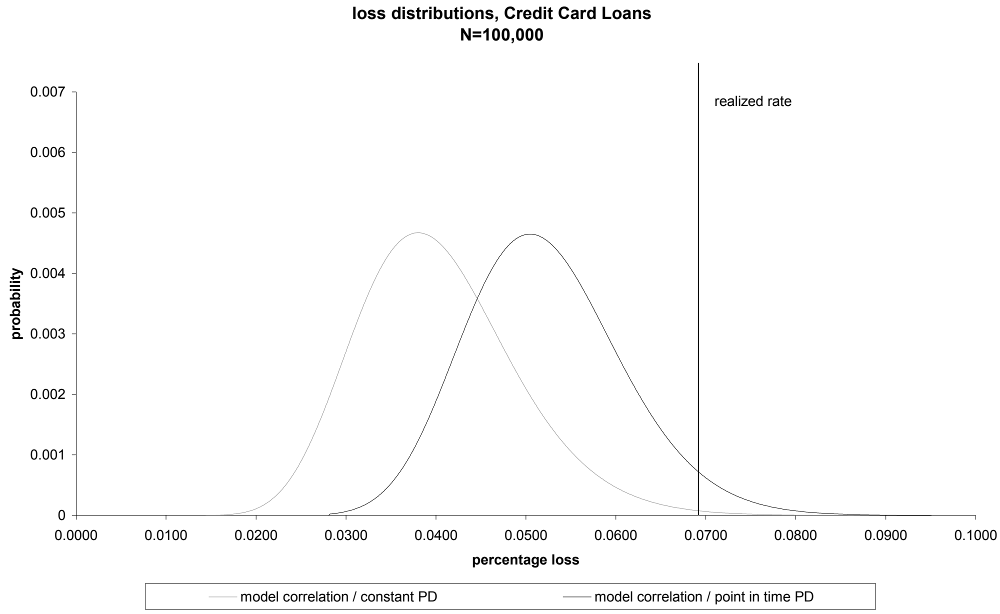

Exhibit 10: Forecasted distributions of potential losses for year 2002; Other Consumer Loans

N=100,000 borrowers each; losses are given as a percentage of portfolio value; estimates for PD and correlations from model (3) and model (* 6) are used*

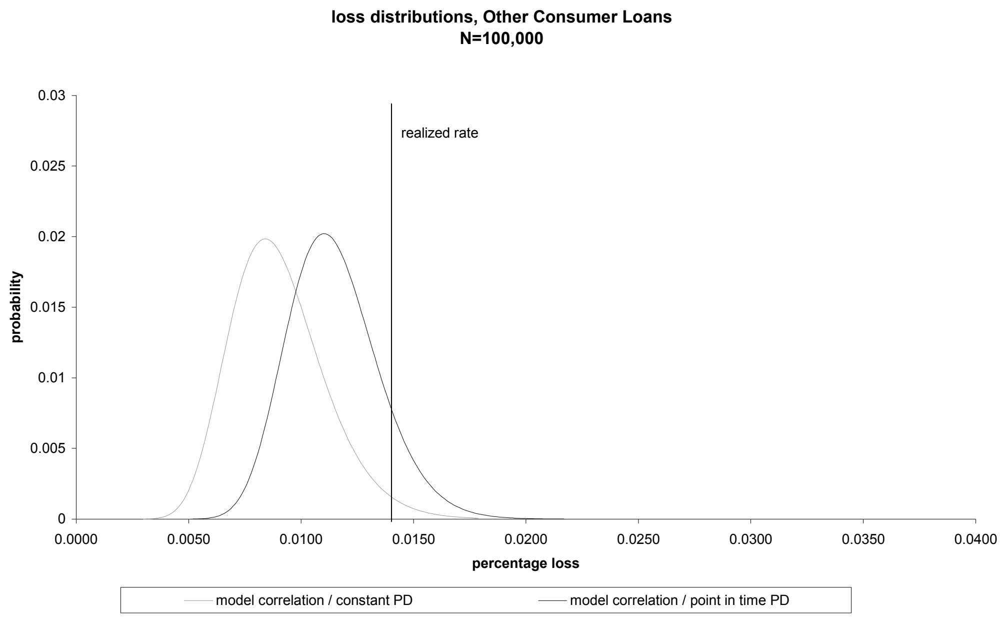

Exhibit 11: Aggregated forecasted loss distributions for year 2002

N=100,000 borrowers within each exposure class; losses are given as a percentage of portfolio value; Basel II correlations and estimates for correlations from model (3) and (* 12) are used*

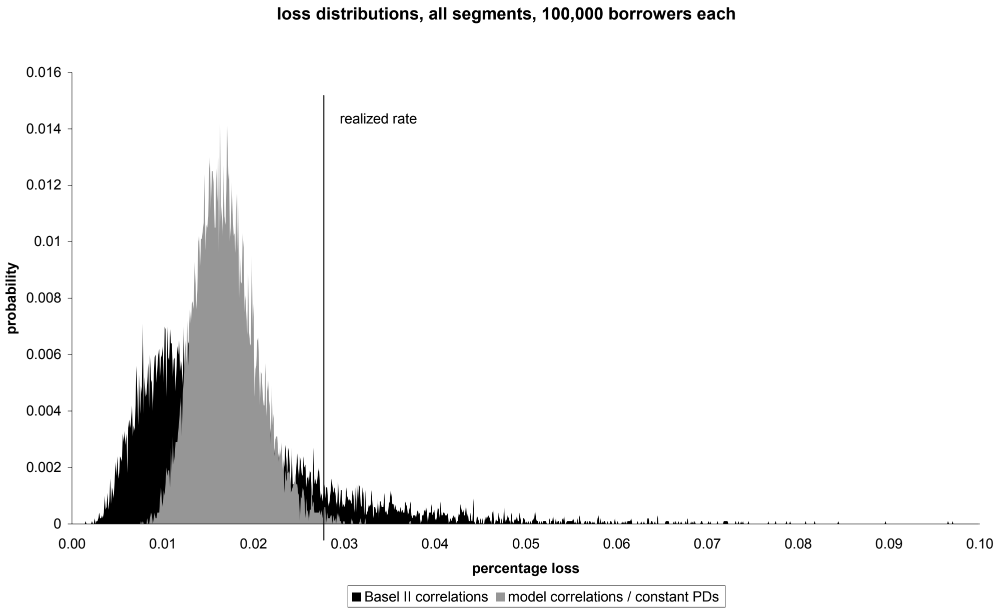

Exhibit 12: Aggregated forecasted loss distributions for year 2002

N=100,000 borrowers within each exposure class; losses are given as a percentage of portfolio value; estimates for PD and correlations from model (3) and model (* 6) and (* 12) are used*